Flow in pipes (fully developed) laminar:

\[ \Delta P_f = \frac{F_f}{P} = \frac{(P_1 - P_2)}{P} = \frac{32 \mu v (L_2 - L_1)}{D^2} \]

\[ F_f = 4PA \nu^2 / ZD \]

\[ F_{flam} = 16 / Re \]

Turbulent Flow

Use friction factor charts (Hood vs Fanning f)

Problem types:
1. Given pipe info & flow, find losses (\( \Delta P_f, h_f, F_f \))
2. Given pipe info & \( \Delta P \), find flow rate (Q)
3. Given flow rate & losses, design & select pipe.

Assume \( f \) to start!

In turbulent flow: \( V_x = V_{x_{max}} \left( \frac{R - x}{R} \right)^n \) \( n \approx 1/7 \)

but \( V_x = \frac{q}{C(Re)} \)

Friction Losses

1. \( \frac{\Delta P_f}{\rho} = \frac{\Delta h}{g_c} \left[ \right. \) ft-lbf/ft
2. \( \frac{\Delta P_f}{g} = \Delta h \left[ \right. \) ft (oz m)
3. \( \Delta P_f \left[ \right. \) atm (nc Pa or psi etc)
4. \( \frac{q}{g_c} \left[ \right. \) ft-lbf (ft-lbf/ft-lbf g_c)

Turb Flow losses: \( h_L = 32 fL g_c Q^2 / \pi^2 g D^5 \)

Friction losses in valves & fittings (minor losses)

Contractions

\[ K_e = 1.0 \sqrt{V^2 / 2} \]

\[ K_e = 0.65 \sqrt{V^2 / 2} \]

\[ K_e = C_1 \sqrt{V^2 / 2} \] where

\[ C_1 \quad \epsilon / D \]

\[ 0.65 \quad 0.0 \]

\[ 0.12 \quad 0.1 \]

\[ 0.04 \quad 0.2 \]
Expansions

\[ \frac{K_{ex}}{\text{Fr}} = 1.0 \frac{V^2}{2} \]

Other

A large variety of other fittings, valves, couplings, T's etc. have tabulated coefficients (see Perry's or other Fluid ref book)

Noncircular cross-sections

\[ \frac{De}{De} = 4 \cdot R_u \quad \text{where} \quad R_u = \frac{\text{cross-sectional area (flaw)}}{\text{wetted perimeter}} \]

For circular \( De = D \)

For annular \( De = D_1 - D_2 \)

For rectangular \( D = \frac{2ab}{a+b} \)

Drag Forces

\[ C_D = \frac{F_D}{A_{proj}} \quad ; \quad F_D = C_D A_p \rho \frac{V_o^2}{2} \]

For creeping flow (stokes flow) \( Re < 0.1 \quad C_D = 24/Re \)

\[ F_D = 3 \pi \mu \rho V_0 \]

Other regimes

\[ C_D = 0.2 \quad Re > 5 \times 10^5 \]

\[ C_D = 0.4a \quad 10^3 < Re < 2 \times 10^5 \]

Other \( Re \): Use \( C_D \) vs \( Re \) charts

\[ A_p (sphere) = \frac{\pi D^2}{2} \]

\[ A_p (cyl + flow) = L D_p \]

Drag on Flat plates

\[ F_D = C_F \frac{A_{shear}}{\frac{P V^2}{2}} \quad \text{use graphs for } C_F \text{ vs } Re \text{ values} \]
Packed Beds (not fluidization)

**Def's:**
- \( e \): void fraction = vol of voids / vol of bed
- \( a_v \): spec. area of particles
  \[ a_v = \frac{Sp}{Up} = \frac{\text{surface area sphere}}{\text{vol of particles}} = \frac{EJ}{A} \, \text{ft}^{-1} \]
- \( 1 - e \): solids fraction
- \( a_L = a_v (1 - e) = 6/D_p (1 - e) \)
- \( \zeta \): total area of bed
- \( \bar{v} \): average interstitial velocity
- \( v' \): superficial velocity = \( \bar{v} \cdot e \)
- \( u' = Q/A_{\text{bed (empty)}} \)
- \( k_H = \frac{e}{\zeta} = \frac{e}{6D_p} \frac{C(1 - e)}{G} \)
- \( N_{Re} = \frac{4D_p v' \rho}{6 (1 - e) \mu} \)

**Pressure drop in packed beds**

\[ \Delta P = \frac{32 \bar{v} \Delta L}{(4 \zeta H)^2} = \frac{150 \mu \bar{v} \Delta L (1 - e)^2}{e^3 D_p} \quad \text{Re} < 10 \quad \text{(laminar)} \]

\[ \Delta P = \frac{1.75 \rho \bar{v}^2 \Delta L (1 - e)}{D_p e^3} \quad \text{Re} \geq 1000 \]

All purpose

\[ \Delta P = \frac{D_p \rho}{G^2} \frac{D_p e^3}{L (1 - e)} = \frac{150}{N_{Re, p}} + 1.75 \]

**Non-spherical particles**

mixtures of different particles

\[ \frac{x_1 / \phi_1 D_p + x_2 / \phi_2 D_p + \ldots}{\text{fraction}} = D_{pm} \]

Sphericity \( \phi = \frac{\text{surf area of sphere}}{\text{surf area of particle}} \)

\[ a_v = \frac{6}{\phi_5 D_p} \quad a = \frac{6}{\phi_5 D_p (1 - e)} \]
Pumps, Fans, Blowers

\[ \Delta P: \text{pump} \quad \frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} + \eta_h p = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g} \]

\text{losses here}

\text{elevation change usually small}

Use of pump curves

\[ C_Q = \text{discharge coeff} = \frac{Q}{n D^3} \]

\[ C_H = \text{head coeff} = \frac{\Delta H}{(D^2 n^2 / g)} \]

\[ C_p = \text{power coeff} = \frac{P}{\rho D^5 n^3} \]

\[ n_s \text{pump} = \frac{H g}{\omega} \quad \text{or} \quad \frac{H g}{\omega e} \]

\text{depending on work units}