Parametric Fault Diagnosis of Nonlinear Analog Circuits using Polynomial Coefficients

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Abstract

We propose a method for diagnosis of parametric faults in analog circuits using polynomial coefficients of the circuit model [15]. As a sequel to our recent work [14], where circuit response is modeled as polynomial for uncovering parametric faults in nonlinear circuits, we propose diagnosis of such faults using sensitivity of coefficients of the estimated polynomial to circuit parameters. The proposed method requires no design for test hardware as might be added to the circuit by some other methods. The proposed method is illustrated for a benchmark elliptic filter. It is shown to uncover several parametric faults causing deviations as small as 5% from the nominal values.

1. Introduction

Several methods have been proposed for parametric fault testing of analog circuits [1, 2, 3, 4, 5, 13]. A prominent method in the industry is the $\mathrm{I_{DDQ}}$ testing where quiescent current from the supply rail is monitored and sizable deviations from its expected value are used for classifying the circuit as faulty or good. However, this requires augmentation of the CUT. For example, in the simplest case a regulator supplying power to any sizable circuit has to be augmented with a current sensing resistor and an ADC (for digital output). Subsequently, analysis is performed on the sensed current. $\mathrm{I_{DDQ}}$ is found suitable only for catastrophic faults as the current drawn from the supply may be distinguishable when there is some "large enough" fault to change the quiescent current by a distinguishable amount. For example, with resistor $\mathrm{R_2}$ being open in Figure 1, the current drawn from supply can change by 50% of its nominal quiescent value. Such faults can typically be found by monitoring $\mathrm{I_{DDQ}}$ using a current sensor. However, parametric deviations, say, less than 10% from their nominal value cannot be observed using this scheme. This is especially so for the very deep submicron circuits where the leakage currents can be comparable to the defect induced current [6]. It is therefore useful to develop a method to detect parametric faults while testing with little or no circuit augmentation.

To address the issue of parametric deviation, we would typically need more observables to have an idea about the parametric drift in circuit parameters. This would mean an increase in the complexity of the sensing circuit. However, we would also want minimal augmentation to tap any of the internal circuit nodes or currents. To overcome these seemingly contrasting requirements the method intended should have some way of "seeing through" the circuit with only the outputs and inputs at its disposal. References [7, 11] give such strategies for linear circuits as described earlier.

To extend this idea to general non-linear circuits, while it is also remains applicable to linear circuits, we adopt a strategy, where in, we express the function of the circuit as a polynomial using a Taylor series expansion [10] in terms of input voltage v_{in}, about the point v_{in} = 0 as follows:

$$v_{out} = f(v_{in}) = f(0) + \frac{f'(0)}{1!}v_{in} + \frac{f''(0)}{2!}v_{in}^2 + \frac{f'''(0)}{3!}v_{in}^3 + \cdots + \frac{f^{(n)}(0)}{n!}v_{in}^n + \cdots \tag{1}$$

where $f(x)$ is a real function of x. Note that in the above expansion the point of expansion of v_{in} can be about any operating voltage of the circuit at desired frequencies.

This method is very general as any analog circuit can be tested using this model. The technique applies equally well to linear circuits, which are a subclass of the general non-linear circuits considered in this paper. The accuracy, resolution and observability of faults uncovered depends on the degree of expansion of the coefficients in (2). Ignoring the higher order terms in (1), we can expand v_{out} up to the n^{th} power of v_{in}, which gives us the approximation in

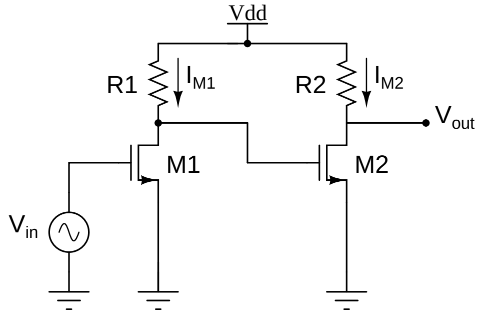

Figure 1. Cascade amplifier

(2). In order to increase the available observables to better track down parametric faults we can expand v_{out} at *multiple frequencies*. Thus, we will have $m \times (n+1)$ observables where m is the number of tones (frequencies) including DC at which v_{out} is expanded and n is the degree of expansion [8]:

$$v_{out} = a_0 + a_1 v_{in} + a_2 v_{in}^2 + \cdots + a_n v_{in}^n \tag{2}$$

where $a_0, a_1, a_2, \ldots, a_n$ are all real functions of circuit parameters $p_k \forall\, k$.
The special case of DC test, that detects a subset of faults, was given in our recent paper [15]. Further, we assume that normal parameter variations (normal drift) in a good circuit are within a fraction α of their nominal value, where $\alpha << 1$. That is, every parameter p_i is allowed to vary within the range $p_{k,nom}(1-\alpha) < p_k < p_{k,nom}(1+\alpha)\ \forall k$, where $p_{k,nom}$ is the nominal value of parameter p_k. Whenever one or more of the coefficient values slip outside its individual hypercube we get a different set of coefficients reflecting a detectable fault. Therefore, equation (3) describes the hypercube for all parameters that correspond to either good machine values or undetectable parametric faults [7, 2, 13]:

$$a_{i,\min} < a_i < a_{i,\max} \quad \forall\, i, \quad 0 \le i \le n \tag{3}$$

This paper is organized as follows. In Section 2, we describe the problem at hand and discuss the proposed solution with an example. In section 3, we generalize the solution to an arbitrarily large circuit. Section 4 presents the simulation results for some standard circuits. Section 5 describes fault diagnosis based on sensitivity of polynomial coefficients to circuit parameters and we conclude in section 6.

2. Problem Description and Sketch of Solution

We shall first give an illustrative example of calculation of limits for polynomial coefficients for a simple circuit using MOS transistors. We shall follow this up with MSDF values for the circuit parameters.

Example. Two stage amplifier
Consider the cascade amplifier shown in Figure 1. The output voltage $\mathrm{V_{out}}$ in terms of input voltage results in a fourth degree polynomial equation as follows:

$$v_{out} = a_0 + a_1 v_{in} + a_2 v_{in}^2 + a_3 v_{in}^3 + a_4 v_{in}^4 \tag{4}$$

where the constants a_0, a_1, a_2, a_3 are defined symbolically in (5) for M1 and M2 operating in saturation region. Nominal values of $\mathrm{V_{DD}}$=1.2V, $\mathrm{V_T}$= 400mV, $\left(\frac{W}{L}\right)_1 = \frac{1}{2}\left(\frac{W}{L}\right)_2 = 20$, and $\mathrm{K} = 100\mu\mathrm{A/V^2}$ are substituted to get coefficients in terms of parameters $\mathrm{R_1}$ and $\mathrm{R_2}$ as given by (6).

$$\begin{aligned}
a_0 &= V_{DD} - R_2 K \left(\tfrac{W}{L}\right)_2 \left\{ \begin{array}{l} (V_{DD}-V_T)^2 + \\ R_1^2 K^2 \left(\frac{W}{L}\right)_1^2 V_T^4 - \\ 2(V_{DD}-V_T) R_1 \left(\frac{W}{L}\right)_1 V_T^2 \end{array} \right\} \\
a_1 &= R_2 K \left(\tfrac{W}{L}\right)_2 \left\{ \begin{array}{l} 4R_1^2 K^2 \left(\frac{W}{L}\right)_1^2 V_T^3 \\ +2(V_{DD}-V_T) R_1 K \left(\frac{W}{L}\right)_1 V_T \end{array} \right\} \\
a_2 &= R_2 K \left(\tfrac{W}{L}\right)_2 \left\{ \begin{array}{l} 2(V_{DD}-V_T) R_1 K \left(\frac{W}{L}\right)_1 \\ -6R_1^2 K^2 \left(\frac{W}{L}\right)_1^2 V_T^2 \end{array} \right\} \\
a_3 &= 4V_T K^3 \left(\tfrac{W}{L}\right)_1^2 \left(\tfrac{W}{L}\right)_2^2 R_1^2 R_2 \\
a_4 &= -K^3 \left(\tfrac{W}{L}\right)_1^2 \left(\tfrac{W}{L}\right)_2^2 R_1^2 R_2
\end{aligned} \tag{5}$$

$$\begin{aligned}
a_0 &= 1.2 - R_2 \left(\begin{array}{l} 2.56 \times 10^{-3} + 1.024 \times 10^{-7} R_1^2 \\ -5.12 \times 10^{-4} R_1 \end{array} \right) \\
a_1 &= 4.096 \times 10^{-9} R_1^2 R_2 + 5.12 \times 10^{-6} R_1 R_2 \\
a_2 &= 1.28 \times 10^{-5} R_1 R_2 - 1.536 \times 10^{-8} R_1^2 R_2 \\
a_3 &= 2.56 \times 10^{-8} R_1^2 R_2 \\
a_4 &= 1.6 \times 10^{-8} R_1^2 R_2
\end{aligned} \tag{6}$$

To find the limiting values of the coefficient a_0 we assume the parameters $\mathrm{R_1}$ and $\mathrm{R_2}$ deviate by fractions x and y from their nominal values, respectively. Maximizing a_0 we have the objective function as given by (7), subject to constraints in (8–12). Note that here we have set out to find MSDF of $\mathrm{R_1}$. Similar approach can be used to find the MSDF of $\mathrm{R_2}$.

$$1.2 - R_{2,nom}(1+y) \left\{ \begin{array}{l} 2.56 \times 10^{-3} + \\ 1.024 \times 10^{-7} R_{1,nom}^2 (1+x)^2 \\ -5.12 \times 10^{-4} R_{1,nom}(1+x) \end{array} \right\} \tag{7}$$

Table 1. MSDF for cascade amplifier of Figure 1 with α = 0.05.

Circuit parameter	%upside MSDF	%downside MSDF
Resistor R_1	10.3	7.4
Resistor R_2	12.3	8.5

$$\begin{aligned} &4.096 \times 10^{-9} R_{1,nom}^2 (1+x)^2 R_{2,nom} \ (1+y) \\ &+5.12 \times 10^{-6} R_{1,nom}(1+x) R_{2,nom}(1+y) \\ &\quad = 4.096 \times 10^{-9} R_{1,nom}^2 (1+\rho)^2 R_{2,nom} \\ &\quad + 5.12 \times 10^{-6} R_{1,nom}(1+\rho) R_{2,nom} \end{aligned} \tag{8}$$

$$\begin{aligned} &1.28 \times 10^{-5} R_{1,nom}(1+x) R_{2,nom}(1+y) \\ &-1.536 \times 10^{-8} R_{1,nom}^2 (1+x)^2 R_{2,nom} \ (1+y) \\ &\quad = 1.28 \times 10^{-5} R_{1,nom}(1+\rho) R_{2,nom} \\ &\quad - 1.536 \times 10^{-8} R_{1,nom}^2 (1+\rho)^2 R_{2,nom} \end{aligned} \tag{9}$$

$$\begin{aligned} &2.56 \times 10^{-8} R_{1,nom}^2 (1+x)^2 R_{2,nom}(1+y) \\ &\quad = 2.56 \times 10^{-8} R_{1,nom}^2 (1+\rho)^2 R_{2,nom} \end{aligned} \tag{10}$$

$$\begin{aligned} &1.6 \times 10^{-8} R_{1,nom}^2 (1+x)^2 R_{2,nom}(1+y) \\ &\quad = 1.6 \times 10^{-8} R_{1,nom}^2 (1+\rho)^2 R_{2,nom} \end{aligned} \tag{11}$$

$$-\alpha \le x, y \le \alpha \tag{12}$$

The extreme values for x and y on solving the set of equations (8–12) are obtained as, x = $-\alpha$ and y = $-\alpha$, this gives us the MSDF value for R_1, as ρ in (13).

$$\rho = (1-\alpha)^{1.5} - 1 \approx 1.5\alpha - 0.375\alpha^2 \tag{13}$$

Table 1 gives the MSDF for R_1 and R_2 based on above calculation.

3. Generalization

In general, the calculation as described above cannot be done for an arbitrarily large circuit. Such circuits are handled by obtaining a nominal numeric polynomial expansion of the fault free circuit. This is done by sweeping the input voltage across all possible values and noting the corresponding output voltages using any of the standard circuit simulators like SPICE. Now, the output voltage is plotted against the input voltage. A polynomial is fitted to this curve and the coefficients of this polynomial are taken to be the nominal coefficients of the desired polynomial. The circuit is simulated for different drifts in the parameter values at equally spaced points from inside the hypercube enclosing each circuit parameter, spaced at a suitably chosen resolution(=ϵ). Polynomial coefficients are obtained for each of these simulations. The maximum and the minimum

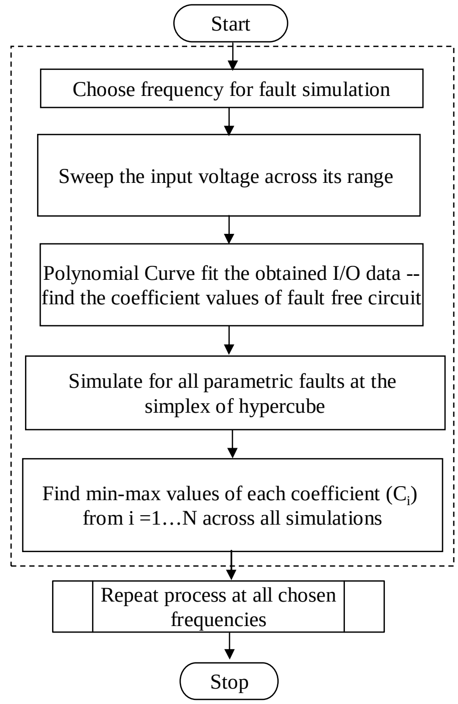

Figure 2. Flow chart showing fault simulation process and bounding of coefficients.

values of a coefficient in this search are taken as the limiting values on that coefficient. This process of modeling the circuit as a polynomial expansion and obtaining limit values on coefficients is repeated at "key" frequencies of interest. For example, the cut-off frequency in case of a non-linear filter can be a good candidate for such characterization. Once the limit values on all coefficients have been determined the CUT is subjected to full range of input at DC and each of the "key" frequencies. Its response to input sweep is curve fitted to a polynomial of order same as the fault free circuit. If there are any coefficients that lay outside the limit values of corresponding coefficients of the fault free circuit, we can conclude the CUT is faulty. The converse is also true with a high probability that is inversely proportional to coefficient of uncertainty ϵ. Flow chart in Figure 2 summarizes the process of numerically finding the polynomial and finding the bounds on coefficients. Flow chart in Figure 3 outlines the procedure to test CUT using the described method.

4. Experimental Results

We subjected an elliptic filter shown in Figure 4 to Polynomial Coefficient based test. The circuit parameter values are as in the benchmark circuit maintained by Stroud

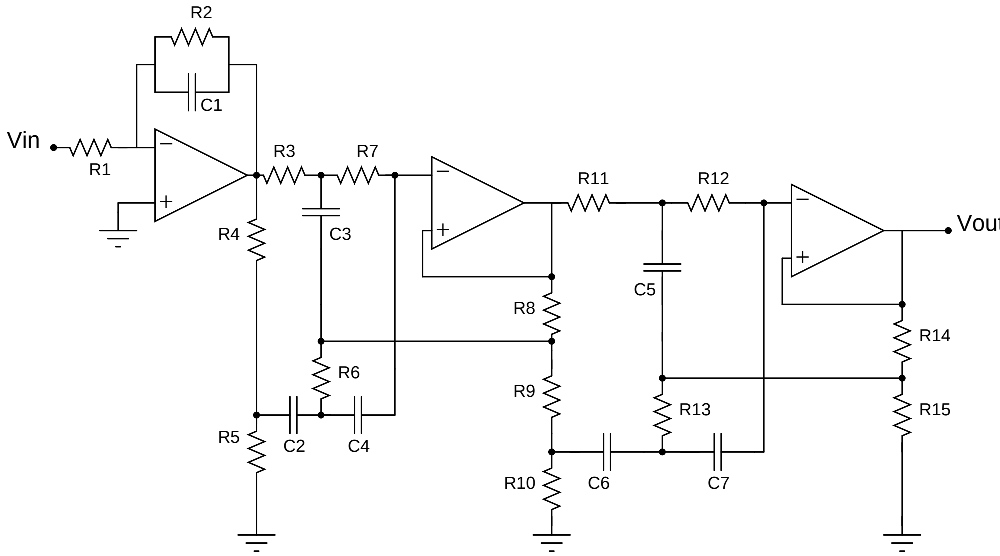

Figure 4. Elliptic filter

et al. [9]. We simulated the circuit at four different frequencies. Two of them were chosen close to its 3-dB cut-off frequency (f_c), which is 1000Hz. The estimated polynomial expansion obtained by curve fitting the I/O data at DC and the frequencies f=100Hz, 900Hz, 1000Hz, 1100Hz are given by equations 14 through 18. The plot tracing I/O response with the curve-fit polynomial at frequency of 1000Hz is shown in Figure 5. The combinations of parameter values leading to maximum values of the coefficients for the tone at 1000Hz are shown in Table 2. Parameter combinations leading to minimum values of coefficients is also found.

$$v_{out} = 4.5341 - 3.498v_{in} - 2.5487v_{in}^2 + 2.1309v_{in}^3 - 0.50514v_{in}^4 + 0.039463v_{in}^5 \quad (14)$$

$$v_{out} = 3 + 7.9v_{in} - 11v_{in}^2 + 4.4v_{in}^3 - 0.78v_{in}^4 + 0.049v_{in}^5 \quad (15)$$

$$v_{out} = 2.5 + 5.4v_{in} - 8.6v_{in}^2 + 4v_{in}^3 - 0.77v_{in}^4 + 0.054v_{in}^5 \quad (16)$$

$$v_{out} = 1.1707 + 2.4132v_{in} - 3.8777v_{in}^2 + 1.8035v_{in}^3 - 0.3465v_{in}^4 + 0.023962v_{in}^5 \quad (17)$$

$$v_{out} = 0.23 + 0.48v_{in} - 0.74v_{in}^2 + 0.34v_{in}^3 - 0.063v_{in}^4 + 0.0043v_{in}^5 \quad (18)$$

5. Fault Diagnosis

Fault diagnosis using sensitivity of output to circuit parameters has been investigated in the literature [16]. We have extended that approach exploiting the sensitivity of polynomial coefficients to circuit parameters. The advantage of the new approach is an improved fault diagnosis without circuit augmentation. Sensitivity of i^{th} coefficient C_i to k^{th} parameter p_k is represented by $S_{p_k}^{C_i}$ and is given by:

$$S_{Pk}^{Ci} = \frac{p_k}{C_i} \frac{\partial C_i}{\partial p_k} \quad (19)$$

Each of these sensitivities maps the circuit parameters to polynomial coefficients. Figure 6 shows a possible scenario.

5.1. Computation of Sensitivities

Numerical computation of sensitivities given by (19) is accomplished by introducing fractional drifts (=α) in each component (p_k $\forall k$); simulating the circuit and measuring the fractional drift in each coefficient of the polynomial resulting from curve fitting operation. This way the numerical sensitivities are computed and a dictionary is maintained for sensitivities. The complexity in computation of sensitivities is linear in the number N of circuit parameters, i.e., $O(N)$.

5.2. Diagnosing Parametric Faults

Restricting ourselves to single parametric faults, we find the descending order of sensitivities of coefficients (with respect to circuit parameter) that have exceeded their limiting values. The parameter with highest sensitivity is said to be at fault with a probability $\mathrm{P}(\delta \mathrm{p_k} | \delta \mathrm{C_i})$ (which can be interpreted as the confidence in diagnosing fault), given by (20), where δp_k is the suspected drift in parameter p_k and δC_i is

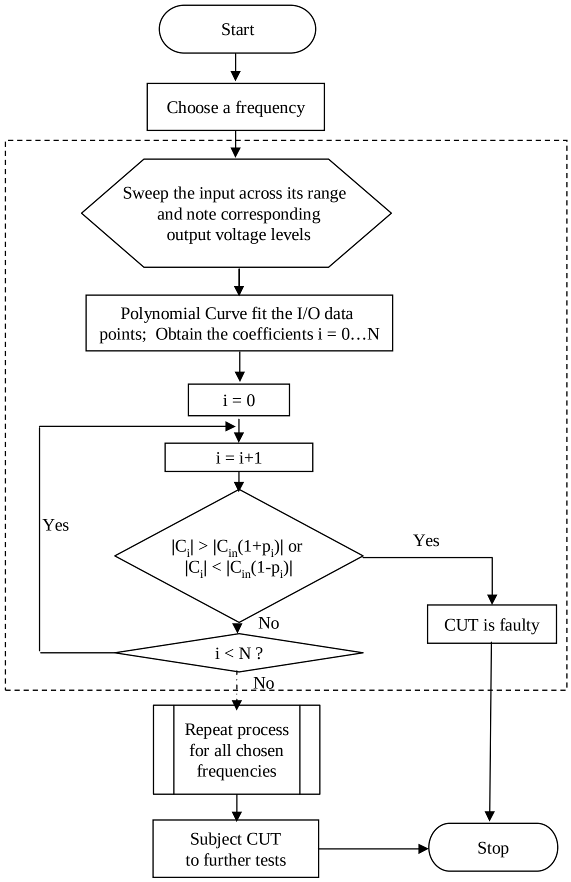

Figure 3. Flow chart outlining test procedure for CUT.

the measured drift in coefficient.

$$\mathrm{P}(\delta \mathrm{p_k}|\delta \mathrm{C_i}) = \phi\left(\frac{S_{p_k}^{C_i}\delta p_k}{\delta C_i}\right) \qquad (20)$$

Note that ϕ in (20) is a probability measure[12], dependent on $\delta p_k, \delta C_i$ and $S_{p_k}^{C_i}$. For example, if sensitivity of some coefficient, say $\mathrm{C_1}$ to parameter p_1 is 5%, measured drift in coefficient value is 10% and we suspect that the parameter drift is 10% then the probability of this being true, by assuming ϕ to be an exponential probability measure, is $e^{\frac{-.05\times.1}{.1}} = .95$

5.3. Fault Deduction

At each frequency, the above process of diagnosis is repeated. This gives the set of fault sites above a certain confidence level at each of these frequencies. The intersection of sets of fault sites at all the frequencies (and at DC) gives a fault site with much higher confidence level. That is, if the confidence of diagnosis of a fault site at one frequency

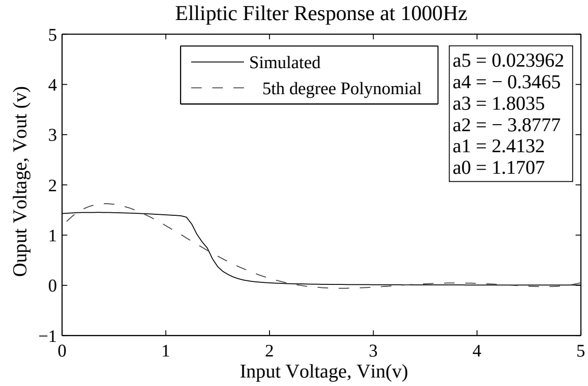

Figure 5. Curve-fitting polynomial with coefficients at frequency = 1000Hz.

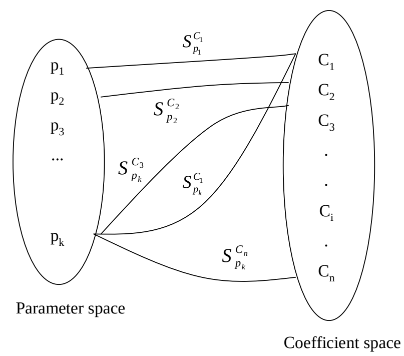

Figure 6. Mapping showing one possible relation between various parameters and coefficients.

is say $\mathrm{P_i}$, then the resulting confidence level after diagnosis at all the frequencies is as follows[12]:

$$\mathrm{P} = 1 - \prod_{\mathrm{i=1}}^{\mathrm{i=N}} (1 - \mathrm{P_i}) \qquad (21)$$

where N is the number of frequencies (including DC) at which the circuit is diagnosed.

The single parametric faults for the elliptic filter in Figure 4 were diagnosable with confidence levels up to 60% at each frequency. The resulting confidence level after fault deduction from the four frequencies at which it was diagnosed is about 98.9%. The diagnosis results are tabulated in Table 3 for several injected single parametric faults. Another observation worthy of mention here is that the cardinality of set of fault sites detected at frequencies close to cut-off frequency is greater than that at frequencies closer to DC. This can be attributed to higher sensitivity of coefficients to circuit parameters at these frequencies. As a result, fault coverage is better by observing coefficient drifts at frequencies close to

Table 2. Parameter combinations leading to Max Values of coefficients with α = 0.05 at 1000Hz.

Circuit Parameters (Resistance in Ω,Capacitance in Farad)						
Nominal Values	a_0	a_1	a_2	a_3	a_4	a_5
$R_1 = 19.6k$	18.6k	18.6k	20.5k	20.5k	20.5k	18.6k
$R_2 = 196k$	205k	205k	205k	205k	186k	186k
$R_3 = 147k$	139k	139k	154k	139k	139k	139k
$R_4 = 1k$	950	950	1.05k	1.05k	1.05k	1.05k
$R_5 = 71.5$	75	67	75	67	67	75
$R_6 = 37.4k$	35k	39k	39k	35k	35k	39k
$R_7 = 154k$	146k	146k	161k	161k	146k	146k
$R_8 = 260$	247	273	273	247	247	273
$R_9 = 740$	703	777	703	703	777	703
$R_{10} = 500$	475	525	525	475	525	525
$C_1 = 2.67n$	2.5n	2.5n	2.5n	2.5n	2.5n	2.5n
$C_2 = 2.67n$	2.5n	2.8n	2.8n	2.5n	2.8n	2.8n
$C_3 = 2.67n$	2.8n	2.8n	2.8n	2.5n	2.8n	2.8n
$C_4 = 2.67n$	2.5n	2.8n	2.5n	2.5n	2.5n	2.5n
$C_5 = 2.67n$	2.5n	2.5n	2.5n	2.5n	2.5n	2.8n
$C_6 = 2.67n$	2.5n	2.8n	2.5n	2.8n	2.5n	2.8n
$C_7 = 2.67n$	2.5n	2.8n	2.8n	2.8n	2.8n	2.5n

Table 3. Parametric Fault Diagnosis with Confidence Levels of $\approx$ 98.9%

Injected fault	Diagnosed fault sites at				Deduced fault site
	100Hz	900Hz	1000Hz	1100Hz	
R_1 down 15%	R_1	R_1, R_2	R_1, R_2, C_1	R_1, C_1	R_1
R_2 down 5%	R_2, C_1	R_2, R_3, C_1	R_2, R_3	R_2, C_1	R_2
R_3 up 10%	R_3, C_3	R_3, R_4, C_3	R_3	R_3, C_3	R_3
R_4 down 20%	R_1, R_4	R_2, R_4, C_1	R_1, R_2, R_4	R_1, R_2, R_4	R_4
R_5 up 15%	R_5, C_2	R_4, R_5	R_4, R_5, C_2	R_5, R_6, C_3	R_5
R_7 down 10%	R_7, C_3	R_3, R_7	R_3, R_6, R_7	R_3, R_7, C_3	R_7
R_{10} up 15%	R_{10}	R_{10}, C_6	R_{10}	R_{10}, C_6	R_{10}
R_{11} down 10%	R_{11}	R_{11}, C_5	R_{11}, R_{12}	R_{11}, R_{12}, C_5	R_{11}
R_{13} up 5%	R_{13}, C_5	R_{13}, C_7	R_{13}, C_6	R_{13}, C_5	R_{13}
R_{14} up 20%	R_{14}	R_{14}, R_{15}	R_{14}, R_{15}	R_{14}, R_{15}	R_{14}
R_{15} up 5%	R_{13}, R_{15}	R_{14}, R_{15}	R_{15}, C_5	R_{14}, R_{15}	R_{15}
C_1 down 10%	R_2, C_1	R_2, C_1	R_2, C_1	R_2, C_1	C_1
C_2 up 10%	R_5, C_2	C_2, C_4	C_2	C_2	C_2
C_4 down 10%	R_6, C_4	C_2, C_4	C_2, C_4	C_2, C_4	C_4
C_5 up 5%	C_5	R_{12}, C_5	C_5	C_5	C_5
C_7 up 15%	C_6, C_7	C_7	C_6, C_7	C_6, C_7	C_7

f_c. However these frequencies tend to be unfavourable for diagnosis as more than one parameter is likely to have displaced the coefficients out of their respective hypercubes. We can overcome this by looking at the set of fault sites obtained at much lower frequencies than f_c (here DC and 100Hz).

6. Conclusion

A new approach for test and diagnosis of non-linear circuits based on polynomial expansion of the circuit function has been proposed. By expanding polynomial coefficients at critical frequencies the fault coverage is significantly improved, yielding a minimum size of detectable faults in some parameters as low as 5%. The method has been extended to sensitivity based fault diagnosis with probabilistic confidence levels in parameter drifts. We have also demonstrated diagnosis of several parametric faults with confidence levels up to 98.9% in the benchmark elliptic filter.

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