Establishment of a Sliding Mode in a Nonlinear System by Tuning the Parameters of a Fuzzy Controller

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Abstract: In this paper, a novel method for the establishment of a sliding mode in a nonlinear system is presented. The method discussed aims to minimize a cost measure, which is a function of the switching surface. For this purpose, an adaptive fuzzy controller is selected and the parameters of the defuzzifier are adjusted such that the cost is minimized and the system is enforced to behave in sliding mode. The paper considers a 2-DOF direct drive robotic manipulator as the test bed and a standard fuzzy system is used as the controller. The results obtained clearly stipulate that a sliding motion can be achieved by appropriately tuning the parameters of the controller.

I. INTRODUCTION

Systems having structural uncertainties or a known complicated structure are difficult to control. Modeling of the uncertainties or handling the deterministic complexity are typical problems frequently encountered in the field of systems and control engineering. From this point of view, the designer is generally in pursuit of best utilization of what is known about the system in hand. Sometimes the knowledge about the system is best expressed in words. If the response of the system can be expressed in certain regions of the input space, the designer can come up with some IF-THEN statements, which constitute the basis of the fuzzy inference systems.

Fuzzy Inference Systems are the most popular constituent of the computational intelligence area because of their ability to represent human expertise in the form of *IF antecedent THEN consequent* statements. In this domain, the system behavior is modeled through the use of linguistic descriptions. Although the earliest work by Prof. Zadeh on fuzzy systems was not paid as much attention as it deserved in early 1960s, since then the methodology has become a well-developed framework. The typical architectures of fuzzy inference systems are those introduced by Wang [1-2], Takagi and Sugeno [3], and Jang [4]. In [1], a fuzzy system having Gaussian membership functions, product inference rule and weighted average defuzzifier is constructed and has become the standard method in most applications. Takagi and Sugeno change the defuzzification procedure where dynamic systems are used for this purpose. The potential advantage of the method is that, under certain constraints, the stability of the system can be studied. Jang *et al* [4] propose an Adaptive Neuro Fuzzy Inference System (ANFIS), in which polynomials are used in the defuzzification stage. This structure is commonly seen in the related literature. The choice concerning the order of the polynomials and the variables to be used in the defuzzifier are left to the designer.

In control engineering practice, stability and robustness are of crucial importance. Because of this, the implementation-oriented control engineering expert has always been in pursuit of a design, which provide accuracy as well as insensitivity to environmental disturbances and structural uncertainties. At this point, it must be emphasized that these ambiguities can never be modeled accurately. When the designer tries to minimize the ambiguities by the use of a detailed model, then the design becomes so tedious that its cost increases dramatically. A suitable way of tackling with uncertainties without the use of complicated models is to introduce Variable Structure Systems (VSS) theory based components into the system structure.

Variable Structure Control (VSC) has successfully been applied to a wide variety of systems having uncertainties in the representative system models. The philosophy of the control strategy is simple, being based on two goals. First, the system is forced towards a desired dynamics, second, the system is maintained on that differential geometry. In the literature, the former dynamics is named the reaching mode, while the latter is called the sliding mode. The control strategy borrows its name from the latter dynamic behavior, and is called *Sliding Mode Control (SMC)*.

Earliest notion of SMC strategy was constructed on a second order system in the late 1960s by Emelyanov [5]. The work stipulated that a special line could be defined on the phase plane, such that any initial state vector could be driven towards the plane and then be maintained on it, while forcing the error dynamics towards the origin. Since then, the theory has greatly been improved and the sliding line has taken the form of a multidimensional surface, called the *sliding surface* around which a switching control action takes place.

Numerous contributions to VSS theory have been made during the last decade, some of them are as follows: Hung *et al* [6] has reviewed the control strategy for linear and nonlinear systems. In [6], the switching schemes putting the differential equations into canonical forms and generating simple SMC based controls are considered in detail. Gao *et al* [7], apply the SMC scheme to robotic manipulators and discuss the quality of the scheme. The latest studies

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consider this robustness property by equipping the system with computationally intelligent methods. In [8] and [9], fuzzy inference systems are proposed for SMC scheme. A standard fuzzy system is studied and the relevant robustness analyses are carried out. Particularly, the work presented in [8] emphasizes that the robustness and stability properties of intelligent control strategies can be studied through the use SMC theory. It is shown in the paper in this way that the approach is robust i. e. it can compensate the deficiencies caused by poor modeling of plant dynamics and external disturbances.

In [10-11], it is demonstrated that the theory of VSS can well be used for the purpose of learning. The method discussed is based on the stabilization of gradient based training strategies with the aim of reducing the parametric change effort.

The method discussed in this paper is first proposed by Sira-Ramirez and Colina-Morles for learning in adaptive linear elements (ADALINE) [12]. The paper gives the example of an inverse dynamics identification of a Kapitsa pendulum with a single ADALINE. Yu *et al* discuss the same algorithm for ADALINE with the improvement on uncertainty bound adaptation [13]. The strategy adopted is based on the adaptive adjustment of uncertainty bounds. The approach presented in this paper is similar to what is discussed in [12] and [13] except the extension is specific to models having linear combiners at the output and used particularly for sliding mode control purposes.

This paper is organized as follows: Second section describes the fuzzy system used in the study, the following section presents the derivation of the learning algorithm. In the fourth section how the SMC design is performed is discussed. The next section describes the plant used as the test bed. In the sixth section simulation results are presented. Conclusions constitute the last part of the paper.

2. STANDARD FUZZY SYSTEMS

This section considers the standard fuzzy system approach introduced in [1-2] as the computationally intelligent architecture. The architecture utilized in this study uses bell shaped membership functions as described by (1).

$$\mu_{ij}(u_j) = \frac{1}{1 + \left| \frac{u_j - c_{ij}}{a_{ij}} \right|^{2b_{ij}}} \tag{1}$$

In above, c_{ij} defines the center of the membership function, a_{ij} and b_{ij} characterize the slope and flatness of the function respectively. The structure of the fuzzy system is illustrated in Fig. 1, for which the following type of a rule base structure is adopted.

IF u_1 *is* U_1 *AND* u_2 *is* U_2 *AND ... AND* u_m *is* U_m
THEN $F = y_i$

In the *IF* part of this representation, the lowercase variables denote the inputs and the uppercase variables stand for the fuzzy sets corresponding to the domain of each linguistic label. *THEN* part is comprised of the prescribed decision in the form of a scalar number.

The overall realization performed by the system considered is given in (2), where weighted average defuzzifier is used with algebraic product aggregation method.

$$F = \frac{\sum_{i=1}^{R} y_i \prod_{j=1}^{m} \mu_{ij}(u_j)}{\sum_{i=1}^{R} \prod_{j=1}^{m} \mu_{ij}(u_j)} = \sum_{i=1}^{R} y_i w_{ni} \tag{2}$$

$$w_{ni} = \frac{\prod_{j=1}^{m} \mu_{ij}(u_j)}{\sum_{k=1}^{R} \prod_{j=1}^{m} \mu_{kj}(u_j)} \tag{3}$$

In above, R is the number of rules contained in the rule base and m is the number of inputs. If the vector of firing strengths denoted by w is normalized and the resulting vector is represented by w_n, the ith entry of this vector can be expressed as given in (3). In the application example discussed, parameter tuning is performed on the y variables of the fuzzy system.

3. DERIVATION OF THE LEARNING ALGORITHM

In this section, it is assumed that the physical constraints on the controller outputs put a bound on adjustable parameter magnitudes ($y<B_y$), time derivative of the normalized firing strengths ($\dot{w}_n<B_{\dot{w}n}$) and the time derivative of the desired output of the fuzzy controller ($\dot{F}_d<B_{\dot{F}d}$).

If one defines the adjustable parameter vector as given in (4), defining the error at the output of the controller as in (5), the Lyapunov function in (6) could be a suitable function for describing the learning performance. The time derivative of the function is as given by (7).

$$y = [y_1 \quad y_2 \quad \dots \quad y_R]^T \tag{4}$$

$$e_c = F - F_d \tag{5}$$

$$V = \frac{1}{2} e_c^2 \tag{6}$$

$$\dot{V} = \dot{e}_c e_c \tag{7}$$

where

$$\dot{e}_c = \dot{F} - \dot{F}_d \tag{8}$$

In order to evaluate the expression in (7), we have the following relations:

$$F = y^T w_n \tag{9}$$

$$\dot{F} = \dot{y}^T w_n + y^T \dot{w}_n \tag{10}$$

Theorem: If the dynamic defuzzification strategy is adopted as described in (11), the learning strategy becomes stable in the sense of Lyapunov [14].

$$\dot{y} = -\frac{w_n}{w_n^T w_n} k sign(e_c) \tag{11}$$

Proof: If (11) is substituted into (7) with the aid of (5), (8), (9) and (10), the error dynamics in (12) is obtained. Using the bounds of the uncertainties mentioned at the beginning of the section leads to (13).

$$\dot{e}_c = -k sign(e_c) + y^T \dot{w}_n - \dot{F}_d \tag{12}$$

$$\begin{aligned} \dot{V} &= -k|e_c| + \left(y^T \dot{w}_n - \dot{F}_d\right) e_c \\ &< \left(B_y B_{\dot{w}_n} + B_{\dot{F}_d} - k\right)|e_c| \end{aligned} \tag{13}$$

In order to have a negative time derivative for the Lyapunov function in (6), the parameter k must satisfy the following relation.

$$k > B_y B_{\dot{w}_n} + B_{\dot{F}_d} \tag{14}$$

4. SLIDING MODE CONTROL DESIGN

The analysis presented in the previous section aims to minimize the Lyapunov function in (6), which is an instantaneous cost measure used in most neuro-fuzzy control strategies. It is apparent that the use of the presented analysis in control applications entails the desired values of the controller outputs. Therefore, for the applications in which the desired signals are available, the method can easily be used without any modification.

In this part, parallel to the philosophy of variable structure controller design procedure, a switching function is defined and described by (15). The symbol e seen in (15) is the discrepancy between the reference state value and observed state value.

$$s = \dot{e} + \lambda e \tag{15}$$

If one replaces e_c of (11) with s of (15), it is straightforward to prove that the Lyapunov function in (16) is minimized in time and its time derivative is enforced to have negative values due to the adjustment strategy in (11).

$$V = \frac{1}{2} s^2 \tag{6}$$

For this case, the selection of k values must be reasonably large for compensating the bounds introduced with the new selection. The details of the analysis are not included due to the space limit.

5. PLANT MODEL

In the simulations the dynamic model of a two degrees of freedom direct drive robotic manipulator, which is illustrated in Fig. 2, is used as the test bed. Since the dynamics of such a mechatronic system is modeled by nonlinear and coupled differential equations, precise output tracking becomes a difficult objective due to the strong interdependency between the variables involved. Besides, the ambiguities on the friction related dynamics in the plant model make the design much more complicated. Therefore the methodology adopted must be intelligent in some sense. The general form of robot dynamics is described by (17) where $M(\theta)$, $V(\theta,\dot{\theta})$, τ and f_c stand for the state varying inertia matrix, vector of Coriolis terms, applied torque inputs and friction terms respectively. The plant parameters are given in Table 1 in standard units.

$$M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) = \tau - f_c \tag{17}$$

If the angular positions and angular velocities are described as the state variables of the system, four coupled and first order differential equations can define the model. In (18) and (19), the terms seen in (17) are given explicitly.

$$M(\theta) = \begin{bmatrix} p_1 + 2p_3 cos(\theta_2) & p_2 + p_3 cos(\theta_2) \\ p_2 + p_3 cos(\theta_2) & p_2 \end{bmatrix} \tag{18}$$

$$V(\theta,\dot{\theta}) = \begin{bmatrix} -\dot{\theta}_2\left(2\dot{\theta}_1 + \dot{\theta}_2\right) p_3 \sin(\theta_2) \\ \dot{\theta}_1^2 p_3 \sin(\theta_2) \end{bmatrix} \tag{19}$$

In above, $p_1 = 2.0857+0.0576M_p$, $p_2 = 0.1168+0.0576M_p$ and $p_3 = 0.1630+0.0862M_p$. Here M_p denotes the payload mass. The details of the plant model are presented in [15].

6. SIMULATION RESULTS

In the simulation studies presented, the plant introduced in the fifth section is controlled by the proposed control scheme. The aim is to produce some torque signals that create a sliding motion in the phase plane for each link. As the controller, the architecture discussed in the second section is adopted with nine rules (R=9) and two inputs (m=2) for each link. The structure of the control system is as illustrated in Fig. 3, in which the plant is in an ordinary feedback loop. The membership functions of the controller have been selected as that illustrated in Fig.4. Based on the tracking error vector, first the value of $s(e,\dot{e})$ is evaluated and this quantity is applied to the adjustment mechanism. In evaluating the value of the quantity s, the slope of the switching surface (λ) has been set to 0.2.

In practical implementations a number of difficulties are encountered, which make it difficult to achieve an accurate trajectory tracking. The simulation studies carried out address these difficulties. The first difficulty to be alleviated is the varying payload conditions, which introduces abrupt changes in the dynamics of the system under control. As the reference trajectory illustrated in Fig. 5. implies, the motion starts with no payload. At time t=2 sec, a payload of 2 kg is grasped and released at time t=5 sec. The same variation is repeated at time t=9 sec and t=12 sec. After the time t=15, the manipulator is kept motionless. The second difficulty is the existence of observation noise. The information used by the controller is corrupted by a Gaussian distributed random noise having zero mean and variance equal to 33e-6. The peak magnitude of the noise signal is within ±1 with probability very close to unity. The third difficulty is the nonzero positional initial conditions. In order to demonstrate the reaching mode performance of the algorithm, the first link is moved to $\pi/50$ radians and the second link is moved to $-\pi/50$ radians initially.

Under these conditions, the state tracking error graph in Fig. 6 is obtained. The trend in position and velocity errors clearly stipulate that the algorithm is able to achieve precise tracking objective. The motion in the phase plane is illustrated in the top row of Fig. 7. The upper subplots of the figure show that for both links, after a fast reaching mode, a sliding mode is enforced and is maintained by producing a suitable control signal. In the bottom row of the figure, it is shown for both links that the Lyapunov function in (16) is minimized. In order to show the minimization activity of the algorithm presented, the vertical axes for these subplots are selected as logarithmic. It is seen that some small magnitude spikes occur in time and they are dampened out quickly. We relate these spikes to the difficulties stated above. What should be emphasized as a last point is the torque signal produced by the controller. As seen in Fig. 8, the controller outputs are directly applied to the manipulator without exceeding the limits of the applicable control ranges. The control signal has a smooth characteristic, which does not violate the potential limits of the actuators.

During the simulations k_1, k_2 and the sampling rate have been set to 10000, 1000 and 2.5 msec respectively. Furthermore, in order to reduce the chattering effect in the sliding mode, the function in (20) has been used instead of the sign function in the dynamic defuzzification strategy described in (11).

$$sign(e_c) \approx \frac{e_c}{|e_c| + 0.05} \qquad (20)$$

7. CONCLUSIONS

In this paper, a novel method for establishing a sliding motion in the phase plane of a nonlinear plant is discussed. The method is based on the adoption of a dynamic defuzzification strategy in a fuzzy controller having the standard architecture. It is seen that the algorithm discussed is able to compensate deficiencies caused by the imperfect observations of the state variables, abruptly changing plant dynamics, initial condition errors and complex plant dynamics. From these points of view, the method proposed is highly promising for control purposes.

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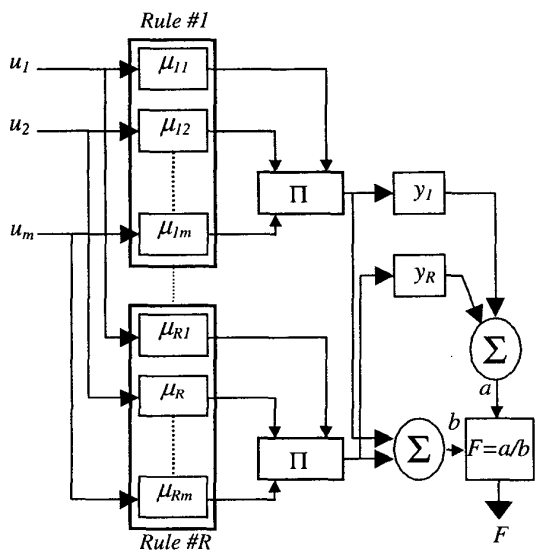

Figure 1. Architecture of the fuzzy system

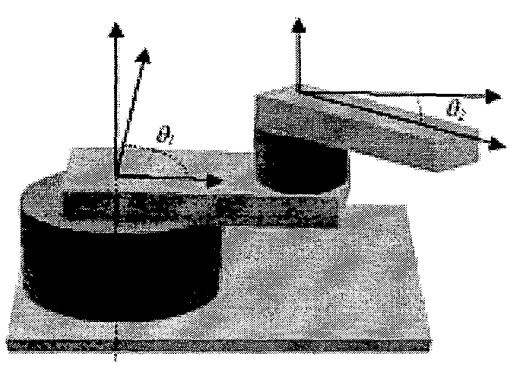

Figure 2. Physical structure of the manipulator

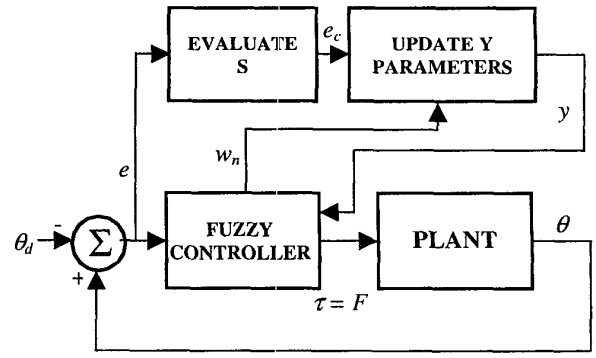

Figure 3. Structure of the control system

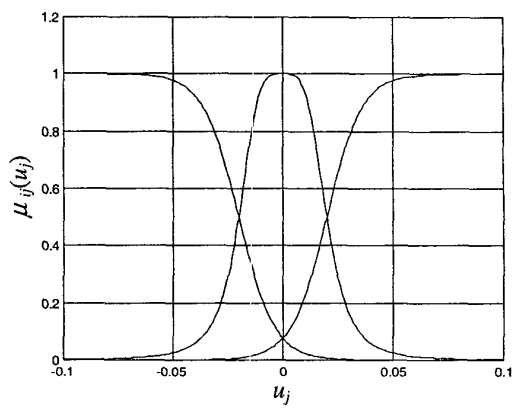

Figure 4. Membership functions used in the simulations

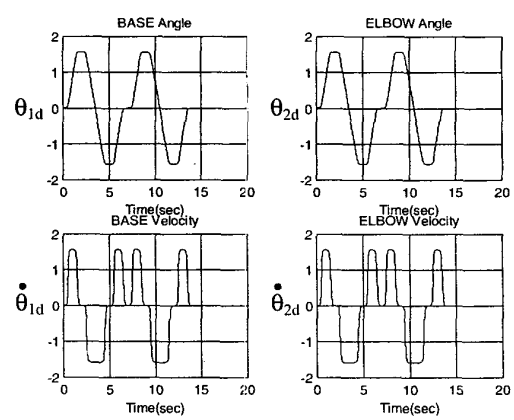

Figure 5. Reference state trajectories

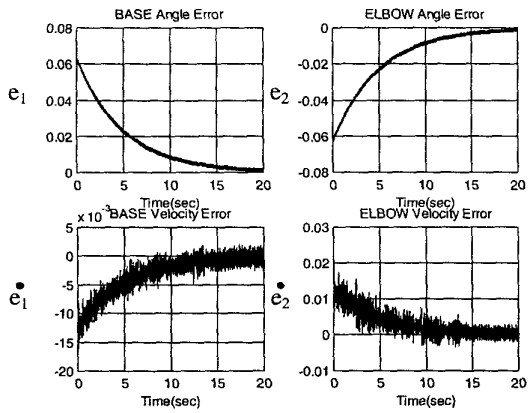

Figure 6. State tracking errors

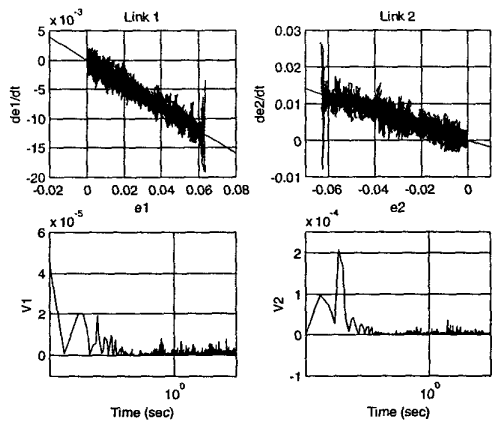

Figure 7. Motion in phase plane for each link (top row) and the behavior of the Lyapunov function in time (bottom row)

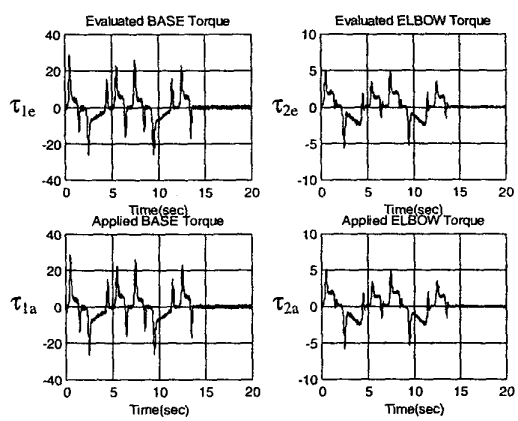

Figure 8. Evaluated and applied torque signals

Table 1. Manipulator Parameters

Motor 1 Rotor Inertia	0.2670	I_1
Arm 1 Inertia	0.3340	I_2
Motor 2 Rotor Inertia	0.0075	I_3
Motor 2 Stator Inertia	0.0400	I_{3C}
Arm 2 Inertia	0.0630	I_4
Motor 1 Mass	73.000	M_1
Arm 1 Mass	9.7800	M_2
Motor 2 Mass	14.000	M_3
Arm 2 Mass	4.4500	M_4
Arm 1 Length	0.3590	L_1
Arm 2 Length	0.2400	L_2
Arm 1 Center of Gravity	0.1360	L_3
Arm 2 Center of Gravity	0.1020	L_4
Axis 1 Friction	5.3000	f_{c1}
Axis 2 Friction	1.1000	f_{c2}
Torque Limit 1	245.00	τ_{1max}
Torque Limit 2	39.200	τ_{2max}