Testing Linear and Non-Linear Analog Circuits Using Moments

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Outline

1. Motivation
2. Moment Based Test
3. Generalization
4. Results
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4. Results
Ideal Test For An Analog Circuit

Wish list for an analog circuit test scheme

- Suitable for large class of circuits
- Detects sufficiently small parametric faults – high sensitivity
- Small area overhead – requires little circuit augmentation
- Large number observables – handy in diagnosis
- Low test time
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- Suitable for large class of circuits
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- Large number observables – handy in diagnosis
- Low test time
- Low design complexity of the input signal
Problem statement

Evaluate probability moments of output as a metric for testing analog circuits with Gaussian noise as the input excitation
- Gaussian noise as input has minimum design effort
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Basic Idea

Circuit is a function $f(.)$ transforming random variable $X$ to a random variable $Y$. This implies circuit can be characterized by the statistics of output($Y$), such as probability density function and moments for a given input random variable probability distribution.

Circuit specifications can be related to moments.
Probability Moment
Definition

Probability moment of a random variable

The $n^{th}$ moment, $\mu_n$ for all $n = 1 \cdots N$ of a continuous random variable $X \geq 0$, and having a pdf given by $f(X)$, is defined as

$$\mu_n = \int_{X=0}^{\infty} X^n f(X) \, dX$$
Calculating probability moment of a random variable

Let \( f(X) = e^{-X} \) for all \( X \geq 0 \)
Probability Moment
A Quick Example

Calculating probability moment of a random variable

Let \( f (X) = e^{-X} \) for all \( X \geq 0 \)
The \( n^{th} \) moment of \( X \), \( \mu_n \) for all \( n = 1 \cdots N \)

\[
\mu_n = \int_0^\infty X^n f (X) \, dX
\]

\[
= \int_0^\infty X^n e^{-X} \, dX
\]

\[
= \Gamma (n + 1) = n!
\]

\[\implies \mu_1 = 1, \mu_2 = 2, \mu_3 = 6, \mu_4 = 24, \cdots\]
Minimum Size Detectable Fault (MSDF)

Definition

Minimum size detectable fault \((\rho)\) of a circuit parameter is defined as the minimum fractional deviation that forces at least one of the moments out of its fault free range.
Minimum size detectable fault (ρ) of a circuit parameter is defined as the minimum fractional deviation that forces at least one of the moments out of its fault free range.

Minimum fractional deviation, ρ, in a circuit element, of nominal value g, such that $g \rightarrow g(1 \pm \rho)$, causes, at least one of the moments, $\mu_i$ to violate the following inequality:

$$\mu_{i,\text{min}} < \mu_i < \mu_{i,\text{max}} \quad \forall \mu_i, \ 1 \leq i \leq n$$
RC Filter
MSDF Calculation - An Example

\[ \mu^2 = \frac{\pi}{4} \frac{N_0}{R C} \]

where:
- \( N_0 \): Input noise power spectral density,
- \( R \): Resistance,
- \( C \): Capacitance.

For a fractional deviation \( \rho \) in \( R \), such that
\[ \mu^2 - \mu_0^2 \geq \mu_0^2, \]
gives
\[ \rho = 4 \mu_0 (\pi - 4) \frac{C R}{N_0}. \]
\[ \mu_2 = \frac{N_o \pi}{4RC} \]

\(N_o\): Input noise power spectral density, \(R\): Resistance, \(C\): Capacitance.

For a fractional deviation \(\rho\) in \(R\), such that \(\bar{\mu}_2 - \mu_2 \geq \mu_0\), gives
RC Filter

MSDF Calculation - An Example

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\( N_o \): Input noise power spectral density, \( R \): Resistance, \( C \): Capacitance.

For a fractional deviation \( \rho \) in \( R \), such that \( \mu_2 - \mu_2 \geq \mu_0 \), gives

\[ \rho = \frac{4\mu_0 CR}{N_o \pi - 4\mu_0 CR} \]
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Test Setup

$V_n^2 = 4kTR$

$+$

- $\underline{\text{CUT}}$

$\text{ATE}$

$\text{ADC}$

$\text{DSP}$

$\text{ROM}$

Golden Moments

Compute & Compare

Pass/Fail
Fault Simulation

1. Start
2. Apply inputs, sampled from a Gaussian probability density function
3. Record output values for each of these inputs and estimate the output probability density function (PDF)
4. Compute moments ($\mu_i$) of the estimated PDF up to the desired order (say N)
5. Repeat steps 1-3, with circuit component values sampled uniformly in their fault free tolerance range
6. Find min-max values of each moment ($\mu_i$) from $i = 1 \cdots N$ across all simulations
7. Stop
Test Procedure

1. Start
2. Apply inputs, sampled from a Gaussian probability density function
3. Record output values for each of these inputs and estimate the output probability density function (PDF)
4. Compute the moments of the estimated output PDF
5. $\mu_i > \mu_{i,\text{max}}$ or $\mu_i < \mu_{i,\text{min}}$. Yes or No?
6. If yes, conclude circuit under test is faulty. If not, repeat the test for next moment
7. If all coefficients are inside the bounds, subject circuit under test to further tests. Stop
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Results – Benchmark Elliptic Filter
Parameter combinations leading to maximum values of moments with $\gamma = 0.05$

<table>
<thead>
<tr>
<th>Circuit Parameter (ohm, nF)</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
<th>$\mu_3$</th>
<th>$\mu_4$</th>
<th>$\mu_5$</th>
<th>$\mu_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1 = 19.6k$</td>
<td>19.6k</td>
<td>20.58k</td>
<td>19.6k</td>
<td>20.58k</td>
<td>20.58k</td>
<td>18.62k</td>
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<tr>
<td>$R_2 = 196k$</td>
<td>186.2k</td>
<td>205.8k</td>
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<td>$R_3 = 147k$</td>
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<td>139.65k</td>
<td>139.65k</td>
<td>154.35k</td>
<td>147k</td>
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<td>$R_4 = 1k$</td>
<td>1050</td>
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<td>950</td>
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<td>950</td>
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<tr>
<td>$R_5 = 71.5$</td>
<td>75.075</td>
<td>67.925</td>
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### Results

Elliptic Filter - Fault Simulation

Parameter combinations leading to minimum values of moments with $\gamma = 0.05$

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## Results

### Elliptic Filter - Fault Detection

#### Fault detection for some injected faults

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<tr>
<th>Circuit Parameter</th>
<th>Out of bound moment</th>
<th>Fault detected?</th>
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<tbody>
<tr>
<td>R₁ down 12%</td>
<td>μ₁, μ₃</td>
<td>Yes</td>
</tr>
<tr>
<td>R₂ down 10%</td>
<td>μ₄</td>
<td>Yes</td>
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<tr>
<td>R₃ up 12%</td>
<td>μ₁, μ₂</td>
<td>Yes</td>
</tr>
<tr>
<td>R₅ up 10%</td>
<td>μ₄</td>
<td>Yes</td>
</tr>
<tr>
<td>R₇ up 15%</td>
<td>μ₅, μ₆</td>
<td>Yes</td>
</tr>
<tr>
<td>R₁₁ up 15%</td>
<td>μ₃</td>
<td>Yes</td>
</tr>
<tr>
<td>R₁₂ down 15%</td>
<td>μ₂, μ₆</td>
<td>Yes</td>
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<tr>
<td>C₄ up 12%</td>
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<td>Yes</td>
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<td>C₅ down 15%</td>
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Conclusion

- Circuit test using probability moments of the output as the test metric was investigated.
- Test procedure was implemented on an elliptic filter, with detection of fault sizes $\approx 12\%$. 
Open Questions

Possible directions for future work

- Optimal order N of moment expansion
- Fault diagnosis using moments
- Other noise distributions for input excitation