Non-Linear Analog Circuit Test and Diagnosis under Process Variation using V-Transform Coefficients

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May 2, 2011
Outline

1. Motivation
2. Coefficient Based Test
3. Fault Classification
4. Results
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4. Results
Fault Classification

Motivation

1. Semiconductor processes at advanced nodes are subject to random variability
   - Poly/thin film resistors - line edge roughness ($\sigma \approx 15\%\mu$)
   - Capacitors - Oxide thickness fluctuation & line edge roughness ($\sigma \approx 20\%\mu$)
## Motivation

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Fault Classification

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   - Poly/thin film resistors - line edge roughness ($\sigma \approx 15\% \mu$)
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2. Faults due to variability can mask or exacerbate failure from conventional defect mechanisms.
   - Dust contamination, Processing Equipment, Material impurity, Clean room contamination, Operator imperfection, etc., (Fault sizes $\mu_{dev} > 50\%$)

3. Distinguishing failure mechanisms between process variation (PV) and conventional ones can possibly help improve yield.
Types Of Faults

- Manufacturing Defect
- Random Variation
Ideal Test For An Analog Circuit

Wish list for an analog circuit test scheme

- Suitable for large class of circuits
- Detects sufficiently small parametric faults – high sensitivity
- Low design complexity of the input signal
- Small area overhead – requires little circuit augmentation
- Large number observables – handy in diagnosis
Motivation Coefficient Based Test Fault Classification Results

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- Suitable for large class of circuits
- Detects sufficiently small parametric faults – high sensitivity
- Low design complexity of the input signal
- Small area overhead – requires little circuit augmentation
- Large number observables – handy in diagnosis
- Aids distinction of small defects from process variation (PV) induced faults – need in advanced tech nodes
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Cascaded Amplifiers – An Example

Two stage amplifier with $4^{th}$ degree non-linearity in $V_{in}$

$$V_{out} = c_0 + c_1 V_{in} + c_2 V_{in}^2 + c_3 V_{in}^3 + c_4 V_{in}^4$$

Polynomial Coefficients

\[ c_0 = V_{DD} - R_2 K \left( \frac{W}{L} \right)_2 \left[ (V_{DD} - V_T)^2 + R_1^2 K^2 \left( \frac{W}{L} \right)_1 V_T^4 \right. \]

\[ -2(V_{DD} - V_T)R_1 \left( \frac{W}{L} \right)_1 V_T^2 \]

\[ c_1 = R_2 K \left( \frac{W}{L} \right)_2 \left[ 4R_1^2 K^2 \left( \frac{W}{L} \right)_1 V_T^3 + 2(V_{DD} - V_T)R_1 K \left( \frac{W}{L} \right)_1 V_T \right] \]

\[ c_2 = R_2 K \left( \frac{W}{L} \right)_2 \left[ 2(V_{DD} - V_T)R_1 K \left( \frac{W}{L} \right)_1 - 6R_1^2 K^2 \left( \frac{W}{L} \right)_1 V_T^2 \right] \]

\[ c_3 = 4V_T K^3 \left( \frac{W}{L} \right)_1 \left( \frac{W}{L} \right)_2 R_1^2 R_2 \]

\[ c_4 = -K^3 \left( \frac{W}{L} \right)_1 \left( \frac{W}{L} \right)_2 R_1^2 R_2 \]
**V-Transform**

**Definition**

\[ V_{C_i} = e^{\gamma C'_i} \quad \forall \ 0 \leq i \leq n \]

\[ \frac{dC'_i}{dp_j} = \left| \frac{dC_i}{dp_j} \right| \quad \forall \ 0 \leq i \leq n \]

- \( C_i \) – i\textsuperscript{th} polynomial coefficient
- \( C'_i \) – i\textsuperscript{th} modified polynomial coefficient
- \( V_{C_i} \) – i\textsuperscript{th} V-Transform coefficient
V-Transform Coefficient – Sensitivity Gain

Sensitivity of coefficients

\[
\frac{S_{p_i}^{V_{C_i}}}{S_{p_i}^{C_i}} = \frac{\left| \frac{dC_i}{dp_i} \right| \gamma e^{\gamma C_i'} \bullet \frac{p_i}{e^{\gamma C_i'}}}{\frac{dC_i}{dp_i} \bullet \frac{p_i}{C_i}} = \gamma C_i
\]

\(\gamma C_i\) – Increased sensitivity over ordinary polynomial coefficients

\(\gamma\) – Sensitivity parameter that can be chosen according to the desired degree of sensitivity
Test Setup

\[ f(\cdot) \]
\[ v_{in} \]
\[ v_{out} \]
\[ \text{Circuit Under Test} \]
\[ \text{Estimate Polynomial Coefficients} \]
\[ a_0 - a_N \]
\[ V_{CO} - V_{CN} \]

- \( v_{ac} \)
- \( V_{bias} \)
- Variable Frequency
- Variable Offset

Motivation Coefficient Based Test Fault Classification Results
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\[ C \leq_{\mathcal{H}_1} C_{th} \]

\( \mathcal{H}_1 \): Fault likely due to manufacturing defect
\( \mathcal{H}_2 \): Fault likely due to process parameter variation
Fault Classification

Summary of steps

- Probability density function of the coefficients are computed by Monte Carlo simulations for fault-free circuits.
- Probability density function of the coefficients are computed by Monte Carlo simulations for faulty circuits.
- Threshold values of coefficients – Boundaries between process variation (PV) and manufacturing defects is estimated for each frequency.
- Confidence of classifying a fault as PV or manufacturing defect is improved by observing one or more coefficients at multiple frequencies.
If $P_i$ is the probability of coefficient being outside its permissible interval due to process variation, then we define confidence in diagnosing CUT to be faulty due to PV, $C$ (N is the total number of coefficients).

$$C = \frac{1}{\prod_{i=1}^{i=N} (1 - P_i)}$$
Results – Benchmark Elliptic Filter
Results - V-Transform Coefficients

\[ V_{C_5} = 1.0402 \quad V_{C_4} = 1.6572 \quad V_{C_3} = 8.4224 \quad V_{C_2} = 12.7904 \quad V_{C_1} = 33.0492 \quad V_{C_0} = 93.1396 \]
Results at DC - Elliptic Filter

Parameter combinations leading to max values of V-Transform coefficients with $\alpha = 0.05$

<table>
<thead>
<tr>
<th>Circuit Parameter, (ohm)</th>
<th>$V_{c0}$</th>
<th>$V_{c1}$</th>
<th>$V_{c2}$</th>
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<th>$V_{c4}$</th>
<th>$V_{c5}$</th>
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<tbody>
<tr>
<td>$R_1 = 19.6k$</td>
<td>18.6k</td>
<td>20.5k</td>
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<tr>
<td>$R_2 = 196k$</td>
<td>186k</td>
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<td>186k</td>
<td>186k</td>
<td>186k</td>
<td>205k</td>
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<tr>
<td>$R_3 = 147k$</td>
<td>139k</td>
<td>154k</td>
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<tr>
<td>$R_4 = 1k$</td>
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<td>1010</td>
<td>1010</td>
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<tr>
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Results at DC - Elliptic Filter

Parameter combinations leading to min values of V-Transform coefficients with $\alpha = 0.05$

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## Results at DC - Elliptic Filter

### Fault detection for some injected faults

<table>
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<tr>
<th>Circuit Parameter</th>
<th>Out of bound polynomial coefficient</th>
<th>Fault detected?</th>
<th>Out of bound V-Transform coefficient</th>
<th>Fault detected?</th>
</tr>
</thead>
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<tr>
<td>R&lt;sub&gt;1&lt;/sub&gt; down 25%</td>
<td>c&lt;sub&gt;3&lt;/sub&gt;, c&lt;sub&gt;4&lt;/sub&gt;</td>
<td>Yes</td>
<td>V&lt;sub&gt;c&lt;sub&gt;0&lt;/sub&gt;&lt;/sub&gt; – V&lt;sub&gt;c&lt;sub&gt;4&lt;/sub&gt;&lt;/sub&gt;</td>
<td>Yes</td>
</tr>
<tr>
<td>R&lt;sub&gt;2&lt;/sub&gt; down 30%</td>
<td>c&lt;sub&gt;2&lt;/sub&gt;</td>
<td>Yes</td>
<td>V&lt;sub&gt;c&lt;sub&gt;2&lt;/sub&gt;&lt;/sub&gt;, V&lt;sub&gt;c&lt;sub&gt;5&lt;/sub&gt;&lt;/sub&gt;</td>
<td>Yes</td>
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<td>R&lt;sub&gt;3&lt;/sub&gt; up 25%</td>
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<tr>
<td>R&lt;sub&gt;7&lt;/sub&gt; up 10%</td>
<td>None</td>
<td>PV (C = 200)</td>
<td>V&lt;sub&gt;c&lt;sub&gt;1&lt;/sub&gt;&lt;/sub&gt;, V&lt;sub&gt;c&lt;sub&gt;2&lt;/sub&gt;&lt;/sub&gt;</td>
<td>Yes</td>
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<tr>
<td>R&lt;sub&gt;11&lt;/sub&gt; up 15%</td>
<td>None</td>
<td>PV (C = 120)</td>
<td>V&lt;sub&gt;c&lt;sub&gt;4&lt;/sub&gt;&lt;/sub&gt;, V&lt;sub&gt;c&lt;sub&gt;5&lt;/sub&gt;&lt;/sub&gt;</td>
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<tr>
<td>R&lt;sub&gt;12&lt;/sub&gt; down 15%</td>
<td>None</td>
<td>PV (C = 90)</td>
<td>V&lt;sub&gt;c&lt;sub&gt;4&lt;/sub&gt;&lt;/sub&gt;, V&lt;sub&gt;c&lt;sub&gt;5&lt;/sub&gt;&lt;/sub&gt;</td>
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- Technique for parametric fault detection in analog circuits – faults as small as 25% were uncovered for an elliptic filter example.
- Addressed parametric fault distinction between process variation induced faults.
- Enhanced technique for uncovering parametric faults by increasing sensitivity of polynomial coefficients to circuit parameters.

Future work

- Technique for optimal choice of frequencies at which CUT ought to be excited
- Optimal order of polynomial expansion as a tradeoff between test time and diagnostic resolution
- Algorithms to predict/map RF & other circuit specifications to polynomial/V-Transform coefficients
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