Name:

Exam #2 ELEC 5760/6760 Mon 4/22/13

Constants: \( \pi = 3.14159 \), \( \varepsilon_0 = 8.854 \text{pF/m} \), \( 1 \text{G} = 9.8 \text{m/s}^2 \), \( 1 \text{atm} = 101.325 \text{kPa} \)

Equations: \( PE = mgh \), \( KE = \frac{1}{2} mv^2 \), \( P_s = \rho gh \), \( F_{ppa} = \frac{\varepsilon_0 \varepsilon_r AV^2}{2d^2} \)

\[
V_{rl} = \sqrt{\frac{8kd^3}{27A\varepsilon_0 \varepsilon_r}}, \quad A_{core} = (\pi)r^2, \quad d = \alpha \left( \frac{m}{k} \right), \quad aS, \quad y(t) = \frac{2m\omega_0}{c^2 \omega_0^2} \cos(\omega_0 t)
\]

\[
P_i = P_f + \frac{\rho v^2}{2}, \quad f_d = \frac{f_s}{\frac{v_{object}}{V_{wave}}}, \quad V_{rms} = \frac{V_{amplitude}}{\sqrt{2}}
\]

Laplace Transforms: \( \mathcal{L}[u(t)] = \frac{c}{s} \), \( \mathcal{L}[\cos at] = \frac{c}{s + a} \), \( \mathcal{L}[ce^{-at}] = \frac{c}{(s + a)^2} \)

Problems:

1) The gravitational acceleration on Mars is 3.8 \text{m/s}^2. If a spacecraft free falls to the planet’s hard surface from a height of 25m, where it completely stops moving 100ms after initial impact, estimate the acceleration due to the shock event in \text{m/s}^2. (10 points)

\[
mgh = \frac{1}{2}mv^2 \rightarrow V = \sqrt{2gh} = \sqrt{2 \times 3.8 \times 25} = 13.784 \text{ m/s}
\]

\[
a \approx \frac{\Delta V}{\Delta t} = \frac{13.784}{0.1} = 137.84 \text{ m/s}^2
\]
Match the question with an answer by writing the letter of the answer in the blank next to the question. **No answer is used more than once.** (20 points)

**Questions**

1) Low-cost electronic medical thermometers use this: \[ \text{L} \]
2) When 2 or more high Q MEMS gyros interfere with each other: \[ \text{A} \]
3) A rigid segment sometimes added to a pressure sensor diaphragm: \[ \text{E} \]
4) This sensor makes use of the Seebeck effect: \[ \text{M} \]
5) This optical instrument can be used to measure displacement: \[ \text{B} \]
6) An optical instrument for identifying an unknown gas: \[ \text{D} \]
7) Force per unit area is also known as: \[ \text{J} \]
8) A commonly used 4 thermotransistor temperature sensor: \[ \text{N} \]
9) The terms martensite and austenite are associated with this type of actuator: \[ \text{F} \]
10) A type of thermal actuator made of two materials with different CTE’s: \[ \text{C} \]

**Answers to choose from**

- A. Intermode Coupling
- B. Interferometer
- C. Bimorph
- D. Spectrometer
- E. Boss
- F. Shape Memory Alloy
- G. Doppler Shift
- H. Membrane
- I. Quadrature Error
- J. Pressure
- K. Gap-closing Actuator
- L. Thermistor
- M. Themocouple
- N. PTAT

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2) At a typical rock concert (110dB sound level), how long until you have permanent hearing loss (circle one answer): (5 points)

- Microseconds
- Less than 10 minutes
- More than 1 Hour
- Never

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3) A MEMS submarine is being used to monitor the cooling fluid in an industrial transformer. The transformer fluid (liquid) has a density of 2 g/cm$^3$. The sub is in motion and measures the total pressure (1960.4 Pa) and the static pressure (1960 Pa) that it experiences, using gage pressure sensors. For $1\text{G}=9.8 \text{ m/s}^2$, estimate the velocity of the sub in mm/s? (10 points)

\[ \delta = \left( 2 \frac{g}{cm^3} \right) \left( \frac{11g}{1000g} \right) \left( \frac{100cm}{1m} \right)^2 = 2000 \text{ Kg/m}^3 \]

\[ P_t = P_s + \frac{\rho v^2}{2} \rightarrow \frac{\rho v^2}{2} = P_t - P_s \rightarrow v = \sqrt{\frac{2(P_t-P_s)}{\rho}} \]

\[ = \left( \frac{2(1960.4-1960)}{2000} \right) \]

\[ = 0.02 \text{ m/s} \]

\[ = 20 \text{ mm/s} \]

4) For the sub in (3), what is the depth of the sub in mm, ignoring ambient atmospheric pressure? Use $1\text{G}=9.8 \text{ m/s}^2$. (10 points)

\[ P_s = \rho gh \rightarrow h = \frac{P_s}{\rho g} = \frac{1960}{(2000)(9.8)} = 0.1 \text{ m} = 100 \text{ mm} \]
5) You are standing motionless on a train track. You hear a train's horn. You know that the frequency of a stationary train's horn is 2KHz and that the speed of sound in air that day is 350m/s. But the sound you hear from the train is measured at 2100Hz. What is the magnitude of the speed of the train in km/hr? (5 points)

$$f_d = \frac{f_s}{1 + \frac{v_{obj}}{v_{wave}}} \Rightarrow 1 + \frac{v_{obj}}{v_{wave}} = \frac{f_s}{f_d} \Rightarrow v_{obj} = v_{wave} \left( \frac{f_s}{f_d} - 1 \right)$$

$$= 350 \left( \frac{2000}{2100} - 1 \right)$$

$$= -16.67 \text{ m/s} \times \frac{3600}{1000} = 60 \text{ km/hr}$$

magnitude of speed = 60 km/hr

6) Is the train in (5) moving toward you or away from you? (5 points)

toward

7) A parallel plate actuator (PPA) consists of two square electrodes, 500μm on a side, separated by 10μm, in a vacuum. If one electrode is connected to ground and the other electrode is connected to a high frequency AC voltage (100V amplitude), what is the average force produced by the PPA? (10 points)

$$f = \frac{\epsilon_0 \varepsilon_r AV_{rms}^2}{2 X_0^2} = \frac{(8.85 \times 10^{-12})(1)(500 \times 10^{-6})^2 (\frac{100}{V_0})^2}{2 (10 \times 10^{-6})^2}$$

$$= 55,338 \mu N$$
8) What is the measurand for a MEMS pressure sensor? (5 points)

\[ \text{pressure} \]

9) What is the stable operating range for a parallel plate actuator, in terms of its rest gap, \( x_0 \)? (5 points)

\[ 0 \leq x < \frac{x_0}{3} \]

10) Consider an open-loop MEMS accelerometer with a natural frequency of 1kHz. What acceleration produces a 1\( \mu \)m displacement? (5 points)

\[ d = aS = \frac{a}{\omega_n^2} \rightarrow a = \omega_n^2 d \]
\[ = (2\pi1000)^2 (1\times10^{-6}) \]
\[ = 39.48 \text{ m/s}^2 \]

11) A certain MEMS gyroscope has a mass of 4\( \mu \)g, \( c=0.1\times10^{-6}\text{Kg/s}, f_n=10\text{KHz}, A_x=1\mu\text{N} \). What angular rate input (in \textdegree/s) produces an amplitude of motion along the sense axis of 10\( \mu \)m? (5 points)

\[ |\gamma| = \frac{2m\omega_n A_x}{c^2 \omega_n} \Rightarrow \omega = \frac{|\gamma| \omega_n^2}{2mA_x} = \frac{0.1\times10^{-6} \times (2\pi 1000)^2}{2 (4\times10^{-9}) (1\times10^{-6})} \]
\[ = 0.785 \text{ rad/s} \times \frac{180}{\pi} \]
\[ = 45 \text{ \textdegree/s} \]

12) What does PTAT stand for? (5 points)

Proportional To Absolute Temperature
Bonus Question (10 points)

A certain open loop MEMS accelerometer has the following transfer function:

\[ G(s) = \frac{1}{s^2 + 6s + 1} \rightarrow \omega_n = 1 \text{ rad/s}, \quad \frac{\omega_n}{\xi} = 6 \rightarrow Q = \frac{\sqrt{3}}{6} \rightarrow \text{open loop} \]

It has been placed in a closed-loop controller (shown below) to realize a closed-loop accelerometer. What is the Q for the open loop accelerometer and what is the Q for the closed-loop accelerometer? Show all steps in your calculations.

\[ E = A - 8V \quad (i) \]
\[ V = EG \quad (ii) \]
\[ \therefore V = G(A - 8V) \]
\[ V\left(\frac{1}{\xi} + 8\right) = A \]
\[ \frac{V}{A} = \frac{1}{\xi} + 8 = \frac{1}{s^2 + 6s + 1} + 8 = \frac{1}{s^2 + 6s + 9} \]

\[ \omega_n^2 = 9 \rightarrow \omega_n = 3 \text{ rad/s} \]
\[ \frac{\omega_n}{\xi} = 6 \rightarrow Q = \frac{\omega_n}{\xi} = \frac{3}{6} = \frac{1}{2} \]

\[ Q = \frac{1}{2} \rightarrow \text{closed loop} \]