HW-4 solutions:

6.14

\[ Z_0 = 50 \, \Omega \]
\[ Z_L = 50 \, \Omega \]
\[ l = \frac{1}{8} \lambda \]

\[ Z_L = 50 - j25 \, \Omega \]

- Calculate \( \Gamma_L \)
- Calculate SWR
- Calculate \( Z_{in} \) at the sending end.

(a) \( \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - j25 + 50}{100 - j25} = \frac{-j25}{100 - j25} \)

\[ = 0.2425 \angle -104^\circ \]

(b) \( \text{SWR} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = 1.64 \)

(c) \[ Z_{in} = Z_0 \left[ \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right] \]

\[ \beta l = \frac{2\pi}{\frac{1}{8}} \lambda = \frac{2\pi}{\frac{1}{4}} = \frac{\pi}{8} = 45^\circ \Rightarrow \tan 45^\circ = 1 \]
It should say frequency \( f = 600 \, \text{MHz} \) in the line above.
\[ Z_{in} = Z_0 \left[ \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right] \]

\[ \beta l = \frac{\omega}{0.8} \times 30 \times 10^{-2} = \frac{2\pi \times 600 \times 10^6 \times 0.3}{0.8 \times 3 \times 10^8} \]

\[ \beta l = 1.5 \pi = 270^\circ \]

\[ \tan 270^\circ \rightarrow \infty \]

\[ Z_{in} \approx Z_0 \left[ \frac{Z_L}{Z_0} \right] = Z_L \]

When \( (\tan \beta l) \rightarrow \infty \), \( Z_{in} \rightarrow Z_L \)

Under this condition, voltage at the receiving end and sending end would be the same.

\[ V_{in} = V_L = V_g \frac{Z_{in}}{Z_L} = \frac{100 + j125}{75 + 100 + j125} \]

\[ = 4.466 \angle 15.8^\circ \text{ volts} \]