

V-Transform: An Enhanced Polynomial Coefficient Based DC Test for Non-Linear Analog Circuits

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7th IEEE East-West Design Test Symposium
Moscow, Russia



Sep 21, 2009

Analog Circuit Testing

To determine catastrophic (open or short) faults and fractional deviations in circuit components from their nominal values.

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To determine catastrophic (open or short) faults and fractional deviations in circuit components from their nominal values.

In this talk

To propose an enhanced method to detect fractional deviations of circuit components from their nominal values in a large class of circuits.

Outline

- 1 Motivation
- 2 Our Idea
- 3 Generalization
- 4 Results
- 5 Conclusion

Motivation

To Develop an Analog Circuit test scheme

- Suitable for large class of circuits

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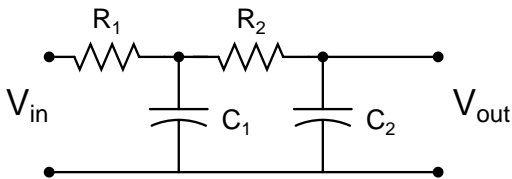
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Previous Approaches

Important previous techniques

- I_{DDQ} based test – Intrusive, Area overhead is high
[Chakravarty '97]
- Signal flow graph – Complexity order is high
[Bushnell et al. '97]
- Transfer function based test – Valid only for LTI systems
[Savir and Guo '03]
- Digital assisted analog test – Intrusive
[Tim Cheng et al. '06]
- Polynomial coefficient based test – Lower sensitivity
[Sindia et al. '09]

Transfer Function Coefficient Based Test



Second order low pass filter

$$H(s) = \frac{1}{(R_1 R_2 C_1 C_2) s^2 + (R_1 C_1 + (R_1 + R_2) C_2) s + 1}$$

Our Earlier Work

Taylor series expansion of circuit function about $v_{in} = 0$

$$V_{out} = f(v_{in})$$

$$V_{out} = f(0) + \frac{f'(0)}{1!} v_{in} + \frac{f''(0)}{2!} v_{in}^2 + \frac{f'''(0)}{3!} v_{in}^3 + \cdots + \frac{f^{(n)}(0)}{n!} v_{in}^n + \cdots$$

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Ignoring the higher order terms we have

$$v_{out} \approx a_0 + a_1 v_{in} + a_2 v_{in}^2 + \cdots + a_n v_{in}^n$$

where every $a_i \in \mathfrak{R}$ and is bounded between its extreme values for

$$a_{i,\min} < a_i < a_{i,\max} \quad \forall i \quad 0 \leq i \leq n$$

Our Earlier work (Contd..)

In a nutshell

- Find the V_{out} v/s V_{in} relationship at DC

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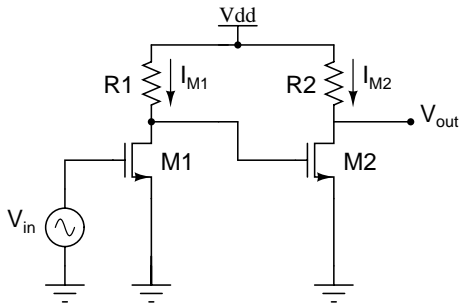
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Our Earlier work (Contd..)

In a nutshell

- Find the V_{out} v/s V_{in} relationship at DC
- Compute the coefficients of fault-free circuit
- Repeat the same for CUT by curve fitting the I/O response
- Compare each of the obtained coefficients with fault-free circuit range
- Classify CUT as **Good** or **Bad**

Cascaded Amplifiers – An Example



Two stage amplifier with 4th degree non-linearity in V_{in}

$$v_{out} = a_0 + a_1 v_{in} + a_2 v_{in}^2 + a_3 v_{in}^3 + a_4 v_{in}^4$$

Polynomial Coefficients

$$a_0 = V_{DD} - R_2 K \left(\frac{W}{L} \right)_2 \left[\begin{array}{l} (V_{DD} - V_T)^2 + R_1^2 K^2 \left(\frac{W}{L} \right)_1^2 V_T^4 \\ -2(V_{DD} - V_T) R_1 \left(\frac{W}{L} \right)_1 V_T^2 \end{array} \right]$$

$$a_1 = R_2 K \left(\frac{W}{L} \right)_2 \left[4R_1^2 K^2 \left(\frac{W}{L} \right)_1^2 V_T^3 + 2(V_{DD} - V_T) R_1 K \left(\frac{W}{L} \right)_1 V_T \right]$$

$$a_2 = R_2 K \left(\frac{W}{L} \right)_2 \left[2(V_{DD} - V_T) R_1 K \left(\frac{W}{L} \right)_1 - 6R_1^2 K^2 \left(\frac{W}{L} \right)_1^2 V_T^2 \right]$$

$$a_3 = 4V_T K^3 \left(\frac{W}{L} \right)_1^2 \left(\frac{W}{L} \right)_2^2 R_1^2 R_2$$

$$a_4 = -K^3 \left(\frac{W}{L} \right)_1^2 \left(\frac{W}{L} \right)_2^2 R_1^2 R_2$$

V-Transform Coefficient – An Introduction

Definition

$$V_{C_i} = e^{\gamma C'_i} \quad \forall 0 \leq i \leq n$$

$$\frac{dC'_i}{dp_j} = \left| \frac{dC_i}{dp_j} \right| \quad \forall 0 \leq i \leq n$$

C_i – i^{th} polynomial coefficient

C'_i – i^{th} modified polynomial coefficient

V_{C_i} – i^{th} V-Transform coefficient

V-Transform Coefficient – Sensitivity gain

Sensitivity of coefficients

$$\frac{S_{p_i}^{V_{C_i}}}{S_{p_i}^{C_i}} = \frac{\left| \frac{dC_i}{dp_i} \right| \gamma e^{\gamma C_i} \cdot \frac{p_i}{e^{\gamma C_i}}}{\frac{dC_i}{dp_i} \cdot \frac{p_i}{C_i}} = \gamma C_i$$

γC_i – Increased sensitivity over ordinary polynomial coefficients

γ – Sensitivity parameter that can be chosen according to the desired degree of sensitivity

MSDF Calculation

Definition

Minimum Size Detectable Fault(ρ) of a circuit parameter is defined as its minimum fractional deviation to force at least one of the polynomial coefficients out of its fault free range

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Overview of MSDF calculation of R_1 with $V_{DD}=1.2V$, $V_T=400mV$, $\left(\frac{W}{L}\right)_1 = \frac{1}{2} \left(\frac{W}{L}\right)_2 = 20$, and $K = 100\mu A/V^2$

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Maximize a_0

$$\left\{ 1.2 - R_{2,nom}(1+y) \begin{pmatrix} 2.56 \times 10^{-3} + R_{1,nom}^2(1+x)^2 1.024 \times 10^{-7} \\ -5.12 \times 10^{-4} R_{1,nom}(1+x) \end{pmatrix} \right\}$$

subject to a_1, a_2, a_3, a_4 being in their fault free ranges and

$$-\alpha \leq x, y \leq \alpha$$

MSDF Calculation (contd..)

Assuming single parametric faults, ρ for R_1

$$\rho = (1 - \alpha)^{1.5} - 1 \approx 1.5\alpha - 0.375\alpha^2$$

MSDF Calculation (contd..)

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MSDF for Cascaded Amplifier with $\alpha = 0.05$

| Circuit parameter | %upside MSDF | %downside MSDF |
|-------------------|--------------|----------------|
| Resistor R_1 | 10.3 | 7.4 |
| Resistor R_2 | 12.3 | 8.5 |

Let us Generalize

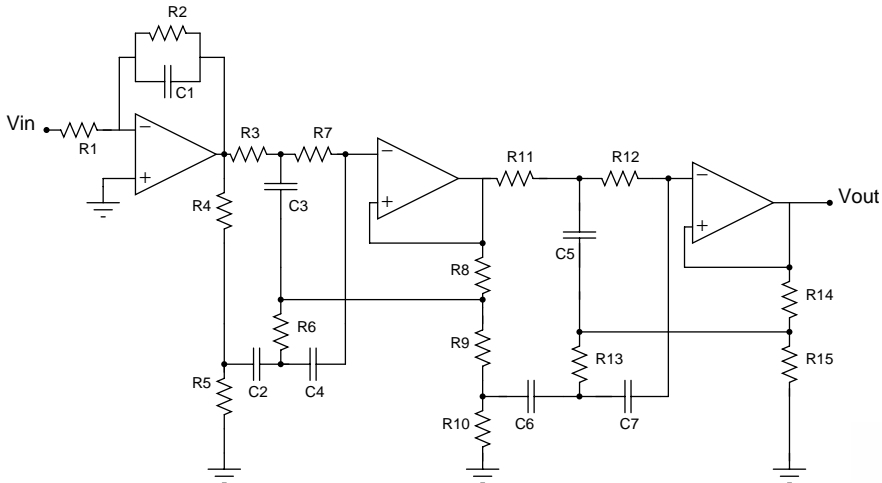
Generalization – Fault Simulation

- 1 **Start**
- 2 Choose frequency for fault simulation
- 3 Apply sweep to input and note corresponding output voltage levels
- 4 Polynomial Curve fit the obtained I/O data – find the coefficient values of fault free circuit. Compute corresponding V-Transform coefficients.
- 5 Simulate for all parametric faults at the simplex of hypercube
- 6 Find min-max values of each coefficient (C_i) from $i = 1 \dots N$ across all simulations. Compute the V-Transform coefficient V_{C_i} bounds.
- 7 Repeat process at all chosen frequencies
- 8 **Stop**

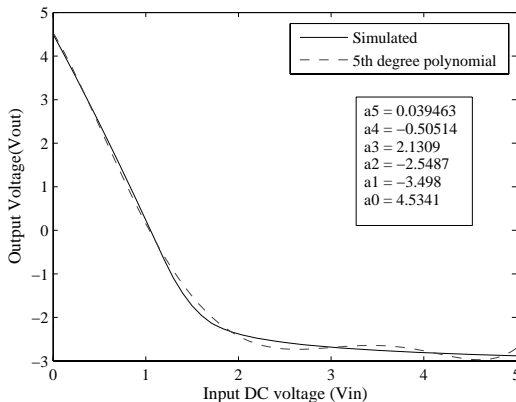
Generalization – Test Procedure

- 1 **Start**
- 2 Choose a frequency
- 3 Sweep input and note corresponding output voltage levels
- 4 Polynomial Curve fit the obtained I/O data. Obtain coefficients $C_i \forall i = 1 \dots N$.
- 5 Obtain corresponding V_{C_i}
- 6 Consider the coefficient V_{C_i} , starting from $i = 1 \dots N$
- 7 $|V_{C_i}| > |V_{C_{i,max}}|$ or $|V_{C_i}| < |V_{C_{i,min}}|$? Yes or No
- 8 If Yes, conclude **CUT is faulty**. If not, repeat the test for next coefficient.
- 9 If all coefficients are inside the bounds, subject CUT to further tests. **Stop**

Results – Benchmark Elliptic filter

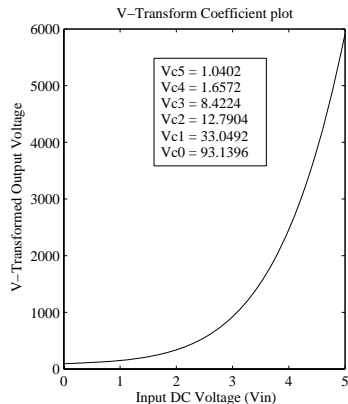
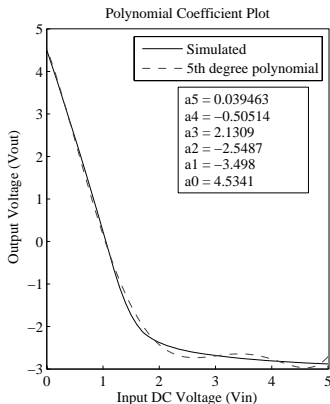


Results - Curve fitting at DC



$$V_{out} = 4.5341 - 3.498v_{in} - 2.5487v_{in}^2 + 2.1309v_{in}^3 - 0.50514v_{in}^4 + 0.039463v_{in}^5$$

Results - V-Transform coefficients



$$V_{C_5} = 1.0402 \quad V_{C_4} = 1.6572 \quad V_{C_3} = 8.4224 \quad V_{C_2} = 12.7904 \quad V_{C_1} = 33.0492 \quad V_{C_0} = 93.1396$$

Results at DC - Elliptic filter

Parameter Combinations Leading to Max Values of V-Transform Coefficients with $\alpha = 0.05$

| Circuit Parameter, (ohm) | V_{C_0} | V_{C_1} | V_{C_2} | V_{C_3} | V_{C_4} | V_{C_5} |
|--------------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| $R_1 = 19.6k$ | 18.6k | 20.5k | 20.5k | 20.5k | 18.6k | 18.6k |
| $R_2 = 196k$ | 186k | 205k | 186k | 186k | 186k | 205k |
| $R_3 = 147k$ | 139k | 154k | 154k | 154k | 139k | 154k |
| $R_4 = 1k$ | 950 | 1010 | 1010 | 1010 | 1010 | 1010 |
| $R_5 = 71.5$ | 70 | 80 | 80 | 70 | 80 | 70 |
| $R_6 = 37.4k$ | 37.4k | 37.4k | 37.4k | 37.4k | 37.4k | 37.4k |
| $R_7 = 154k$ | 161k | 161k | 146k | 161k | 146k | 146k |
| $R_{11} = 110k$ | 115k | 115k | 104k | 115k | 104k | 104k |
| $R_{12} = 110k$ | 104k | 115k | 104k | 104k | 104k | 104k |

Results at DC - Elliptic filter

Parameter Combinations Leading to Min Values of V-Transform coefficients with $\alpha = 0.05$

| Circuit Parameter, (ohm) | V_{C_0} | V_{C_1} | V_{C_2} | V_{C_3} | V_{C_4} | V_{C_5} |
|--------------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| $R_1 = 19.6k$ | 20.5k | 18.6k | 18.6k | 20.5k | 20.5k | 20.5k |
| $R_2 = 196k$ | 205k | 186k | 205k | 205k | 205k | 186k |
| $R_3 = 147k$ | 150k | 139k | 139k | 146k | 154k | 139k |
| $R_4 = 1k$ | 1010 | 950 | 950 | 950 | 950 | 950 |
| $R_5 = 71.5$ | 80 | 70 | 70 | 80 | 70 | 80 |
| $R_6 = 37.4k$ | 39.2k | 39.2k | 39.2k | 39.2k | 35.5k | 39.2k |
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| $R_{11} = 110k$ | 104k | 104k | 115k | 104k | 115k | 115k |
| $R_{12} = 110k$ | 115k | 104k | 115k | 115k | 115k | 115k |

Results at DC - Elliptic filter

Fault detection results for some injected faults

| Circuit Parameter | Out of bound polynomial coeff. | Fault detected? | Out of bound V-Transform coeff. | Fault detected? |
|-------------------------|--------------------------------|-----------------|---------------------------------|-----------------|
| R ₁ down 15% | a_3, a_4 | Yes | $V_{C_0} - V_{C_4}$ | Yes |
| R ₂ down 10% | a_2 | Yes | V_{C_2}, V_{C_5} | Yes |
| R ₃ up 5% | a_3 | Yes | $V_{C_1}, V_{C_2}, V_{C_3}$ | Yes |
| R ₄ down 15% | a_0 | Yes | $V_{C_0} - V_{C_4}$ | Yes |
| R ₅ up 10% | a_4 | Yes | V_{C_0}, V_{C_4} | Yes |
| R ₇ up 5% | None | No | V_{C_1}, V_{C_2} | Yes |
| R ₁₁ up 5% | None | No | V_{C_4}, V_{C_5} | Yes |
| R ₁₂ down 5% | None | No | V_{C_4}, V_{C_5} | Yes |

Conclusion and Future work

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- Technique for parametric fault detection in analog circuits – faults as small as 5% were uncovered for benchmark elliptic filter
- An Enhanced technique for uncovering parametric faults by increasing sensitivity of polynomial coefficients to circuit parameters

Future work

- Multitone testing of circuits based on V-Transform coefficients

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Future work

- Multitone testing of circuits based on V-Transform coefficients
- Diagnosis of parametric faults based on V-Transform coefficients
- Techniques for optimal choice of frequencies at which CUT could be excited

Acknowledgments

- Prof. R Vittala Rao, CEDT, IISc
- Pawan Kumar, SERC, IISc
- Pramod Subramanyan, SERC, IISc

Thanks for your Attention!