**Introduction**

In this course, the solid body will be treated as a “material continuum.”

The concept of continuum originates in number theory. For example, if you consider two real numbers, a third can be imaged to exist between the two. By repeating this process, one can realize the existence of a ‘continuum’ of numbers. Similar arguments can be used to imagine space and time continua. The quantities such as mass, density, etc. are similar to the definition of material continuum.

\[
\text{Density at } Q \Rightarrow \rho = \lim_{n \to \infty} \left( \frac{m_n}{V_n} \right)
\]

**Other assumptions** made regarding the solid in this course are:

- Material is isotropic (properties are independent of the coordinate system)
- Material is homogeneous (properties do not vary spatially)
- Material is elastic (material points return to their original state when loads are removed)
Definition of ‘stress’ at a point

Consider a solid subjected to forces $F_1$, $F_2$, $F_3$ and supported, as shown. Let the solid be in equilibrium.

Let Q be a generic point in the solid.

To define ‘stress’ at Q, consider a plane that divides the solid into two.

Consider the lower-half (marked ‘2’) of the solid with Q located on the exposed surface. (The lower half of the solid is not in equilibrium anymore.)

Let the surface (plane) be represented by its unit normal vector $n$ as shown ($n$, $t$, $s$ form a second rectangular coordinate system at Q)
There are infinite number of planes that can be passed through Q and so there are infinite ways of defining stress at Q!! Totality (set) of all these is called the "STATE OF STRESS" at Q.

To restore equilibrium, consider internal reactions (restoring forces) on the exposed plane plane containing Q.

Let \( \mathbf{R} \) denote the resultant of all the restoring forces acting over a differential area \( \Delta A \) surrounding Q, as shown.

Then, stress at Q on a plane defined by \( \mathbf{n} \)

\[
\sigma = \lim_{\Delta A \to 0} \left( \frac{\mathbf{R}}{\Delta A} \right)
\]

Note that \( \mathbf{n} \) and \( \sigma \) are not acting in the same direction. And, definition of stress at Q is intricately linked with the plane on which Q resides.

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Stress Components on an arbitrary plane

Consider a Cartesian coordinate systems \(x, y, z\) with \(Q\) as the origin and let \(\mathbf{e}_i\) denote unit vectors in the \(i\)th direction.

Let \(ABC\) denote an arbitrary plane whose normal is \(\mathbf{n}\) and passing through \(Q\) but at a differential distance \(\delta h\) via \(Q'\), as shown.

Now, stress \(\mathbf{n}\) on \(ABC\) can be decomposed into Cartesian components as:

\[
\sigma_x^n, \sigma_y^n, \sigma_z^n
\]

Alternatively, \(\sigma_{nx}, \sigma_{ny}, \sigma_{nz}\) are denoted as \(\sigma_{nx}, \sigma_{ny}, \sigma_{nz}\) by moving the superscript to the first subscript position.

If the coordinate system \(n, s, t\) are replaced by, say, \(x', y', z'\) such that \(\mathbf{σ} \equiv \mathbf{σ'}\), then Cartesian stress components will be \(\sigma_{x'x}, \sigma_{x'y}, \sigma_{x'z}\).
Stress Components on Cartesian planes

Similar arguments can be used to denote stress components on the x-, y- and z-planes, respectively, as shown.

That is, stress $\sigma$ on QBC can be decomposed into components as:

\[
\sigma_x \equiv \sigma_{xx}, \quad \sigma_y \equiv \sigma_{xy}, \quad \sigma_z \equiv \sigma_{xz}
\]
Cartesian stress components in 3D as a stress matrix

\[
\sigma^{3D} = \begin{pmatrix}
\sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma_{zz}
\end{pmatrix}_Q
\]