Test and Diagnosis of Analog Circuits using Moment Generating Functions

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Outline

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Ideal Test Signature For An Analog Circuit

Wish list for an analog circuit test signature

- Suitable for large class of circuits

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- Large number of observables – handy in diagnosis
- Low test time
- Low design complexity of the input signal

In This Talk

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1. Evaluate probability moments of output as a metric for testing analog circuits with Gaussian noise as the input excitation
   - AWGN as input requires minimum signal design effort

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   - AWGN as input requires minimum signal design effort
2. Use probability moments as a metric for parametric fault diagnosis in analog circuits

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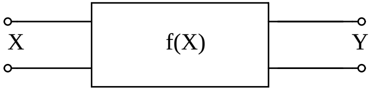
Basic Idea



Random Variable Transformation

Premise

- Circuit is a function $f(.)$ transforming random variable X to a random variable Y.
  This implies circuit can be characterized by the statistics of output (Y), such as probability density function and moments for a given input random variable probability distribution
- Circuit specifications can be related to moments

Probability Moment

Definition

Probability moment of a random variable

The n^{th} moment, μ_n for all $n = 1 \cdots N$ of a continuous random variable $X \geq 0$, and having a pdf given by $f(X)$, is defined as

$$\mu_n = \int_{X=0}^{\infty} X^n f(X) \, \mathrm{d}X$$

Probability Moment

A Quick Example

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The n^{th} moment of X, μ_n for all $n = 1 \cdots N$

$$\begin{aligned} \mu_n &= \int_0^\infty X^n f(X)\,\mathrm{d}X \\ &= \int_0^\infty X^n e^{-X}\,\mathrm{d}X \\ &= \Gamma(n+1) = n! \end{aligned}$$

$$\implies \mu_1 = 1, \mu_2 = 2, \mu_3 = 6, \mu_4 = 24, \cdots$$

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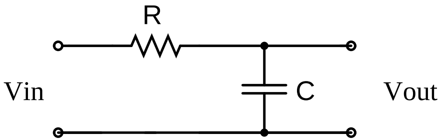
Minimum fractional deviation, ρ, in a circuit element, of nominal value g, such that $g \to g(1 \pm \rho)$, causes, at least one of the moments, μ_i to violate the following inequality

$$\mu_{i,\text{min}} < \mu_i < \mu_{i,\text{max}} \quad \forall \mu_i,\ 1 \leq i \leq n$$

RC Filter

MSDF Calculation - An Example



RC Filter

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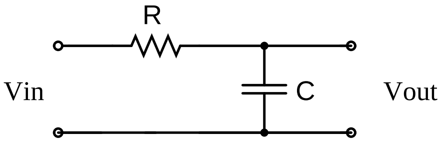

$$\mu_2 = \frac{N_o \pi}{4RC}$$

N_o: Input noise power spectral density, R: Resistance, C: Capacitance.

RC Filter

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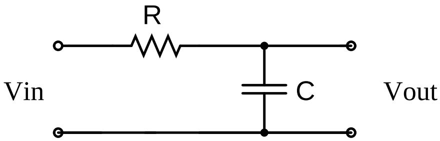

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A fractional deviation ρ in R, such that $\overline{\mu_2} - \mu_2 \geq \mu_0$, results in

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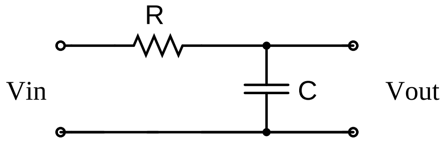

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N_o: Input noise power spectral density, R: Resistance, C: Capacitance.

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$$\rho = \frac{4\mu_0 CR}{N_o \pi - 4\mu_0 CR}$$

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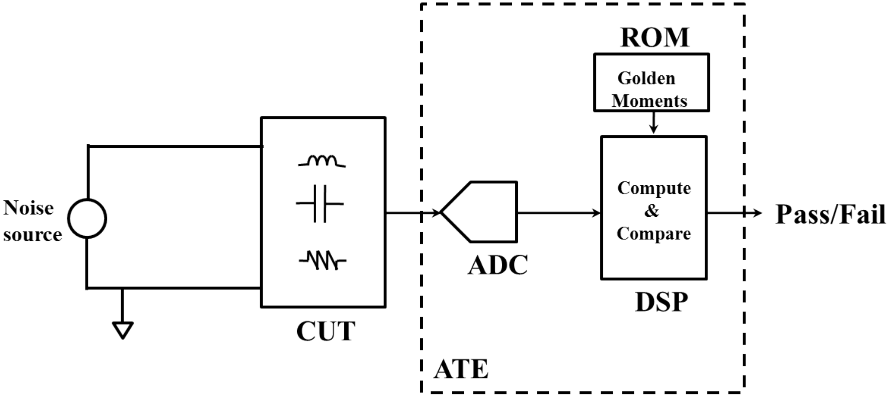
Test Setup


ROM
Golden Moments
Noise source
CUT
ADC
Compute & Compare
DSP
Pass/Fail
ATE

Fault Simulation

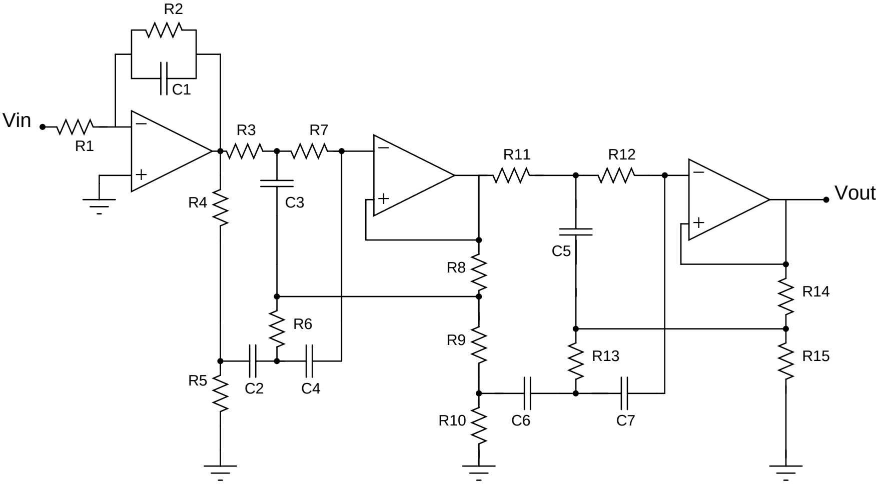
1. Start
2. Apply inputs, sampled from a Gaussian probability density function
3. Record output values for each of these inputs and estimate the output probability density function (PDF)
4. Compute moments(μ_i) of the estimated PDF up to the desired order (say N)
5. Repeat steps 1-3, with circuit component values sampled uniformly in their fault free tolerance range
6. Find min-max values of each moment (μ_i) from $i = 1 \cdots N$ across all simulations
7. Stop

Test Procedure

1. Start
2. Apply inputs, sampled from a Gaussian probability density function
3. Record output values for each of these inputs and estimate the output probability density function (PDF)
4. Compute the moments of the estimated output PDF
5. $\mu_i > \mu_{i,max}$ or $\mu_i < \mu_{i,min}$. Yes or No ?
6. If yes, conclude circuit under test is faulty. If not, repeat the test for next moment
7. If all coefficients are inside the bounds, subject circuit under test to further tests. Stop

Outline

Results – Benchmark Elliptic Filter



Results

Elliptic Filter - Fault Simulation

Parameter combinations leading to maximum values of moments with γ = 0.05

Circuit Parameter (ohm, nF)	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6
$R_1 = 19.6k$	19.6k	20.58k	19.6k	20.58k	20.58k	18.62k
$R_2 = 196k$	186.2k	205.8k	205.8k	205.8k	186.2k	186.2k
$R_3 = 147k$	139.65k	147k	139.65k	139.65k	154.35k	147k
$R_4 = 1k$	1050	1050	950	1000	1050	950
$R_5 = 71.5$	75.075	67.925	75.075	71.5	75.075	67.925
$R_6 = 37.4k$	37.4k	37.4k	37.4k	39.27k	39.27k	37.4k
$R_7 = 154k$	154k	154k	154k	146.3k	154k	154k
$C_3 = 2.67$	2.8035	2.8035	2.67	2.5365	2.5365	2.5365
$C_4 = 2.67$	2.8035	2.67	2.5365	2.67	2.5365	2.67
$C_5 = 2.67$	2.8035	2.67	2.8035	2.5365	2.5365	2.67
$C_6 = 2.67$	2.8035	2.5365	2.8035	2.5365	2.67	2.8035
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$R_3 = 147k$	147k	154.35k	154.35k	139.65k	154.35k	154.35k
$R_4 = 1k$	950	1000	1050	950	1050	950
$R_5 = 71.5$	67.925	71.5	75.075	75.075	67.925	71.5
$R_6 = 37.4k$	39.27k	37.4k	35.53k	39.27k	35.53k	35.53k
$R_7 = 154k$	146.3k	161.7k	154k	161.7k	154k	154k
$C_3 = 2.67$	2.8035	2.8035	2.8035	2.8035	2.5365	2.8035
$C_4 = 2.67$	2.8035	2.8035	2.8035	2.8035	2.8035	2.67
$C_5 = 2.67$	2.67	2.8035	2.67	2.8035	2.8035	2.67
$C_6 = 2.67$	2.8035	2.5365	2.8035	2.5365	2.67	2.8035
$C_7 = 2.67$	2.67	2.5365	2.8035	2.67	2.67	2.8035

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Elliptic Filter - Fault Detection

Fault detection for some injected faults

Circuit Parameter	Out of bound moment	Fault detected?
R_1 down 12%	$\mu_3,\ \mu_1$	Yes
R_2 down 10%	μ_4	Yes
R_3 up 12%	$\mu_1,\ \mu_2$	Yes
R_5 up 10%	μ_4	Yes
R_7 up 15%	$\mu_5,\ \mu_6$	Yes
R_{11} up 15%	μ_3	Yes
R_{12} down 15%	$\mu_2,\ \mu_6$	Yes
C_4 up 12%	μ_4	Yes
C_5 down 15%	$\mu_1,\ \mu_6$	Yes

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Fault Diagnosis using Moments

In a nutshell

- Create a mapping between catastrophic faults and moments displaced by them

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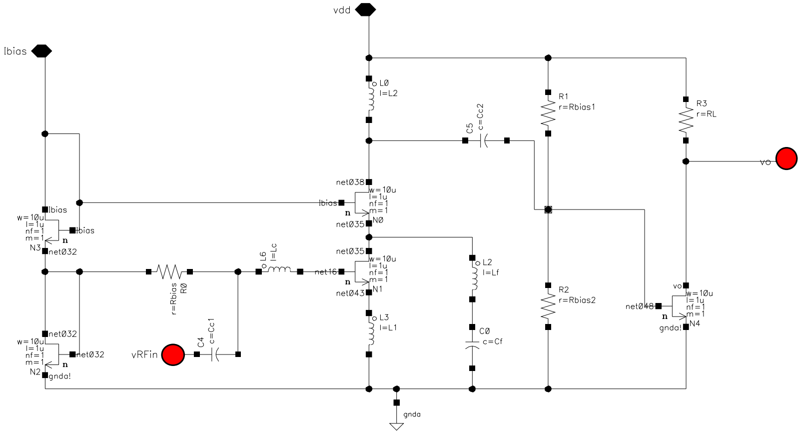
Fault Diagnosis using Moments

In a nutshell

- Create a mapping between catastrophic faults and moments displaced by them
- Faults causing deviation of unique set of moments are diagnosable
- Faults that share the same set of failing moments are not uniquely diagnosable, but result in a smaller set for further investigation – Expanding further into higher order moments can resolve the problem

Fault Diagnosis Example

Low Noise Amplifier - Schematic



Results

Low Noise Amplifier - Fault Diagnosis

Fault diagnosis of some catastrophic faults

Component (ohm, nH, fF)	Nature of fault	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	Uniquely Diagnosable ?
$L_1 = 1.5$	short	✓	✓	✓	✓	✓	✓	Yes
$L_2 = 1.5$	short	✓		✓		✓	✓	Yes
$C_{C2} = 100$	short	✓	✓		✓	✓	✓	Yes
$R_{bias1} = 100k$	short	✓	✓		✓			Yes
$N_0(D - S)$	short	✓					✓	Yes
$N_1(D - S)$	short		✓		✓			Yes
$R_{bias} = 10$	short		✓	✓	✓	✓	✓	No (2)[a]
$L_C = 1$	short		✓	✓	✓	✓	✓	No (2)
$R_{bias} = 10$	open	✓	✓	✓				Yes
$R_{bias1} = 100k$	open		✓	✓		✓		Yes
$N_0(D - S)$	open	✓		✓			✓	Yes
$N_1(D - S)$	open	✓		✓	✓		✓	Yes

[a] μ_7 deviates with R_{bias} short, but not L_C short

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Conclusion

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- Circuit test using probability moments of the output as a test metric was proposed
- Test procedure was implemented on an elliptic filter, with detection of faults of sizes $\approx 12\% - 15\%$
- Catastrophic fault diagnosis based on moments was proposed

Open Questions

Possible directions for future work

- Optimal order to which moments of the output are to be expanded
- Fault diagnosis using sensitivity of moments to circuit parameters as opposed to moments themselves
- Other noise distributions for input excitation