Combined Bending and Axial Loading

- Combined bending and axial tension

\[
\frac{f_t}{F_t^*} + \frac{f_b}{F_b^*} \leq 1.0 \quad \text{and} \quad \frac{f_b - f_t}{F_b^{**}} \leq 1.0
\]

- \( F_b^* \) = tabulated bending design value multiplied by all adjustments except \( C_L \)

- \( F_b^{**} \) = tabulated bending design value multiplied by all adjustments except \( C_V \)

Combined Bending and Axial Loading

- Combined bending and axial compression

\[
\left[ \frac{f_c}{F_c'} \right]^2 + \frac{f_{b1}}{F_{b1}'} \left[ 1 - \left( \frac{f_c}{F_{cE1}} \right) \right] + \frac{f_{b2}}{F_{b2}'} \left[ 1 - \left( \frac{f_c}{F_{cE2}} \right) - \left( \frac{f_b}{F_{bE}} \right) \right]^2 \leq 1.0
\]

- \( f_{b1} \) = actual edgewise bending stress

- \( f_{b2} \) = actual flatwise bending stress
Combined Bending and Axial Loading

- Combined bending and axial compression

\[ f_{c1} < F_{cE1} = \frac{K_{cE} E'}{I_{s1} d_1} \text{ for uniaxial or biaxial bending} \]

\[ f_{c2} < F_{cE2} = \frac{K_{cE} E'}{I_{s2} d_2} \text{ for biaxial bending} \]

\[ f_{b1} < F_{BE} = \frac{K_{BE} E'}{(R_B)^2} \text{ for biaxial bending} \]

Beam-Column Design Example

- Given:
  - 10-ft-long Wall Studs
  - No. 1 Douglas fir - Larch 2x4 spaced 16 in. OC
  - Axial loads of 1000 lb. Snow and 500 lb. Dead; side loads of 25 psf from Wind.
  - Wall sheathing provides lateral support for width (1.5 in. dimension)
Solution:
• Check load combinations from Standard Building Code
  • for bending:  D + W
  • for compression:  D + S
  • for combined bending and compression:
    • D + S + 1/2W

Design Data:  No. 1 Douglas fir Larch 2x4

A = 5.25 in²  \quad F_b = 1000 \text{ psi}
Sxx = 3.063 in³  \quad F_c = 1500 \text{ psi}
lxx = 5.359 in⁴  \quad E = 1.7 \times 10^6 \text{ psi}
Design Data: No. 1 Douglas fir Larch 2x4

- $C_D = 1.15$ for snow load, 1.6 for wind load
- $C_M = 1.0$
- $C_t = 1.0$
- $C_L = 1.0$ (full lateral support by wall sheathing)
- $C_P = ?$ (to be determined)
- $C_F = 1.5$ for $F_b$
- $C_F = 1.15$ for $F_c$
- $C_r = 1.15$ (16 in. spacing with sheathing)

Check Bending Stress:

$$f_b = \frac{M}{S} \leq F_b'$$

$$f_b = \left[ \frac{25 \text{ lb/ft}^2}{144 \text{ in.}^2} \left( \frac{1 \text{ ft}^2}{16 \text{ in.}} \right) \left( 120 \text{ in.} \right)^2 \right] \frac{8 \left( 3.063 \text{ in.}^3 \right)}{}$$

$$f_b = 1633 \text{ psi}$$
• Check Bending Stress:

\[ F'_b = F_b C_D C_F C_r \]
\[ F'_b = (1000 \text{ psi})(1.6)(1.15)(1.15) \]

\[ F'_b = 2760 \text{ psi} \]

Conclusion: stud is acceptable in bending stress
Design Data: No. 1 Douglas fir Larch 2x4

A = 5.25 in²  \hspace{1cm} F_b = 1000 psi
Sxx = 3.063 in³  \hspace{1cm} F_c = 1500 psi
Ixx = 5.359 in⁴  \hspace{1cm} E = 1.7 \times 10^6 psi

Design Data: No. 1 Douglas fir Larch 2x4

C_D = 1.15 for snow load, 1.6 for wind load
C_M = 1.0
C_t = 1.0
C_L = 1.0 (full lateral support by wall sheathing)
C_p = ? (to be determined)
C_F = 1.5  \text{ for } F_b
C_F = 1.15 \text{ for } F_c
C_r = 1.15 (16 in. spacing with sheathing)
• **Column Stability Factor,** $C_p$
  
  • Effective length for column pinned at each end
  
  $$l_e = Ke l = (1.0)(120 \text{ in}) = 120 \text{ in}$$
  
  • Slenderness Ratio, $l_e/d$
  
  $$\frac{l_e}{d} = \frac{120 \text{ in.}}{3.5 \text{ in.}} = 34.3$$

$$C_p = \frac{1 + \left(\frac{F_{cE}}{F_c^*}\right)}{2c} - \sqrt{\left[1 + \left(\frac{F_{cE}}{F_c^*}\right)\right]^{-2} \left(\frac{F_{cE}}{F_c^*}\right)}$$

$$F_{cE} = \frac{K_{cE}E'}{\left(\frac{l_e}{d}\right)^2}$$

$$F_c^* = F_c C_D C_M C_t C_F C_i$$
• **Column Stability Factor,** $C_p$

\[
E' = E C_M C_T C_T = 1.7 \times 10^6 \text{ psi}
\]

\[
F_{ce} = \frac{K_{ce} E'}{(l_b^2/d)} = \frac{(0.3) \left(1.7 \times 10^6 \text{ psi}\right)}{(34.3)^2} = 433 \text{ psi}
\]

\[
F_g^* = F_g C_D C_p = (1500 \text{ psi})(1.15)(1.15) = 1984 \text{ psi}
\]

• **Column Stability Factor,** $C_p$

\[
C_p = 1 + \left( \frac{433 \text{ psi}}{1984 \text{ psi}} \right) - \left[ \frac{1 + \left( \frac{433 \text{ psi}}{1984 \text{ psi}} \right)}{2 \left(0.8\right)} \right]^2 - \left( \frac{433 \text{ psi}}{1984 \text{ psi}} \right)
\]

\[
C_p = 0.207
\]
• **Check Compression Stress:**

\[
f_c = \frac{P}{A} \leq F_c'
\]

\[
f_c = \frac{1500 \text{ lb.}}{5.25 \text{ in}^2}
\]

\[
f_c = 286 \text{ psi}
\]

\[
F_c' = F_c C_D C_P C_F
\]

\[
F_c' = (1500 \text{ psi})(1.15)(0.207)(1.15)
\]

\[
F_c' = 411 \text{ psi}
\]

\[
f_c = 286 \text{ psi} < F_c' = 411 \text{ psi}
\]

**Conclusion:** stud is acceptable in compression stress
• Check Combined Bending and Compression Stress:
  • use \( D + S + 1/2W \)

\[
\left( \frac{f_c}{F_c'} \right)^2 + \frac{f_{b1}}{F_{b1}'} \left[ 1 - \left( \frac{f_c}{F_{cE1}} \right) \right] \leq 1.0
\]

\[
\left( \frac{286 \text{ psi}}{417 \text{ psi}} \right)^2 + \frac{1633 \text{ psi}}{2} \left[ 1 - \left( \frac{286 \text{ psi}}{433 \text{ psi}} \right) \right] = 1.34
\]
• Check Combined Bending and Compression Stress:
  • combined stress index = 1.34 > 1.0
  • beam-column is not sized properly
  • if beam-column had been acceptable in combined stress, shear, deflection, and bearing should be checked also

Design for Bearing

• Bearing parallel-to-grain:
  • based on net bearing area:

\[
f_g = \frac{P}{bd} \leq F_g'
\]

• \( F_g \) applies to end-to-end bearing (if cut square)
• When \( f_g > 0.75 \ F_g' \), bearing should be on a metal strap or similar material
Special Component Design
Provisions

- Provisions for specific components are listed in Parts 4, 5, and 6 of the NDS
- Sawn lumber provisions are in Part 4
- Glulam provisions are in Part 5
- Round Timber Piles are in Part 6
  - Design values are listed here
  - Adjustment factors listed here