4-5. The assembly consists of a steel rod \( CB \) and an aluminum rod \( BA \), each having a diameter of 12 mm. If the rod is subjected to the axial loadings at \( A \) and at the coupling \( B \), determine the displacement of the coupling \( B \) and the end \( A \). The unstretched length of each segment is shown in the figure. Neglect the size of the connections at \( B \) and \( C \), and assume that they are rigid. \( E_{st} = 200 \text{ GPa}, E_{al} = 70 \text{ GPa}. \)

\[
\delta_B = \frac{PL}{AE} = \frac{12 \times 10^3(3)}{\frac{4}{3}(0.012)^2(200)(10^9)} = 0.00159 \text{ m} = 1.59 \text{ mm}
\]

\[
\delta_A = \sum \frac{PL}{AE} = \frac{12 \times 10^3(3)}{\frac{4}{3}(0.012)^2(200)(10^9)} + \frac{18 \times 10^3(2)}{\frac{4}{3}(0.012)^2(70)(10^9)}
\]

\[= 0.00614 \text{ m} = 6.14 \text{ mm} \]

Ans: \( \delta_B = 1.59 \text{ mm}, \delta_A = 6.14 \text{ mm} \)
4–10. The assembly consists of two 10-mm diameter red brass C83400 copper rods AB and CD, a 15-mm diameter 304 stainless steel rod EF, and a rigid bar G. If the horizontal displacement of end F of rod EF is 0.45 mm, determine the magnitude of P.

**Internal Loading:** The normal forces developed in rods EF, AB, and CD are shown on the free-body diagrams in Figs. a and b.

**Displacement:** The cross-sectional areas of rods EF and AB are

\[ A_{EF} = \frac{\pi}{4} (0.015^2) = 56.25(10^{-6})\pi \text{ m}^2 \]

\[ A_{AB} = \frac{\pi}{4} (0.012^2) = 25(10^{-6})\pi \text{ m}^2. \]

\[
\delta_F = \sum \frac{P L}{A E} = \frac{P_{EF} L_{EF}}{A_{EF} E_{st}} + \frac{P_{AB} L_{AB}}{A_{AB} E_{br}}
\]

\[
0.45 = \frac{4P(450)}{56.25(10^{-6})\pi(193)(10^6)} + \frac{P(300)}{25(10^{-6})\pi(101)(10^6)}
\]

\[
P = 4967 \text{ N} = 4.97 \text{ kN}
\]

**Ans:**

\[
P = 49.7 \text{ kN}
\]
4-13. The rigid bar is supported by the pin-connected rod CB that has a cross-sectional area of 14 mm\(^2\) and is made from 6061-T6 aluminum. Determine the vertical deflection of the bar at D when the distributed load is applied.

\[
\zeta + \Sigma M_A = 0; \quad 1200(2) - T_{CB}(0.6)(2) = 0
\]

\[
T_{CB} = 2000 \text{ N}
\]

\[
\delta_{B/C} = \frac{PL}{AE} = \frac{(2000)(2.5)}{14 \times 10^{-6} \times 68.9 \times 10^5} = 0.0051835
\]

\[
(2.5051835)^2 = (1.5)^2 + (2)^2 - 2(1.5)(2) \cos \theta
\]

\[
\theta = 90.248^\circ
\]

\[
\theta = 90.248^\circ - 90^\circ = 0.2478^\circ = 0.004324 \text{ rad}
\]

\[
\delta_D = \theta r = 0.004324(4000) = 17.3 \text{ mm}
\]

Ans.

\[
\delta_D = 17.3 \text{ mm}
\]
4-17. The hanger consists of three 2014-T6 aluminum alloy rods, rigid beams $AC$ and $BD$, and a spring. If the vertical displacement of end $F$ is 5 mm, determine the magnitude of the load $P$. Rods $AB$ and $CD$ each have a diameter of 10 mm, and rod $EF$ has a diameter of 15 mm. The spring has a stiffness of $k = 100 \text{ MN/m}$ and is unstretched when $P = 0$.

**Internal Loading:** The normal forces developed in rods $EF$, $AB$, and $CD$ and the spring are shown in their respective free-body diagrams shown in Figs. a, b, and c.

**Displacements:** The cross-sectional areas of the rods are

$$A_{EF} = \frac{\pi}{4}(0.015^2) = 56.25 \times 10^{-6} \pi \text{ m}^2$$

and

$$A_{AB} = A_{CD} = \frac{\pi}{4}(0.01^2) = 25 \times 10^{-6} \pi \text{ m}^2.$$

$$\delta_{F/E} = \frac{F_{EF} L_{EF}}{A_{EF} E_d} = \frac{P(450)}{56.25 \times 10^{-6} \pi (73.1)(10^6)} = 34.836 \times 10^{-6} P$$

$$\delta_{B/A} = \frac{F_{AB} L_{AB}}{A_{AB} E_d} = \frac{(P/2)(450)}{25 \times 10^{-6} \pi (73.1)(10^6)} = 39.190 \times 10^{-6} P$$

The positive signs indicate that ends $F$ and $B$ move away from $E$ and $A$, respectively. Applying the spring formula with

$$k = \left[ \frac{100(10^3)}{1 \text{ kN/m}} \right] \left[ \frac{1000 \text{ N}}{1 \text{ kN}} \right] \left[ \frac{1 \text{ m}}{1000 \text{ mm}} \right] = 100(10^3) \text{ N/mm}.$$

$$\delta_{E/B} = \frac{F_{sp}}{k} \left( \frac{P}{100(10^3)} = -10 \times 10^{-6} P = 10 \times 10^{-6} P$$

The negative sign indicates that $E$ moves towards $B$. Thus, the vertical displacement of $F$ is

$$\delta_{F/A} = \delta_{B/A} + \delta_{F/E} + \delta_{E/B}$$

$$5 = 34.836 \times 10^{-6} P + 39.190 \times 10^{-6} P + 10 \times 10^{-6} P$$

$$P = 59 \ 505.71 \text{ N} = 59.5 \text{ kN} \quad \text{Ans.}$$
4–19. Collar $A$ can slide freely along the smooth vertical guide. If the vertical displacement of the collar is 0.035 in. and the supporting 0.75 in. diameter rod $AB$ is made of 304 stainless steel, determine the magnitude of $P$.

**Internal Loading:** The normal force developed in rod $AB$ can be determined by considering the equilibrium of collar $A$ with reference to its free-body diagram, Fig. $a$.

$$\sum F_y = 0; \quad -F_{AB}(\frac{4}{5}) - P = 0 \quad F_{AB} = -1.25 P$$

**Displacements:** The cross-sectional area of rod $AB$ is

$$A_{AB} = \frac{\pi}{4}(0.75^2) = 0.4418 \text{in}^2; \text{and the initial length of rod } AB \text{ is}$$

$$L_{AB} = \sqrt{2^2 + 1.5^2} = 2.5 \text{ft. The axial deformation of rod } AB \text{ is}$$

$$\delta_{AB} = \frac{F_{AB}L_{AB}}{A_{AB}E_{st}} = \frac{-1.25P(2.5)(12)}{0.4418(28.0)(10^3)} = -0.003032P$$

The negative sign indicates that end $A$ moves towards $B$. From the geometry shown in Fig. $b$, we obtain $\theta = \tan^{-1}(\frac{1.5}{2}) = 36.87^\circ$. Thus,

$$\delta_{AB} = (\delta_A)_V \cos \theta$$

$0.003032P = 0.035 \cos 36.87^\circ$

$$P = 9.24 \text{ kip} \quad \text{Ans.}$$
4-33. The steel pipe is filled with concrete and subjected to a compressive force of 80 kN. Determine the average normal stress in the concrete and the steel due to this loading. The pipe has an outer diameter of 80 mm and an inner diameter of 70 mm. \( E_{st} = 200 \text{ GPa}, \ E_c = 24 \text{ GPa} \).

\[
+ \sum F_y = 0; \quad P_{st} + P_{con} - 80 = 0
\]

\[
\delta_{st} = \delta_{con}
\]

\[
\frac{P_{st} L}{\pi (0.08^2 - 0.07^2) (200) (10^3)} = \frac{P_{con} L}{\pi (0.07^2) (24) (10^3)}
\]

\[
P_{st} = 2.5510 P_{con}
\]

Solving Eqs. (1) and (2) yields

\[
P_{st} = 57.47 \text{ kN} \quad P_{con} = 22.53 \text{ kN}
\]

\[
\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{57.47 (10^3)}{\pi (0.08^2 - 0.07^2)} = 48.8 \text{ MPa}
\]

\[
\sigma_{con} = \frac{P_{con}}{A_{con}} = \frac{22.53 (10^3)}{\pi (0.07^2)} = 5.85 \text{ MPa}
\]

**Ans:**

\( \sigma_{st} = 48.8 \text{ MPa}, \ \sigma_{con} = 5.85 \text{ MPa} \)
4–39. The load of 2800 lb is to be supported by the two essentially vertical A-36 steel wires. If originally wire \( AB \) is 60 in. long and wire \( AC \) is 40 in. long, determine the cross-sectional area of \( AB \) if the load is to be shared equally between both wires. Wire \( AC \) has a cross-sectional area of 0.02 in\(^2\).

\[
T_{AC} = T_{AB} = \frac{2800}{2} = 1400 \text{ lb}
\]

\[
\delta_{AC} = \delta_{AB}
\]

\[
\frac{1400(40)}{(0.02)(29)(10^6)} = \frac{1400(60)}{A_{AB}(29)(10^6)}
\]

\[
A_{AB} = 0.03 \text{ in}^2
\]

Ans:

\[
A_{AB} = 0.03 \text{ in}^2
\]
4–45. The bolt has a diameter of 20 mm and passes through a tube that has an inner diameter of 50 mm and an outer diameter of 60 mm. If the bolt and tube are made of A-36 steel, determine the normal stress in the tube and bolt when a force of 40 kN is applied to the bolt. Assume the end caps are rigid.

Referring to the **FBD** of left portion of the cut assembly, Fig. a

\[ \sum F_x = 0; \quad 40(10^3) - F_b - F_t = 0 \quad (1) \]

Here, it is required that the bolt and the tube have the same deformation. Thus

\[ \delta_t = \delta_b \]

\[ \frac{F_t(150)}{\frac{1}{2}(0.06^2 - 0.05^2)} \frac{1}{200(10^9)} = \frac{F_b(160)}{\frac{1}{2}(0.02^2)} \frac{1}{200(10^9)} \]

\[ F_t = 2.9333 F_b \quad (2) \]

Solving Eqs (1) and (2) yields

\[ F_b = 10.17 (10^3) \text{ N} \quad F_t = 29.83 (10^3) \text{ N} \]

Thus,

\[ \sigma_b = \frac{F_b}{A_b} = \frac{10.17(10^3)}{\frac{1}{2}(0.02^2)} = 32.4 \text{ MPa} \quad \text{Ans.} \]

\[ \sigma_t = \frac{F_t}{A_t} = \frac{29.83 (10^3)}{\frac{1}{2}(0.06^2 - 0.05^2)} = 34.5 \text{ MPa} \quad \text{Ans.} \]

Ans:

\[ \sigma_b = 32.4 \text{ MPa}, \sigma_t = 34.5 \text{ MPa} \]
4–47. The support consists of a solid red brass C83400 copper post surrounded by a 304 stainless steel tube. Before the load is applied the gap between these two parts is 1 mm. Given the dimensions shown, determine the greatest axial load that can be applied to the rigid cap \( A \) without causing yielding of any one of the materials.

Require,

\[
\delta_{st} = \delta_{br} + 0.001
\]

\[
\frac{F_{st}(0.25)}{\pi[(0.05)^2 - (0.04)^2]} = \frac{F_{br}(0.25)}{\pi(0.03)^2(101)(10^9)} + 0.001
\]

\[
0.45813 F_{st} = 0.87544 F_{br} + 10^6
\]

\[
+ \sum F_y = 0; \quad F_{st} + F_{br} - P = 0
\]

(1)

(2)

Assume brass yields, then

\[
(F_{br})_{max} = \sigma_y A_{br} = 70(10^6)(\pi)(0.03)^2 = 197920.3 \text{ N}
\]

\[
(\epsilon_y)_{br} = \frac{\sigma_y}{E} = \frac{70.0(10^6)}{101(10^9)} = 0.6931(10^{-3}) \text{ mm/mm}
\]

\[
\delta_{br} = (\epsilon_y)_{br} L = 0.6931(10^{-3})(0.25) = 0.1733 \text{ mm} < 1 \text{ mm}
\]

Thus only the brass is loaded.

\[
P = F_{br} = 198 \text{ kN}
\]

\text{Ans.}

\[
\text{Ans:} \quad P = 198 \text{ kN}
\]
4-51. The rigid bar supports the uniform distributed load of 6 kip/ft. Determine the force in each cable if each cable has a cross-sectional area of 0.05 in², and \( E = 31(10^3) \) ksi.

\[ \zeta + \Sigma M_A = 0; \quad T_{CB} \left( \frac{2}{\sqrt{5}} \right) - 54(4.5) + T_{CD} \left( \frac{2}{\sqrt{5}} \right) = 0 \]

\[ \theta = \tan^{-1} \frac{6}{6} = 45^\circ \]

\[ L_{BC}^2 = (3)^2 + (8.4853)^2 - 2(3)(8.4853) \cos \theta' \]

Also,

\[ L_{DC}^2 = (9)^2 + (8.4853)^2 - 2(9)(8.4853) \cos \theta' \]

Thus, eliminating \( \cos \theta' \).

\[-L_{BC}^2(0.019642) + 1.5910 = -L_{BC}^2(0.0065473) + 1.001735 \]

\[ L_{BC}^2 = 0.333 L_{DC}^2 + 30 \]

But,

\[ L_{BC} = \sqrt{45 + \delta_{BC}}, \quad L_{DC} = \sqrt{45 + \delta_{DC}} \]

Neglect squares or \( \delta_B \) since small strain occurs.

\[ L_{DC}^2 = (\sqrt{45 + \delta_{BC}})^2 = 45 + 2\sqrt{45} \delta_{BC} \]

\[ L_{DC}^2 = (\sqrt{45 + \delta_{DC}})^2 = 45 + 2\sqrt{45} \delta_{DC} \]

\[ 45 + 2\sqrt{45} \delta_{BC} = 0.333(45 + 2\sqrt{45} \delta_{DC}) + 30 \]

\[ 2\sqrt{45} \delta_{BC} = 0.333(2\sqrt{45} \delta_{DC}) \]

\[ \delta_{DC} = 3\delta_{BC} \]

Thus,

\[ T_{CD} \frac{\sqrt{45}}{AE} = 3 \frac{T_{CB} \sqrt{45}}{AE} \]

\[ T_{CD} = 3 T_{CB} \]

From Eq. (1).

\[ T_{CD} = 27.1682 \text{ kip} = 27.2 \text{ kip} \quad \text{Ans.} \]

\[ T_{CB} = 9.06 \text{ kip} \quad \text{Ans.} \]

\[ T_{CD} = 27.2 \text{ kip, } T_{CB} = 9.06 \text{ kip} \quad \text{Ans.} \]
4-57. The rigid bar is originally horizontal and is supported by two A-36 steel cables each having a cross-sectional area of 0.04 in². Determine the rotation of the bar when the 800-lb load is applied.

Referring to the FBD of the rigid bar Fig. a,

\[ \zeta + \sum M_A = 0; \quad F_{BC} \left( \frac{12}{13} \right) (5) + F_{CD} \left( \frac{3}{5} \right) (16) - 800(10) = 0 \]  

(1)

The unstretched lengths of wires BC and CD are \( L_{BC} = \sqrt{12^2 + 5^2} = 13 \) ft and \( L_{CD} = \sqrt{12^2 + 16^2} = 20 \) ft. The stretch of wires BC and CD are

\[ \delta_{BC} = \frac{F_{BC} L_{BC}}{A E} = \frac{F_{BC} (13)}{A E} \quad \delta_{CD} = \frac{F_{CD} L_{CD}}{A E} = \frac{F_{CD} (20)}{A E} \]

Referring to the geometry shown in Fig. b, the vertical displacement of a point on the rigid bar is \( \delta_v = \frac{\delta}{\cos \theta} \). For points B and D, \( \cos \theta_B = \frac{12}{13} \) and \( \cos \theta_D = \frac{3}{5} \). Thus, the vertical displacements of points B and D are

\[ (\delta_B)_v = \frac{\delta_{BC}}{\cos \theta_B} = \frac{F_{BC}}{12/13} = \frac{169 F_{BC}}{12 A E} \]

\[ (\delta_D)_v = \frac{\delta_{CD}}{\cos \theta_D} = \frac{F_{CD}}{3/5} = \frac{100 F_{CD}}{3 A E} \]

The similar triangles shown in Fig. c gives

\[ \frac{(\delta_B)_v}{5} = \frac{(\delta_D)_v}{16} \]

\[ \frac{1}{5} \left( \frac{169 F_{BC}}{12 A E} \right) = \frac{1}{16} \left( \frac{100 F_{CD}}{3 A E} \right) \]

\[ F_{BC} = \frac{125}{169} F_{CD} \]

Solving Eqs (1) and (2), yields

\[ F_{CD} = 614.73 \text{ lb} \quad F_{BC} = 454.69 \text{ lb} \]

Thus,

\[ (\delta_D)_v = \frac{100(614.73)}{3(0.04) \left[ 29.0 \left( 10^6 \right) \right]} = 0.01766 \text{ ft} \]

Then

\[ \theta = \left( \frac{0.01766}{16 \text{ ft}} \right) \left( \frac{180^\circ}{\pi} \right) = 0.0633^\circ \]

Ans. \( \theta = 0.0633^\circ \)
The assembly has the diameters and material makeup indicated. If it fits securely between its fixed supports when the temperature is $T_1 = 70°F$, determine the average normal stress in each material when the temperature reaches $T_2 = 110°F$.

\[ \Sigma F_x = 0; \quad F_A = F_B = F \]

\[ \delta_{A/D} = 0: \quad -\frac{F(4)(12)}{\pi(6)^2(10.6)(10^6)} = 12.8(10^{-6})(110 - 70)(4)(12) \]
\[ -\frac{F(6)(12)}{\pi(4)^2(15)(10^6)} = 9.60(10^{-6})(110 - 70)(6)(12) \]
\[ -\frac{F(3)(12)}{\pi(2)^2(28)(10^6)} = 9.60(10^{-6})(110 - 70)(3)(12) = 0 \]

\[ F = 277.69 \text{ kip} \]

\[ \sigma_{al} = \frac{277.69}{\pi(6)^2} = 2.46 \text{ ksi} \quad \text{Ans.} \]

\[ \sigma_{br} = \frac{277.69}{\pi(4)^2} = 5.52 \text{ ksi} \quad \text{Ans.} \]

\[ \sigma_{st} = \frac{277.69}{\pi(2)^2} = 22.1 \text{ ksi} \quad \text{Ans.} \]

\[ \sigma_{al} = 2.46 \text{ ksi}, \sigma_{br} = 5.52 \text{ ksi}, \sigma_{st} = 22.1 \text{ ksi} \]
4–78. When the temperature is at 30°C, the A-36 steel pipe fits snugly between the two fuel tanks. When fuel flows through the pipe, the temperatures at ends A and B rise to 130°C and 80°C, respectively. If the temperature drop along the pipe is linear, determine the average normal stress developed in the pipe. Assume each tank provides a rigid support at A and B.

**Temperature Gradient:** Since the temperature varies linearly along the pipe, Fig. a, the temperature gradient can be expressed as a function of \( x \) as

\[
T(x) = 80 + \frac{50}{6}(6 - x) = \left(130 - \frac{50}{6}x\right)^\circ C
\]

Thus, the change in temperature as a function of \( x \) is

\[
\Delta T = T(x) - 30 = \left(130 - \frac{50}{6}x\right) - 30 = \left(100 - \frac{50}{6}x\right)^\circ C
\]

**Compatibility Equation:** If the pipe is unconstrained, it will have a free expansion of

\[
\delta_T = \alpha \int \Delta T \, dx = 12(10^{-6}) \int_0^{6m} \left(100 - \frac{50}{6}x\right) \, dx = 0.0054 \, m = 5.40 \, mm
\]

Using the method of superposition, Fig. b,

\[
(\uparrow) \quad 0 = \delta_T - \delta_F
\]

\[
0 = 5.40 - \frac{F(6000)}{\pi(0.16^2 - 0.15^2)(200)(10^5)}
\]

\[
F = 1 \, 753 \, 008 \, N
\]

**Normal Stress:**

\[
\sigma = \frac{F}{A} = \frac{1 \, 753 \, 008}{\pi(0.16^2 - 0.15^2)} = 180 \, MPa
\]

**Ans:** \( \sigma = 180 \, MPa \)
4-81. The 50-mm-diameter cylinder is made from Am 1004-T61 magnesium and is placed in the clamp when the temperature is $T_1 = 20^\circ$ C. If the 304-stainless-steel carriage bolts of the clamp each have a diameter of 10 mm, and they hold the cylinder snug with negligible force against the rigid jaws, determine the force in the cylinder when the temperature rises to $T_2 = 130^\circ$ C.

\[ +\Sigma F_Y = 0; \quad F_{st} = F_{mg} = F \]
\[ \delta_{mg} = \delta_{st} \]
\[ \alpha_{mg} L_{mg} \Delta T - \frac{F_{mg} L_{mg}}{E_{mg} A_{mg}} = \alpha_{st} L_{st} \Delta T + \frac{F_{st} L_{st}}{E_{st} A_{st}} \]
\[
26(10^{-6})(0.1)(110) - \frac{F(0.1)}{44.7(10^9) \frac{\pi}{4} (0.05)^2} = 17(10^{-6})(0.150)(110) + \frac{F(0.150)}{193(10^9)(2) \frac{\pi}{4} (0.01)^2}
\]
\[ F = 904 \text{ N} \]

\textit{Ans.}
4–83. The wires $AB$ and $AC$ are made of steel, and wire $AD$ is made of copper. Before the 150-lb force is applied, $AB$ and $AC$ are each 60 in. long and $AD$ is 40 in. long. If the temperature is increased by 80°F, determine the force in each wire needed to support the load. Take $E_{st} = 29(10^3)$ ksi, $E_{cu} = 17(10^3)$ ksi, $a_{st} = 8(10^{-6})/°F$, $a_{cu} = 9.60(10^{-6})/°F$. Each wire has a cross-sectional area of 0.0123 in$^2$. 

Equations of Equilibrium:

\[ \begin{align*} \sum F_x &= 0; \quad F_{AC} \cos 45° - F_{AB} \cos 45° = 0 \\ F_{AC} &= F_{AB} = F \\ \sum F_y &= 0; \quad 2F \sin 45° + F_{AD} - 150 = 0 \end{align*} \] (1)

Compatibility:

\[ \begin{align*} (\delta_{AC})_T &= 8.0(10^{-6})(80)(60) = 0.03840 \text{ in.} \\ (\delta_{AC})_T &= \frac{(\delta_{AC})_T}{\cos 45°} = \frac{0.03840}{\cos 45°} = 0.05431 \text{ in.} \\ (\delta_{AD})_T &= 9.60(10^{-6})(80)(40) = 0.03072 \text{ in.} \\ \delta_0 &= (\delta_{AC})_T - (\delta_{AD})_T = 0.05431 - 0.03072 = 0.02359 \text{ in.} \\ (\delta_{AD})_F &= (\delta_{AC})_F + \delta_0 \\ \frac{F_{AD}(40)}{0.0123(17.0)(10^6)} &= \frac{F(60)}{0.0123(29.0)(10^6) \cos 45°} + 0.02359 \\ 0.1913F_{AD} - 0.2379F &= 23.5858 \end{align*} \] (2)

Solving Eq. (1) and (2) yields:

\[ F_{AC} = F_{AB} = F = 10.0 \text{ lb} \quad \text{Ans.} \]

\[ F_{AD} = 136 \text{ lb} \quad \text{Ans.} \]

\[ F_{AC} = F_{AB} = 10.0 \text{ lb}, F_{AD} = 136 \text{ lb} \]