Introduction to MEMS Actuators (actuator converts electrical signal into a non-electrical quantity)

Consider 2 oppositely charged particles in a vacuum

\[ \text{Vacuum} \]

\[ +Q_1 \]

\[ -Q_2 \]

\[ r \]

$r$ is the distance between $Q_1$ and $Q_2$

An electrostatic force exists between the two particles that attempts to bring them into contact

\[ F_e = \frac{kQ_1Q_2}{r^2} \]

where $k = \frac{1}{4\pi\varepsilon_0}$

Consider a parallel plate capacitor:

\[ \text{Electrode surface area} = A \]

\[ F_{el} = \frac{\varepsilon_0 \varepsilon_r AV^2}{2d^2} = \frac{CV^2}{2d} \]

\[ F_{el} \propto V^2 \rightarrow \text{same effect if } V \text{ is pos, neg or AC (RMS avg)} \]

\[ F_{el} \propto \frac{1}{d^2} \]

Book: Ch 5, 6, 7. p. 104

C ↑: $F_{el}$ ↑
Electrostatic Actuators Continued

1.
\[
C = \frac{\varepsilon_0 \varepsilon_r A}{d} \quad \text{\text{and}} \quad F_{el} = \frac{\varepsilon_0 \varepsilon_r A V^2}{2d^2}
\]

Parallel Plate Capacitor \Rightarrow Parallel Plate Actuator (PPA)

A more realistic application:

\[
F_{el} = \frac{\varepsilon_0 \varepsilon_r A U(t)^2}{2(x_0 - x(t))^2} = F(t) \Rightarrow \text{PPA is a "square law device"}
\]

Diff EQ: \( m \ddot{x} + c \dot{x} + kx = F(t) = \frac{\varepsilon_0 \varepsilon_r A V^2}{2(x_0 - x(t))^2} \)

\[
\frac{x}{F(t)} = \frac{\frac{V_0}{s^2 + \frac{c}{m} s + \omega_n^2}}{s^2 + \frac{c}{m} s + \omega_n^2} \Rightarrow \text{LPF}
\]

Let \( U(t) = V_0 \cos(\omega t) \)

\[
U^2(t) = V_0^2 \left[ \frac{1}{2} + \frac{1}{2} \cos(2\omega t) \right]
\]

for \( \omega \gg \omega_n \Rightarrow \text{the mech. sys. cannot respond to } \cos(2\omega t) \)

\[
U(t) \approx \frac{V_0}{\sqrt{2}} \quad \text{for } U^2 = \frac{V_0^2}{2}
\]

\( \Rightarrow \) the rms value of \( U(t) \)
Back to Diff Eq: \[ m^2 \ddot{x} + c \dot{x} + kx = \frac{\varepsilon_r \varepsilon_0 AV^2}{2(x_0-x)^2} \]

Examine case where \( V(t) \) changes very slowly or \( \omega >> \omega_n \)

\[ \Rightarrow \text{so that } \dot{x} \approx 0 \text{ and } x \approx 0 \]

\[ \Rightarrow kx = \frac{\varepsilon_r \varepsilon_0 AV^2}{2(x_0-x)^2} \Rightarrow \text{solve graphically} \]

[Graph showing force \( F \) vs. \( x \)]

2 mathematical solutions exist: \( A + B \)

\( A \rightarrow \) stable equilibrium point

\( B \rightarrow \) unstable "" ""

Increase \( V \) (slowly) \( \Rightarrow F_{el} \) trace moves up \( \Rightarrow A \) & \( B \) move closer together

At a Voltage called the pull-in voltage \( (V_{pi}) \) \( A \) & \( B \) converge into 1 unstable equilibrium point

\[ V_{pi} = \sqrt{\frac{8kx_0^3}{27A\varepsilon_r\varepsilon_0}} \Rightarrow \text{corresponds to } x = \frac{1}{3}x_0 \]

For \( V > V_{pi} \) \( \Rightarrow \) no stable equilibrium point exists and the two electrodes will snap into contact

For an open loop PPA where motion is controlled by \( V \), the PPA has a stable range of motion of \( 0 < x < \frac{1}{3}x_0 \).

This stable range of motion can be increased using closed loop controllers.