LNA Test: A Polynomial Coefficient Approach

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Outline

1. Motivation
2. Our Idea
3. Generalization
4. Results
5. Fault Diagnosis
6. Conclusion
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Motivation

To Develop an Analog Circuit Test Scheme

- Suitable for large class of circuits
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- Detects sufficiently small parametric faults
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Our Idea

**Taylor series expansion** of circuit function in terms of magnitude of input $v_{in}$ at a frequency

\[
v_{out} = f(v_{in})
\]

\[
v_{out} = f(0) + \frac{f'(0)}{1!} v_{in} + \frac{f''(0)}{2!} v_{in}^2 + \frac{f'''(0)}{3!} v_{in}^3 + \cdots + \frac{f^{(n)}(0)}{n!} v_{in}^n + \cdots
\]
Taylor series expansion of circuit function in terms of magnitude of input $v_{in}$ at a frequency

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\[ v_{out} = f(0) + \frac{f'(0)}{1!} v_{in} + \frac{f''(0)}{2!} v_{in}^2 + \frac{f'''(0)}{3!} v_{in}^3 + \cdots + \frac{f^{(n)}(0)}{n!} v_{in}^n + \cdots \]

Ignoring the higher order terms we have

\[ v_{out} \approx a_0 + a_1 v_{in} + a_2 v_{in}^2 + \cdots + a_n v_{in}^n \]

where every $a_i \in \mathbb{R}$ and is bounded between its extreme values for

\[ a_{i,\text{min}} < a_i < a_{i,\text{max}} \quad \forall i \ 0 \leq i \leq n \]
In a Nutshell

- Find the $V_{out}$ v/s $V_{in}$ relationship at frequencies of interest (Eg.: Cutoff, fundamental)
- Compute the coefficients of fault-free circuit
- Repeat the same for CUT by curve fitting the I/O response
- Compare each of the obtained coefficients with fault-free circuit range
- Classify CUT as Good or Bad
Cascaded Amplifiers
An Example

Two stage amplifier with 4\textsuperscript{th} degree non-linearity in $V_{\text{in}}$

\[ v_{\text{out}} = a_0 + a_1 v_{\text{in}} + a_2 v_{\text{in}}^2 + a_3 v_{\text{in}}^3 + a_4 v_{\text{in}}^4 \]
Polynomial Coefficients

\[ a_0 = V_{DD} - R_2 K \left( \frac{W}{L} \right)_2 \left\{ (V_{DD} - V_T)^2 + R_1^2 K^2 \left( \frac{W}{L} \right)_1^2 V_T^4 \right\} - 2 (V_{DD} - V_T) R_1 \left( \frac{W}{L} \right)_1 V_T^2 \]

\[ a_1 = R_2 K \left( \frac{W}{L} \right)_2 \left[ 4 R_1^2 K^2 \left( \frac{W}{L} \right)_1^2 V_T^3 + 2 (V_{DD} - V_T) R_1 K \left( \frac{W}{L} \right)_1 V_T \right] \]

\[ a_2 = R_2 K \left( \frac{W}{L} \right)_2 \left[ 2 (V_{DD} - V_T) R_1 K \left( \frac{W}{L} \right)_1 - 6 R_1^2 K^2 \left( \frac{W}{L} \right)_1^2 V_T^2 \right] \]

\[ a_3 = 4 V_T K^3 \left( \frac{W}{L} \right)_1 \left( \frac{W}{L} \right)_2 R_1^2 R_2 \]

\[ a_4 = -K^3 \left( \frac{W}{L} \right)_1 \left( \frac{W}{L} \right)_2 R_1^2 R_2 \]
**Definition**

**Minimum Size Detectable Fault** \( (\rho) \) of a circuit parameter is defined as its minimum fractional deviation to force at least one of the polynomial coefficients out of its fault free range.
MSDF Calculation

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Minimum Size Detectable Fault ($\rho$) of a circuit parameter is defined as its minimum fractional deviation to force at least one of the polynomial coefficients out of its fault free range.

Overview of MSDF calculation of R1 with $V_{DD}=1.2V$, $V_T=400mV$, $\left(\frac{W}{L}\right)_1 = \frac{1}{2}\left(\frac{W}{L}\right)_2 = 20$, and $K = 100\mu A/V^2$.

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Maximize $a_0$

\[
\begin{align*}
1.2 - R_{2,nom}(1 + y) \left( 2.56\times10^{-3} + R_{1,nom}^2(1 + x)^2 \times 1.024\times10^{-7} \\
-5.12\times10^{-4} R_{1,nom}(1 + x) \right)
\end{align*}
\]

subject to $a_1, a_2, a_3, a_4$ being in their fault free ranges and

$-\alpha \leq x, y \leq \alpha$
Assuming single parametric faults, $\rho$ for $R_1$

$$\rho = (1 + \alpha)^{1.5} - 1 \approx 1.5\alpha - 0.375\alpha^2$$
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$$\rho = (1 + \alpha)^{1.5} - 1 \approx 1.5\alpha - 0.375\alpha^2$$

### MSDF for Cascaded Amplifier with $\alpha = 0.05$

<table>
<thead>
<tr>
<th>Circuit parameter</th>
<th>%upside MSDF</th>
<th>%downside MSDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistor $R_1$</td>
<td>10.3</td>
<td>7.4</td>
</tr>
<tr>
<td>Resistor $R_2$</td>
<td>12.3</td>
<td>8.5</td>
</tr>
</tbody>
</table>
Generalization – Fault Simulation

1. Start
2. Choose a frequency of interest
3. Sweep bias at the input and note corresponding output voltage levels
4. Polynomial curve fit the obtained I/O data – find the coefficient values of fault free circuit
5. Simulate for all parametric faults at the simplex of hypercube
6. Find min-max values of each coefficient ($C_i$) from $i = 1 \cdots N$ across all simulations
7. Stop
Test Setup

Circuit Under Test $f(\cdot)$

\( v_{in} \)

\( v_{ac} \)

\( V_{bias} \)

\( v_{out} \)

\( a_0 - a_N \)

Estimate Polynomial Coefficients

Variable Frequency

Variable Offset
1. Start
2. Sweep bias at the input and note corresponding output voltage levels
3. Polynomial curve fit the obtained I/O data
4. Start with first coefficient
5. Consider next coefficient $C_{i+1}$
6. $|C_i| > |C_{i,\text{max}}|$ or $|C_i| < |C_{i,\text{min}}|$?
   If True go to step 9
7. $i < N$? If True go to step 5
8. Subject CUT to further tests. Stop
9. CUT is faulty. Stop
Outline

1 Motivation
2 Our Idea
3 Generalization
4 Results
5 Fault Diagnosis
6 Conclusion
## Specifications

<table>
<thead>
<tr>
<th>Performance Parameter</th>
<th>Nominal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain (dB)</td>
<td>16</td>
</tr>
<tr>
<td>$\text{IIP}_3$ (dBm)</td>
<td>-18</td>
</tr>
<tr>
<td>Noise figure (dB)</td>
<td>9.1</td>
</tr>
<tr>
<td>$S_{11}$ (dB)</td>
<td>-16.5</td>
</tr>
</tbody>
</table>
Low Noise Amplifier – Schematic
Comparison for parametric fault in $R_L = 100k$ ohm
### Parameter Combinations Leading to Max Values of Coefficients with $\alpha = 0.05$

<table>
<thead>
<tr>
<th>Component (ohm, nH, fF)</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{\text{bias}} = 10$</td>
<td>10</td>
<td>10</td>
<td>10.5</td>
<td>10.5</td>
<td>9.5</td>
<td>10.5</td>
</tr>
<tr>
<td>$L_C = 1$</td>
<td>1</td>
<td>0.95</td>
<td>1.05</td>
<td>0.95</td>
<td>1.05</td>
<td>1</td>
</tr>
<tr>
<td>$C_{C1} = 100$</td>
<td>95</td>
<td>95</td>
<td>95</td>
<td>95</td>
<td>95</td>
<td>105</td>
</tr>
<tr>
<td>$L_1 = 1.5$</td>
<td>1.425</td>
<td>1.5</td>
<td>1.5</td>
<td>1.425</td>
<td>1.575</td>
<td>1.425</td>
</tr>
<tr>
<td>$L_2 = 1.5$</td>
<td>1.5</td>
<td>1.425</td>
<td>1.425</td>
<td>1.575</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>$L_f = 1$</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
<td>1</td>
<td>1.05</td>
<td>1</td>
</tr>
<tr>
<td>$C_f = 100$</td>
<td>105</td>
<td>95</td>
<td>95</td>
<td>105</td>
<td>95</td>
<td>95</td>
</tr>
<tr>
<td>$C_{C2} = 100$</td>
<td>95</td>
<td>100</td>
<td>105</td>
<td>95</td>
<td>95</td>
<td>95</td>
</tr>
<tr>
<td>$R_{\text{bias1}} = 100k$</td>
<td>105k</td>
<td>105k</td>
<td>100k</td>
<td>105k</td>
<td>105k</td>
<td>95k</td>
</tr>
<tr>
<td>$R_{\text{bias2}} = 100k$</td>
<td>105k</td>
<td>95k</td>
<td>100k</td>
<td>95k</td>
<td>95k</td>
<td>95k</td>
</tr>
<tr>
<td>$R_L = 100k$</td>
<td>100k</td>
<td>95k</td>
<td>95k</td>
<td>100k</td>
<td>105k</td>
<td>100k</td>
</tr>
</tbody>
</table>
## Results – Low Noise Amplifier @ 10GHz

Parameter Combinations Leading to Min Values of Coefficients with $\alpha = 0.05$

<table>
<thead>
<tr>
<th>Component (ohm, nH, fF)</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{\text{bias}} = 10$</td>
<td>10</td>
<td>9.5</td>
<td>9.5</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$L_C = 1$</td>
<td>1.05</td>
<td>0.95</td>
<td>0.95</td>
<td>1</td>
<td>1</td>
<td>0.95</td>
</tr>
<tr>
<td>$C_{C1} = 100$</td>
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<td>105</td>
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<td>0.95</td>
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<td>95</td>
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<td>105</td>
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<td>$C_{C2} = 100$</td>
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<td>$R_{\text{bias1}} = 100k$</td>
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<td>100k</td>
</tr>
<tr>
<td>$R_{\text{bias2}} = 100k$</td>
<td>100k</td>
<td>105k</td>
<td>95k</td>
<td>95k</td>
<td>105k</td>
<td>95k</td>
</tr>
<tr>
<td>$R_L = 100k$</td>
<td>95k</td>
<td>100k</td>
<td>95k</td>
<td>100k</td>
<td>105k</td>
<td>95k</td>
</tr>
</tbody>
</table>
## Results – Low Noise Amplifier @ 10GHz

### Results of some Injected Faults

<table>
<thead>
<tr>
<th>Circuit Parameter</th>
<th>Coefficients out of bounds</th>
<th>Detected</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{\text{bias}}$ down 25%</td>
<td>$a_0 - a_4$</td>
<td>Yes</td>
</tr>
<tr>
<td>$L_C$ down 15%</td>
<td>$a_2, a_5$</td>
<td>Yes</td>
</tr>
<tr>
<td>$C_{C1}$ up 10%</td>
<td>$a_1, a_2, a_3$</td>
<td>Yes</td>
</tr>
<tr>
<td>$L_1$ down 25%</td>
<td>$a_0 - a_4$</td>
<td>Yes</td>
</tr>
<tr>
<td>$L_2$ up 15%</td>
<td>$a_0, a_4$</td>
<td>Yes</td>
</tr>
<tr>
<td>$L_f$ up 10%</td>
<td>$a_1, a_2$</td>
<td>Yes</td>
</tr>
<tr>
<td>$C_f$ up 10%</td>
<td>$a_4, a_5$</td>
<td>Yes</td>
</tr>
<tr>
<td>$C_{C2}$ down 10%</td>
<td>$a_4, a_5$</td>
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Fault Diagnosis

Definition

To determine the circuit parameters responsible for deviation of circuit from its desired behavior.
Fault Diagnosis

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Sensitivity based diagnosis

\[ S_{p_k}^{C_i} = \frac{p_k}{C_i} \frac{\partial C_i}{\partial p_k} \]
Possible relation between various parameters and coefficients
## Results – Low Noise Amplifier

### Fault Diagnosis at $f = 10$ GHz

<table>
<thead>
<tr>
<th>Fault injected</th>
<th>Coefficient status</th>
<th>Diagnosed fault sites</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{bias}$ down 25%</td>
<td>$a_0 - a_4$</td>
<td>$R_{bias}$</td>
</tr>
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<td>$a_2, a_5$</td>
<td>$L_C$ or $C_{C1}$</td>
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<td>$C_{C1}$ up 10%</td>
<td>$a_1, a_2, a_3$</td>
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</tr>
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<td>$L_1$ down 25%</td>
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<td>$L_1$</td>
</tr>
<tr>
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<td>$L_2$</td>
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<td>$C_{C2}$</td>
</tr>
<tr>
<td>$C_{C2}$ down 10%</td>
<td></td>
<td></td>
</tr>
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- Technique for parametric fault detection in analog circuits – faults as small as 10% were uncovered for LNA example
- Diagnosis based on Sensitivity of Polynomial Coefficients to circuit parameters
- **Limitation** – Extensive fault simulations required to cover all corner cases
Conclusions and Future Work

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**In Future**

- Neural models to map specifications to polynomial coefficients
- To implement proposed test scheme as BIST by storing polynomial coefficients on chip
Acknowledgments

- Wireless Engineering Research and Education Center (WEREC), Auburn Univ.
- Virendra Singh, Indian Institute of Science, Bangalore
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Thanks for your Attention!
Questions are guaranteed in life; Answers aren't.