patterns. An Eulerian sequence lets one apply all \( k \times 2^k \) APNPs to a neighborhood with \( k \) writes to initialize the neighborhood, and 1 additional write for each of the \( k \times 2^k \) APNPs.

**Testing Neighborhoods Simultaneously.** When a cell is written, we change \( k \) different neighborhoods (Type-1 or Type-2.) We wish to test the neighborhoods simultaneously, using the *tiling* and *two-group* methods.

**Tiling Method.** The *tiling method* totally covers memory with non-overlapping neighborhoods. This is known as a *tiling group*, and the set of all neighborhoods in the group is called the *tiling neighborhoods* [688]. Figure 9.10 [688] depicts this for a 5-element Type-1 neighborhood. Cell 2 is always the base cell, and the deleted neighborhood cells are numbered as shown. Figure 9.11 [688] shows the tiling for a 9-element Type-2 neighborhood, where cell 4 is the base cell. In a Type-1 neighborhood, when we tessellate the neighborhoods, we have \( \frac{n}{2} \) base cells. However, while we are applying all of the test patterns to the \( \frac{n}{2} \) base cells 2, it turns out that we also apply the appropriate patterns to the memory when cells 0 are base cells, cells 1 are base cells, cells 3 are base cells, and cells 4 are base cells. This reduces the pattern length from \( n \times 2^k \) patterns to \( \frac{n}{2} \times 2^k \) patterns. Under this method, each cell is simultaneously a base cell and a deleted neighborhood cell for other base cells (see Figure 9.27 [688].) A similar argument and reduction holds for Type-2 neighborhoods. This argument breaks down when we test ANPSFs and PNPSFs separately, because with those patterns the \( k \) cells in the neighborhood are not treated identically.

![Figure 9.27: Base cell 0, 1, 2, 3, and 4 Type-1 tiling neighborhoods.](image)

**Two-Group Method.** For the *two-group method*, a cell is simultaneously a base cell in one group and a deleted neighborhood cell in the other group, and vice versa (see Figure 9.28 [688].) With this *duality* property, cells are divided into two groups, group-1 and group-2, in a checkerboard pattern. Base cells of group-1 are deleted neighborhood cells of group-2, and vice versa. Each group has \( n/2 \) base cells \( b \) and \( n/2 \) deleted neighborhood cells formed by 4 subgroups \( A, B, C, \) and \( D \). This only works for Type-1 neighborhoods, because in Type-2, 9 cell neighborhoods, there are both middle cells 1, 3, 5, and 7 and corner cells 0, 2, 6, and 8. Unfortunately, duality (a cell is either a base cell or a deleted neighborhood cell) does not hold here.