Bending Member Design

- Check:
  - Flexure
  - Shear
  - Deflection
  - Bearing
Bending Member Design

• Avoid notching whenever possible - especially on tension faces of beams
• NDS states “bending members shall not be notched except as permitted by 4.4.3, 5.4.4, 7.4.4, and 8.4.1.”
• Use gradually tapered notches
• Square cut notches result in greater problems from stress concentrations

Bending Member Design

• See Figure 4A for SAWN LUMBER notch recommendations
  • No notches in middle third of span
  • Interior notches located in outer thirds of span are permitted; depth can’t exceed 1/6 beam depth
  • Interior notches are not permitted on tension side of nominal 4 in. thick members
  • Notches at ends of member over a support can be larger – not greater than ¼ beam depth
Bending Member Design

- Notching requirements for GLUED-LAMINATED TIMBERS:
  - Don’t notch tension side of glulam beams
  - Notches are permitted on tension side at ends of beams over a support
    - Notch depth can’t exceed lesser of 1/10 depth or 3 in.
  - Notches are permitted on compression side of beam only at the ends, and notch depth can’t exceed 2/5 depth of beam
Bending Member Design

• Notching requirements for Wood I-JOISTS:
  • In general, don’t notch I-joists.
    • Consult with manufacturer before notching
  • Notching for STRUCTURAL COMPOSITE LUMBER
    • Use same recommendations as glulam

• Flexural design equations (NDS 3.3.2):

\[
\frac{f_b}{M} = \frac{M}{S} = \frac{Mc}{I} \\
\text{for rectangular beams:} \\
\frac{f_b}{S} = \frac{6M}{bd^2}
\]

\[f_b \leq F'_b = F_b C_D C_M C_t C_L C_F C_v C_{fu} C_1 C_c C_e C_f\]
### Table 4.3.1  Applicability of Adjustment Factors for Sawn Lumber

| F_f' = F_f | x | C_D | C_M | C_L | C_P | C_σ | C_T | - | - | - |
| F_t' = F_t | x | C_D | C_M | C_L | - | C_T | - | C_σ | - | - | - |
| F_v' = F_v | x | C_D | C_M | C_L | - | - | - | C_T | - | - | - |
| F_w' = F_w | x | - | C_M | C_L | - | - | - | C_σ | - | - | - | C_T | - | - | - |
| F_e' = F_e | x | C_D | C_M | C_L | - | C_T | - | C_σ | - | - | - | C_T | - | - | - |
| E = E | x | - | C_M | C_L | - | - | - | C_σ | - | - | - | C_T | - | - | - |

**Example Test Beam**

Sample beam from bending test. The tension face of the beam is shown here at midspan.

Bending failure on tension face of beam
- **Beam stability factor,** $C_L$:
  - when $d < b$, $C_L = 1$
  - when compression edge is fully supported to prevent lateral displacement, $C_L = 1$
  - When sawn lumber bending members are designed in accordance with NDS 4.4.1, $C_L = 1$

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**Bending Member Stability**

- Test beam after load was applied
• Lateral torsional buckling occurs on compression edge of beam

• **Beam stability factor**, $C_L$:  
  - when $d > b$ and no additional lateral support provided beyond that at the bearings, $C_L$ is calculated  
  - find unsupported length, $l_u$  
  - then find effective length, $l_e$, from Table 3.3.3
Beam stability factor, $C_L$:

- find $l_o$
- find slenderness ratio, $R_B$

$$R_B = \sqrt{\frac{l_e d}{b^2}} \leq 50$$

- calculate stability factor, $C_L$:

$$C_L = 1 + \frac{F_{bE}}{F_b} \frac{1.9}{1 + \left( \frac{F_{bE}}{F_b} \right)} - \sqrt{1 + \left( \frac{F_{bE}}{F_b} \right)^2 - \frac{F_{bE}}{F_b} \frac{1.9}{0.95}}$$
• Beam stability factor, $C_L$:

$$C_L = \frac{1 + \left( \frac{F_{bE}}{F_b^*} \right)}{1.9} - \sqrt{1 + \left( \frac{F_{bE}}{F_b^*} \right)^2 - \left( \frac{F_{bE}}{F_b^*} \right)^2}$$

• $F_b^*$ is tabulated bending design value multiplied by all adjustments except $C_{fu}, C_V,$ and $C_L$

$$F_b^* = F_b C_D C_M C_1 C_F C_1 C_c C_c C_t$$

• Beam stability factor, $C_L$:

$$C_L = \frac{1 + \left( \frac{F_{bE}}{F_b^*} \right)}{1.9} - \sqrt{1 + \left( \frac{F_{bE}}{F_b^*} \right)^2 - \left( \frac{F_{bE}}{F_b^*} \right)^2}$$

$$F_{bE} = \frac{K_{bE} E'}{R_B^2} \quad E' = E C_M C_t C_1 C_T$$

• $K_{bE} = 0.439$ for visually graded lumber
• $K_{bE} = 0.561$ for MEL
• $K_{bE} = 0.610$ for MSR, glulam
• Flexural design equations:

\[ f_b = \frac{M}{S} \leq F'_b = F_D C_D C_M C_t C_L C_F C_V C_{fu} C_t C_r C_c C_f \]

Beam Design Example

• Given:
  • Beam supporting roof trusses; 18 ft Span
  • 4 point loads, 6 ft OC; 3200 lb Snow + 1600 lb Dead
  • Lateral restraint at supports and at each load application point

[Diagram of a beam with load applications and labels: 6 ft TYP, 4800 lb TYP, 18 ft]
• **Find:**
  - Southern pine glulam beam size and combination that will carry loads

• **Assume:**
  - Dry conditions
  - Normal temperatures
  - Maximum live load deflection = \( L / 360 \)

**Solution:**
- Max. Bending Moment = 28,800 lb.ft
- Max. Shear Force = 4,800 lb.
- Max. Reaction Force = 9,600 lb.

\[ 4 @ 4,800 \text{ lb.} \]

\[ 9,600 \text{ lb.} \]
• Find trial size first by assuming that we will use a 24F-1.8E stress class; initial guess for $F_b' = 2400$ psi

\[
f_b = \frac{M}{S} \leq F_b' \quad \text{or} \quad \frac{M}{F_b'} \leq S
\]

\[
S \geq \frac{(28,800 \text{ lb.ft}) \left( \frac{12 \text{ in.}}{\text{ft}} \right)}{2400 \text{ psi}} = 144 \text{ in}^3
\]
Trial Size:

- Try a glulam beam with a 3.0 in. width and find size with $S \geq 144 \text{ in}^3$

- From NDS Supplement Table 1D, we can try a 3.0 x 17.875 in. beam ($S = 159.8 \text{ in}^3$)

- This is based on the assumption that we have a 24F-1.8E glulam stress class
Design Data: 24F-1.8E southern pine 3.0 x 17.875

A = 53.63 in²
Sxx = 159.8 in³
Ixx = 1428 in⁴

F_bxx = 2400 psi
F_cxx = 650 psi
F_vxx = 240 psi
E_xx = 1.8 x 10⁶ psi

Table 10  Section Properties of Southern Pine Glued Laminated Timber

<table>
<thead>
<tr>
<th>Depth</th>
<th>Width</th>
<th>A (in²)</th>
<th>Sxx (in³)</th>
<th>Ixx (in⁴)</th>
<th>F_bxx (psi)</th>
<th>F_cxx (psi)</th>
<th>F_vxx (psi)</th>
<th>F_ex (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2 in.</td>
<td>3/4 in.</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
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</tr>
<tr>
<td>3/4 in.</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
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</tr>
<tr>
<td>1.00 in.</td>
<td>1.00</td>
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<td>1.00</td>
<td>1.00</td>
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<td>1.00</td>
</tr>
<tr>
<td>1.50 in.</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
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<td>1.00</td>
</tr>
<tr>
<td>2.00 in.</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>2.50 in.</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>3.00 in.</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

*Note: All values are in standard US customary units.*
Design Data: 24F-1.8E southern pine 3.0 x 17.875

- $C_D = 1.15$ for snow load  
  $C_c = NA$
- $C_M = 1.0$  
  $C_f = NA$
- $C_t = 1.0$  
  $C_H = 1.0$
- $C_L = ?$ (to be determined)  
  $C_b = NA$
- $C_V = ?$ (to be determined)  
  $C_T = NA$
- $C_{fu} = NA$
- $C_i = NA$
- $C_r = NA$

• **Volume Factor, $C_V$**

  $$C_V = K_L \left( \frac{21}{L} \right)^{\frac{1}{20}} \left( \frac{12}{d} \right)^{\frac{1}{20}} \left( \frac{5.125}{b} \right)^{\frac{1}{20}} \leq 1.0$$

  $$C_V = 1.0 \left( \frac{21}{18} \right)^{\frac{1}{20}} \left( \frac{12}{17.875} \right)^{\frac{1}{20}} \left( \frac{5.125}{3.0} \right)^{\frac{1}{20}} = 1.01$$

  Use $C_V = 1.0$
• **Beam Stability Factor,** \( C_L \)

  • Effective length for 2 equal concentrated loads @ 1/3 points

  \[
  l_e = 1.68l_u = (1.68)(72 \text{ in}) = 120.96 \text{ in}
  \]

  • Slenderness Ratio, \( R_B \)

  \[
  R_B = \sqrt{\frac{l_e d}{b^2}} = \sqrt{\frac{(120.96 \text{ in})(17.875 \text{ in})}{(3.0 \text{ in})^2}}
  \]

  \[
  R_B = 15.5
  \]

• **Beam Stability Factor,** \( C_L \)

\[
C_L = 1 + \frac{F_{bE \cdot F_c \cdot F_c} \cdot F_{bE \cdot F_c \cdot F_c}}{1.9} - \sqrt{\left[1 + \frac{F_{bE \cdot F_c \cdot F_c} \cdot F_{bE \cdot F_c \cdot F_c}}{1.9}\right]^2 - \frac{F_{bE \cdot F_c \cdot F_c} \cdot F_{bE \cdot F_c \cdot F_c}}{0.95}}
\]

\[
F_{bE} = \frac{K_{bE}E'}{R_B^2}
\]

\[
F_{b \cdot} = F_b \cdot C_D \cdot C_M \cdot C_t \cdot C_F \cdot C_i \cdot C_r \cdot C_c \cdot C_f
\]
• **Beam Stability Factor,** $C_L$

\[
E' = E C_M C_L C_I C_T = 1.8 \times 10^6 \text{ psi}
\]

\[
F_{bE} = \frac{K_{bE} E'}{R_B^2} = \frac{(0.610)(1.8 \times 10^6 \text{ psi})}{(15.5)^2} = 4,570 \text{ psi}
\]

\[
F_b^* = F_b C_D = (2400 \text{ psi})(1.15) = 2760 \text{ psi}
\]

• **Beam Stability Factor,** $C_L$

\[
C_L = 0.94
\]
Check Bending Stress:

\[ f_b = \frac{M}{S} \leq F_b' \]

\[ f_b = \frac{(28,800 \text{ lb.ft})(12 \text{ in.})}{(159.8 \text{ in}^3)} \]

\[ f_b = 2163 \text{ psi} \]

Check Bending Stress:

\[ F_b' = F_b C_D C_L \]

\[ F_b' = (2400 \text{ psi})(1.15)(0.94) \]

\[ F_b' = 2594 \text{ psi} \]
• Check Bending Stress:

\[ f_b = 2263 \text{ psi} < F'_b = 2594 \text{ psi} \]

**Conclusion:** beam size is acceptable in bending stress