Control of a Ground Vehicle Using Quadratic Programming Based Control Allocation

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Outline

- The Problem and Objectives
- Control Allocation
- Vehicle Model
- Controller Design
- Simulation Results
- Conclusions
Problem and Objectives

We want to track a yaw rate trajectory with a 4 wheeled ground vehicle while minimizing the sideslip angle.

Question: But if there are multiple inputs available, How do you choose which to use and how much?

Answer: Control Allocation!!
Control Allocation (CA)

- CA involves generating a set of effector commands that produce a desired control effect while minimizing the control effort.

- A CA approach is generally used for a redundant set of effectors.

- CA allows for reconfiguration in the event of an effector failure.
Vehicle Model (nonlinear)

Assumptions made:

- The vehicle runs at constant velocity

- No longitudinal weight transfer (pitch is neglected)

- A Pacejka tire model was used to represent nonlinear tire behavior.
## Vehicle Model (linear)

The linear vehicle model in state space form:

\[
\begin{bmatrix}
\dot{\beta} \\
\dot{r}
\end{bmatrix} = \begin{bmatrix}
\frac{-C_o}{mV} & \frac{-C_1}{mV^2} & -1 \\
\frac{-C_1}{I_z} & \frac{-C_2}{VI_z} & 0
\end{bmatrix} \begin{bmatrix}
\beta \\
r
\end{bmatrix} + \begin{bmatrix}
\frac{C_{\alpha f}}{mV} & 0 & 0 \\
\frac{C_{\alpha f}}{I_z} & \frac{T_f}{2I_z} & \frac{T_r}{2I_z}
\end{bmatrix} \begin{bmatrix}
\delta \\
\Delta F_{xf} \\
\Delta F_{xr}
\end{bmatrix}
\]

- \( \delta \) = steering angle
- \( \Delta F_{xf} \) = differential braking force on front axle
- \( \Delta F_{xr} \) = differential braking force on rear axle
Controller Design Approach

Controller has 2 main tasks:

1. Generate Control Effort  \rightarrow State Feedback
2. Generate Effector Commands  \rightarrow Control Allocation

\[ \text{ref} \rightarrow \text{State Feedback} \rightarrow \bar{u} \rightarrow \text{Control Allocation} \rightarrow u \rightarrow \text{Ground Vehicle} \]

"virtual" command
LQR gains are designed for the following modified system:

\[
\begin{bmatrix}
\dot{\beta} \\
\dot{r} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
-\frac{C_o}{mV} & -\frac{C_1}{mV^2} & -1 & 0 \\
-\frac{C_1}{I_z} & -\frac{C_2}{VI_z} & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\beta \\
r \\
\psi
\end{bmatrix}
+ 
\begin{bmatrix}
1 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\bar{u}_\beta \\
\bar{u}_r
\end{bmatrix}
\]

This model is used solely for generating the virtual command effort.

The gain matrix \( K \) is applied to the error vector to produce the overall desired effect.

\[
\begin{bmatrix}
\bar{u}_\beta \\
\bar{u}_r
\end{bmatrix} = -K_{2 \times 3}
\begin{bmatrix}
\beta_{error} \\
r_{error} \\
\psi_{error}
\end{bmatrix}
\]
Command Generation

Physical effector commands are generated by the Control Allocation routine.

**Quadratic Programming (QP)**

Given the virtual command $\bar{u}$, solve for the effector commands $u$

$$\min u: \quad \frac{1}{2} u^T Qu + c^T u$$

subject to:

$$Bu = \bar{u}$$

$$u^- \leq u \leq u^+$$
The Sign Preserving Quadratic Programming (SPQP) problem set up as follows:

\[
\begin{align*}
\text{min } u, \sigma : & & \frac{1}{2} u^T Q_u u + c^T u + \frac{1}{2} Q_\sigma (1 - \sigma_\beta)^2 + \frac{1}{2} Q_\sigma (1 - \sigma_r)^2 \\
\text{subject to: } & & Bu - \Sigma \overrightarrow{u} = 0
\end{align*}
\]

where

\[
\Sigma = \begin{bmatrix}
\sigma_\beta & 0 \\
0 & \sigma_r
\end{bmatrix}
\]

Ignore virtual command \( \mathbf{u}^- \) \( \leq \mathbf{u} \leq \mathbf{u}^+ \)

achieve virtual command

\[
\begin{bmatrix}
u^- \\
0 \\
0 \\
\sigma_\beta \\
\sigma_r
\end{bmatrix} \leq \begin{bmatrix}
u \\
\sigma_\beta \\
\sigma_r
\end{bmatrix} \leq \begin{bmatrix}
u^+ \\
1 \\
1
\end{bmatrix}
\]
Due to the structure of the input matrix $B$, it is necessary to add a 4th “pseudo” input $\nu$.

\[
B = \begin{bmatrix}
\frac{C_{af}}{mV} & 0 & 0 & 1 \\
\frac{C_{af}}{a} & \frac{T_f}{2I_z} & \frac{T_r}{2I_z} & 0
\end{bmatrix}
\]

\[
u = \begin{bmatrix}
\delta \\
\Delta F_{xf} \\
\Delta F_{xr} \\
\nu
\end{bmatrix}
\]

\[
Q = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & Q_{\nu}
\end{bmatrix}
\]

This relaxes the equality constraint placed on the sideslip angle.
Simulation

- The vehicle is simulated traveling at constant velocities of 45, 55, and 65 mph.
- Scenarios examined: nominal, brake failure, and steering failure.
- Penalty on pseudo effector: $Q_v = 10^4, 10^6$.
- The desired yaw rate trajectory simulates a double lane change maneuver.
- Effector limitations: steering angle = $\pm 0.5 \text{ rad}$
  braking = $0.75 \times Fzi$
Results (pseudo effector)
Nominal, 55mph, QP

\[ Q_v = 1e4 \]

- Differential braking force (N)
- Time (seconds)
- Front
- Rear

\[ Q_v = 1e6 \]

- Differential braking force (N)
- Time (seconds)
- Front
- Rear
Results – Nominal, 45mph SPQP

**Steering Input vs. Time**

- **Steering Angle (degrees)**
- **Time (seconds)**

**Differential braking force vs. Time**

- **Differential braking force (N)**
- **Time (seconds)**

*Legend:*
- Front
- Rear
Results – Nominal, 45mph SPQP

**Yaw Rate vs. Time**
- Actual (solid blue line)
- Desired (dashed black line)
- Error (dotted red line)

**Sideslip Angle vs. Time**
- Time (seconds)
Results – Steering failure, 55mph QP
Results – Steering failure, 55mph QP
Conclusions

- QP provides an intelligent reconfiguration of control effort in the event of a failure and also obeys effector limits.

- SPQP gives the best yaw rate tracking but standard QP provides the best sideslip minimization.

- The quadratic penalty placed on the pseudo effector significantly affects the results.
Current Work

- Gain insight for controller tuning through further investigation of the effects of the pseudo effector weight $Q_v$.
- Address differential braking saturation limits. Better estimate of limits using the friction circle.
- Study the effects of additional inputs such as rear steering and individual torque control of each wheel.
Questions

???
Answers – Commands w/ noisy state estimates
Answers – tracking w/ noise

**Yaw Rate vs. Time**

- **Actual**
- **Desired**
- **Error**

**Sideslip Angle vs. Time**

- **Sideslip Angle (degrees)**