**b. Resistive Sensors**

Consider a conductor:

\[
\begin{array}{c}
I \\
\end{array}
\]

Cross-sectional area: \( S \)

Resistance = \( R = \frac{\rho L}{S} \)

where \( \rho \) = resistivity, a material property

\([\rho] = \mu \text{\Omega cm}\)

**i. Temperature Effects**

\( \rho \) varies with temperature:

\[ p(T) = p_0 (1 + \alpha_\tau T + \beta_\tau T^2) \] → for metals

where \( p_0 \) = a resistivity specified at some reference temperature, such as 0°C

\( \alpha_\tau \) and \( \beta_\tau \) = temperature coefficients

\( \alpha_\tau \) = linear temperature coefficient of resistivity

For metals: \( \alpha_\tau \sim 10^{-3} \) and \( \beta_\tau \sim 10^{-7} \)

\[ p(T) = p_0 (1 + \alpha_\tau T) \]

\( \alpha_\tau \) at 20°C for Al: \( p_0 = 2.65 \times 10^{-6} \mu \text{\Omega cm} \) and \( \alpha_\tau = 4.3 \times 10^{-3}/°C \)

This property can be used to make a metal temperature sensor.

More on this later in the course.
Strain Effects

For most materials: if you axially stretch along L, the cross-sectional area ($w^+$) will shrink.

For $S = w^+$

$$R = \rho \frac{L}{w^+}$$

If $L \uparrow$ and $SW \Rightarrow R \uparrow$

- Poisson's Ratio: a ratio of the tendency of a material to get thinner in a transverse direction when subjected to axial stretching.

$$\nu = -\frac{\text{transverse strain}}{\text{axial strain}} = -\frac{E_{\text{trans}}}{E_{\text{axial}}}$$

$[\nu]$ = dimensionless

Typical values for $\nu$: 0.1 to 0.4

**Derivation**

$$R = \rho \frac{L}{w^+} \quad (1)$$

$$dR = \frac{L}{w^+} dp + \frac{\rho}{w^+} dL - \frac{\rho}{w^{+2}} dw - \frac{\rho}{w^{+2}} d\theta$$

$$\frac{dL}{L} = \frac{dL}{\rho} + \frac{dL}{w} - \frac{dw}{w} - \frac{d\theta}{\theta}$$

$$\frac{dL}{L} = \text{axial strain} = \varepsilon_1$$
\[ \frac{dw}{w} = \text{transverse strain} = \varepsilon_w = -V \varepsilon_1, \]
\[ \frac{dt}{t} = \text{transverse strain} = \varepsilon_t = -V \varepsilon_1, \]
\[ \frac{dR}{R} = \frac{dp}{p} + \varepsilon_t + VE_1 + V \varepsilon_1, \]
\[ = \frac{dp}{p} + \varepsilon_t + 2V \varepsilon_1. \]

**Gauge Factor (GF):**

\[ GF = \frac{\Delta R}{R} = \frac{dp/p}{\varepsilon_t} \]

\[ \frac{dp/p}{\varepsilon_t} \rightarrow \text{piezoresistive effect (PE)} \]

\[ 1 + 2V \rightarrow \text{geometric effect (GE)} \]

**GF:**

\[ GF = \frac{\Delta \text{change in resistance}}{\Delta \text{change in length}} \]

A sensor that makes use of the GE is called a strain gauge.

A sensor that makes use of the PE is called a piezoresistor.

**For metals:** GE > PE

**For semiconductors:** PE > GE

From Table 5.1 in textbook:

<table>
<thead>
<tr>
<th>Material</th>
<th>GE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal foils</td>
<td>2-5</td>
</tr>
<tr>
<td>Thin film metals</td>
<td>2</td>
</tr>
<tr>
<td>Single crystal Si</td>
<td>-125 to +200</td>
</tr>
<tr>
<td>Polysilicon</td>
<td>±30</td>
</tr>
</tbody>
</table>

Note: Young's modulus: E

\[ E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\varepsilon} \]

\[[E] = Pa = N/m^2 = [\sigma] = [\varepsilon] \]

**Piezoresistive effect:**

\[ \text{Note error in text, p. 86} \]
Ex: A certain metal strain gauge has a nominal resistance of 1kΩ, and has a GF = 2. If it experiences a 1% axial strain, what does the resistance become?

Solution:

\[ \varepsilon_1 = \frac{\Delta L}{L} = \frac{0.01}{L} \geq 0.01 \%
\]

\[ GF = \frac{\Delta R}{R} \rightarrow \Delta R = R \varepsilon_1, GF = (1000 \times 0.01)(2) = 20 \Omega
\]

\[ R_{new} = R + \Delta R = 1000 + 20 = 1020 \Omega
\]