Polynomial Coefficient Based DC Testing of Non-Linear Analog Circuits

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19th Great Lakes Symposium on VLSI
Boston, MA

May 11, 2009

Analog Circuit Testing

To determine catastrophic (open or short) faults and fractional deviations in circuit components from their nominal values.

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This Work

To propose a method to detect fractional deviations of circuit components from their nominal values in a large class of circuits.

Outline

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To Develop an Analog Circuit Test Scheme

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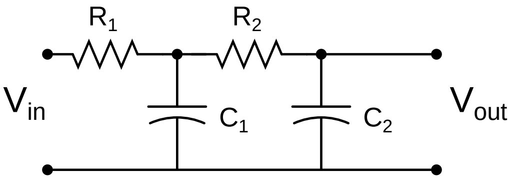
Previous Approaches

Important Previous Techniques

- Signal flow graph – Complexity order is high [Bushnell et. al. '96]
- I_{DDQ} based test – Intrusive, Area overhead is high [Chakravarty '97]
- Transfer function based test – Valid only for LTI systems [Savir et. al. '03]

Transfer Function Coefficient - Based Test



Second order low pass filter

$$H(s) = \frac{1}{(R_1R_2C_1C_2)\,s^2 + (R_1C_1 + (R_1 + R_2)C_2)\,s + 1}$$

Our Idea

Taylor series expansion of circuit function about $v_{in} = 0$

$$v_{out} = f(v_{in})$$
$$v_{out} = f(0) + \frac{f'(0)}{1!}v_{in} + \frac{f''(0)}{2!}v_{in}^2 + \frac{f'''(0)}{3!}v_{in}^3 + \cdots + \frac{f^{(n)}(0)}{n!}v_{in}^n + \cdots$$

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Ignoring the higher order terms we have

$$v_{out} \approx a_0 + a_1 v_{in} + a_2 v_{in}^2 + \cdots + a_n v_{in}^n$$

where every $a_i \in \Re$ and is bounded between its extreme values for

$$a_{i,\min} < a_i < a_{i,\max}\ \forall i\ 0 \leq i \leq n$$

Our Idea (Contd..)

In a Nutshell

- Find the V_{out} v/s V_{in} relationship at DC

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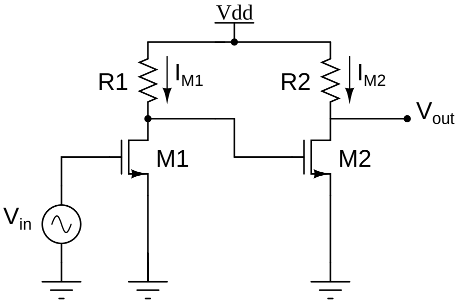
In a Nutshell

- Find the V_{out} v/s V_{in} relationship at DC
- Compute the coefficients of fault-free circuit
- Repeat the same for CUT by curve fitting the I/O response
- Compare each of the obtained coefficients with fault-free circuit range
- Classify CUT as Good or Bad

Cascaded Amplifiers



Two stage amplifier with 4^{th} degree non-linearity in V_{in}

$$v_{out} = a_0 + a_1 v_{in} + a_2 v_{in}^2 + a_3 v_{in}^3 + a_4 v_{in}^4$$

Polynomial Coefficients

$$a_0 = V_{DD} - R_2 K \left(\frac{W}{L}\right)_2 \left[\begin{array}{l} (V_{DD} - V_T)^2 + R_1^2 K^2 \left(\frac{W}{L}\right)_1^2 V_T^4 \\ -2(V_{DD} - V_T) R_1 \left(\frac{W}{L}\right)_1 V_T^2 \end{array} \right]$$

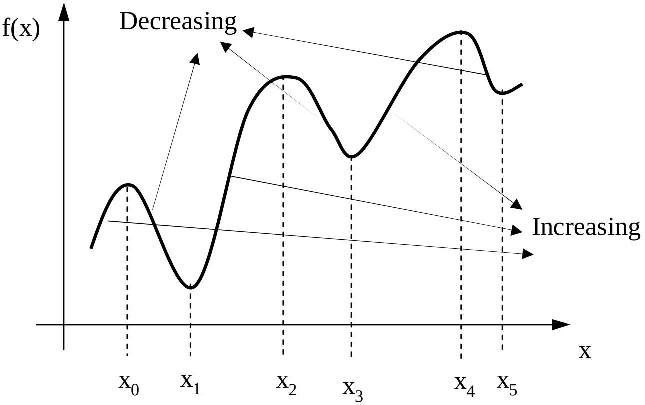
$$a_1 = R_2 K \left(\frac{W}{L}\right)_2 \left[4R_1^2 K^2 \left(\frac{W}{L}\right)_1^2 V_T^3 + 2(V_{DD} - V_T) R_1 K \left(\frac{W}{L}\right)_1 V_T \right]$$

$$a_2 = R_2 K \left(\frac{W}{L}\right)_2 \left[2(V_{DD} - V_T) R_1 K \left(\frac{W}{L}\right)_1 - 6R_1^2 K^2 \left(\frac{W}{L}\right)_1^2 V_T^2 \right]$$

$$a_3 = 4 V_T K^3 \left(\frac{W}{L}\right)_1^2 \left(\frac{W}{L}\right)_2^2 R_1^2 R_2$$

$$a_4 = -K^3 \left(\frac{W}{L}\right)_1^2 \left(\frac{W}{L}\right)_2^2 R_1^2 R_2$$

Nature of Polynomial Coefficients



Non-linear, Non-monotonic function is decomposed into piecewise monotonic functions

MSDF Calculation

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Overview of MSDF calculation of R1 with $\mathrm{V_{DD}}$=1.2V, $\mathrm{V_T}$= 400mV, $\left(\frac{W}{L}\right)_1 = \frac{1}{2}\left(\frac{W}{L}\right)_2 = 20$, and $\mathrm{K} = 100\mu\mathrm{A}/\mathrm{V}^2$

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Maximize a_0

$$\left\{1.2 - R_{2,nom}(1+y)\left(\begin{array}{c} 2.56\text{x}10^{-3} + R_{1,nom}^2(1+x)^2 1.024\text{x}10^{-7} \\ -5.12\text{x}10^{-4} R_{1,nom}(1+x) \end{array}\right)\right\}$$

subject to a_1, a_2, a_3, a_4 being in their fault free ranges and

$$-\alpha \leq x, y \leq \alpha$$

MSDF Calculation (contd..)

Assuming single parametric faults, ρ for R_1

$$\rho = (1-\alpha)^{1.5} - 1 \approx 1.5\alpha - 0.375\alpha^2$$

MSDF Calculation (contd..)

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MSDF for Cascaded Amplifier with α = 0.05

Circuit parameter	%upside MSDF	%downside MSDF
Resistor R_1	10.3	7.4
Resistor R_2	12.3	8.5

Let us Generalize

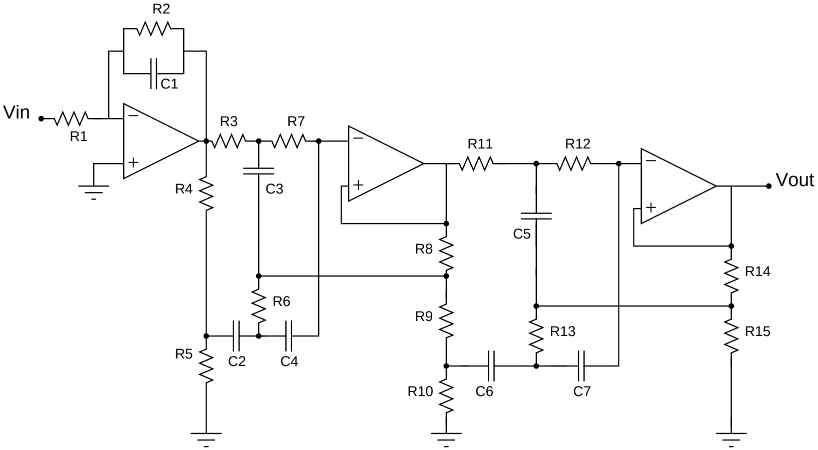
Generalization – Fault Simulation

1. Start
2. Apply DC sweep to input and note corresponding output voltage levels
3. Polynomial Curve fit the obtained I/O data – find the coefficient values of fault free circuit
4. Simulate for all parametric faults at the simplex of hypercube
5. Find min-max values of each coefficient (C_i) from $i = 1 \cdots N$ across all simulations
6. Stop

Generalization – Test Procedure

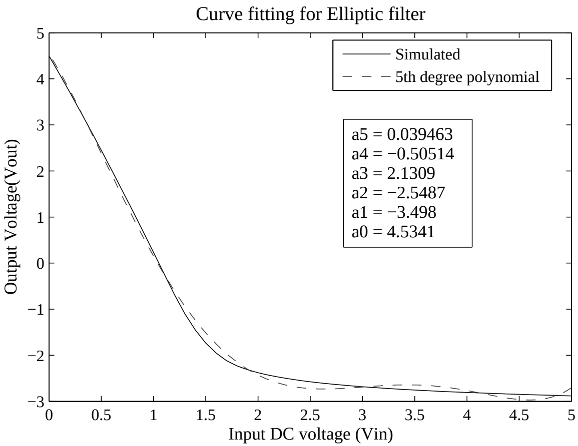
1. Start
2. Apply DC sweep to input and note corresponding output voltage levels
3. Polynomial Curve fit the obtained I/O data
4. Start with first coefficient
5. Consider next coefficient C_{i+1}
6. $|C_i| > |C_{i,max}|$ *or* $|C_i| < |C_{i,min}|$?
   If True go to step 8
7. $i < N$? If True go to step 4
8. Subject CUT to further tests. Stop
9. CUT is faulty. Stop

Results – Elliptic filter



Results - Curve fitting

Curve fit polynomial at 5th order for elliptic filter



$$v_{out} = 4.5341 - 3.498v_{in} - 2.5487v_{in}^2 + 2.1309v_{in}^3 - 0.50514v_{in}^4 + 0.039463v_{in}^5$$

Results – Elliptic filter

Parameter Combinations Leading to Max Values of Coefficients with α = 0.05

Resistance (ohm)	a_0	a_1	a_2	a_3	a_4	a_5
$R_1 = 19.6k$	18.6k	20.5k	20.5k	20.5k	18.6k	18.6k
$R_2 = 196k$	186k	205k	186k	186k	186k	205k
$R_3 = 147k$	139k	154k	154k	154k	139k	154k
$R_4 = 1k$	950	1010	1010	1010	1010	1010
$R_5 = 71.5$	70	80	80	70	80	70
$R_6 = 37.4k$	37.4k	37.4k	37.4k	37.4k	37.4k	37.4k
$R_7 = 154k$	161k	161k	146k	161k	146k	146k
$R_{11} = 110k$	115k	115k	104k	115k	104k	104k
$R_{12} = 110k$	104k	115k	104k	104k	104k	104k

Results – Elliptic filter

Parameter Combinations Leading to Min Values of Coefficients with α = 0.05

Resistance (ohm)	a_0	a_1	a_2	a_3	a_4	a_5
$R_1 = 19.6k$	20.5k	18.6k	18.6k	20.5k	20.5k	20.5k
$R_2 = 196k$	205k	186k	205k	205k	205k	186k
$R_3 = 147k$	150k	139k	139k	146k	154k	139k
$R_4 = 1k$	1010	950	950	950	950	950
$R_5 = 71.5$	80	70	70	80	70	80
$R_6 = 37.4k$	39.2k	39.2k	39.2k	39.2k	35.5k	39.2k
$R_7 = 154k$	146k	146k	161k	146k	161k	161k
$R_{11} = 110k$	104k	104k	115k	104k	115k	115k
$R_{12} = 110k$	115k	104k	115k	115k	115k	115k

Results – Elliptic filter

Results of some Injected Faults

Circuit Parameter	Coefficients out of bounds	Detected
R_1 down 25%	$a_0 - a_4$	Yes
R_2 down 15%	a_2, a_5	Yes
R_3 up 10%	a_1, a_2, a_3	Yes
R_4 down 25%	$a_0 - a_4$	Yes
R_5 up 15%	a_0, a_4	Yes
R_7 up 10%	a_1, a_2	Yes
R_{11} up 10%	a_4, a_5	Yes
R_{12} down 10%	a_4, a_5	Yes

Fault Diagnosis

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To determine the circuit parameters responsible for deviation of circuit from its desired behavior.

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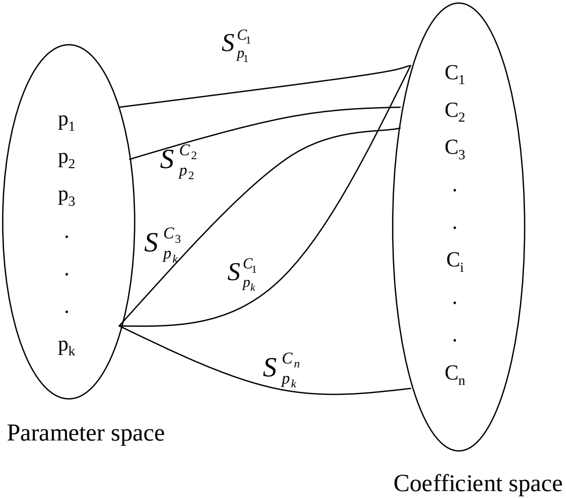
Sensitivity based diagnosis

$$S_{p_k}^{C_i} = \frac{p_k}{C_i} \frac{\partial C_i}{\partial p_k}$$

Fault Diagnosis



Possible relation between various parameters and coefficients

Fault Diagnosis at DC

Fault injected	Coefficient status	Diagnosed fault sites
R_1 down 25%	$a_0 - a_4$	R_1 or R4
R_2 down 15%	a_2, a_5	R_2
R_3 up 10%	a_1, a_2, a_3	R_3
R_4 down 25%	$a_0 - a_4$	R_1 or R4
R_5 up 15%	a_0, a_4	R_5
R_7 up 10%	a_1, a_2	R_7
R_{11} up 10%	a_4, a_5	R_{11} or R_{12}
R_{12} down 10%	a_4, a_5	R_{11} or R_{12}

Conclusions and Future Work

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- Technique for parametric fault detection in analog circuits – faults as small as 10% were uncovered
- Diagnosis based on Sensitivity of Polynomial Coefficients to circuit parameters
- Limitation – Cannot uncover parametric deviations in reactive elements as input is swept at DC

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- Technique for parametric fault detection in analog circuits – faults as small as 10% were uncovered
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In future

- Multi-tone polynomial test to uncover parametric faults in reactive elements (*To appear in NATW 2009*)
- Smart Transforms on Polynomial coefficients to capture monotonicity with high sensitivity

Acknowledgments

- Department of Science and Technology, Govt. of India
- Pawan Kumar, SERC, IISc
- Pramod Subramanyan, SERC, IISc

Thanks for your Attention!