Multi-Tone Testing of Linear and Nonlinear Analog Circuits using Polynomial Coefficients

Suraj Sindia
Centre for Electronic Design and Technology
Indian Institute of Science
Bangalore 560012, India
Email: ssuraj@cedt.iisc.ernet.in

Virendra Singh
Supercomputer Education and Research Centre
Indian Institute of Science
Bangalore 560012, India
Email: viren@serc.iisc.ernet.in

Vishwani D. Agrawal
Department of Electrical and Computer Engineering
Auburn University
Auburn, AL 36849, USA
Email: vagrawal@eng.auburn.edu

***Abstract*—A method of testing for parametric faults of analog circuits based on a polynomial representation of fault-free function of the circuit is presented. The response of the circuit under test (CUT) is estimated as a polynomial in the applied input voltage at relevant frequencies in addition to DC. Classification of CUT is based on a comparison of the estimated polynomial coefficients with those of the fault free circuit. This testing method requires no design for test hardware as might be added to the circuit by some other methods. The proposed method is illustrated for a benchmark elliptic filter. It is shown to uncover several parametric faults causing deviations as small as 5% from the nominal values.**

***Index Terms*—Multi-tone test, Parametric faults, Analog circuit test, Curve fitting, Polynomial coefficient testing.**

I. Introduction

Several methods have been proposed for parametric fault testing of analog circuits [1], [2], [3], [4], [5], [6], [7], [8]. A prominent method in the industry is the $\mathrm{I_{DDQ}}$ testing where quiescent current from the supply rail is monitored and sizable deviations from its expected value are used for classifying the circuit as faulty or good. However, this requires augmentation of the CUT. For example, in the simplest case a regulator supplying power to any sizable circuit has to be augmented with a current sensing resistor and an ADC (for digital output). Subsequently, analysis is performed on the sensed current. $\mathrm{I_{DDQ}}$ is found suitable only for catastrophic faults as the current drawn from the supply may be distinguishable when there is some "large enough" fault to change the quiescent current by a distinguishable amount. For example, with resistor $\mathrm{R_2}$ being open in Figure 1, the current drawn from supply can change by 50% of its nominal quiescent value. Such faults can typically be found by monitoring $\mathrm{I_{DDQ}}$ using a current sensor. However, parametric deviations, say, less than 10% from their nominal value cannot be observed using this scheme. This is especially so for the very deep submicron circuits where the leakage currents can be comparable to the defect induced current [9]. It is therefore useful to develop a method to detect parametric faults while testing with little or no circuit augmentation.

To address the issue of parametric deviation, we would typically need more observables to have an idea about the parametric drift in circuit parameters. This would mean an increase in the complexity of the sensing circuit. However, we would also want minimal augmentation to tap any of the internal circuit nodes or currents. To overcome these seemingly contrasting requirements the method intended should have some way of "seeing through" the circuit with only the outputs and inputs at its disposal. References [10], [11] give such strategies for linear circuits as described earlier.

To extend this idea to general non-linear circuits, while it is also remains applicable to linear circuits, we adopt a strategy, where in, we express the function of the circuit as a polynomial using a Taylor series expansion [12] in terms of input voltage v_{in}, about the point v_{in} = 0 as follows:

$$v_{out} = f(v_{in}) = f(0) + \frac{f'(0)}{1!}v_{in} + \frac{f''(0)}{2!}v_{in}^2 + \frac{f'''(0)}{3!}v_{in}^3 + \cdots + \frac{f^{(n)}(0)}{n!}v_{in}^n + \cdots \quad (1)$$

where $f(x)$ is a real function of x. Note that in the above expansion the point of expansion of v_{in} can be about any operating voltage of the circuit at desired frequencies.

This method is very general as any analog circuit can be tested using this model. The technique applies equally well to linear circuits, which are a subclass of the general non-linear circuits considered in this paper. The accuracy, resolution and observability of faults uncovered depends on the degree of expansion of the coefficients in (2). Ignoring the higher order terms in (1), we can expand v_{out} up to the n^{th} power of v_{in}, which gives us the approximation in (2). In order to increase the available observables to better track down parametric faults we can expand v_{out} at *multiple frequencies*. Thus, we will have $m \times (n+1)$ observables where m is the number of tones (frequencies) including DC at which v_{out} is expanded and n is the degree of expansion [13]:

$$v_{out} = a_0 + a_1 v_{in} + a_2 v_{in}^2 + \cdots + a_n v_{in}^n \quad (2)$$

where $a_0, a_1, a_2, \ldots, a_n$ are all real functions of circuit parameters $p_k \forall\ k$.
The special case of DC test, that detects a subset of faults, was given in our recent paper [14]. Further, we assume that

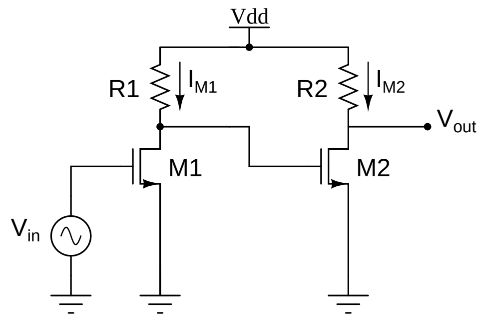

Fig. 1. Cascade amplifier.

normal parameter variations (normal drift) in a good circuit are within a fraction α of their nominal value, where $\alpha << 1$. That is, every parameter p_i is allowed to vary within the range $p_{k,nom}(1-\alpha) < p_k < p_{k,nom}(1+\alpha)\ \forall k$, where $p_{k,nom}$ is the nominal value of parameter p_k. Whenever one or more of the coefficient values slip outside its individual hypercube we get a different set of coefficients reflecting a detectable fault. Therefore, equation (3) describes the hypercube for all parameters that correspond to either good machine values or undetectable parametric faults [10], [2], [8]:

$$a_{i,\min} < a_i < a_{i,\max} \quad \forall\ i, \quad 0 \le i \le n \tag{3}$$

This paper is organized as follows. Section 2 analyzes the coefficients of the polynomial expansion of the function $f(v_{in})$ and determines the detectable fault sizes of parameters. In Section 3, we describe the problem at hand and discuss the proposed solution with an example. In Section 4, we generalize the solution to an arbitrarily large circuit. Section 5 presents the simulation results for some standard circuits and we conclude in Section 6.

II. Preliminaries

The coefficients $a_i\ \forall i\ 0 \le i \le n$ are, in general, non-linear functions of circuit parameters $p_k\ \forall k$. The rationale behind using these coefficients as metrics in classifying CUT as faulty or fault free is based on the dependence of the coefficients on circuit parameters.

A. Analysis of Polynomial Coefficients

We derive several significant results.

Theorem 1: If coefficient a_i is a monotonic function of all parameters, then a_i takes its limiting (maximum and minimum) values when at least one or more of the parameters are at the boundaries of their individual hypercube [14].

Lemma 1: If coefficient a_i is a non-monotonic function of one or more circuit parameters p_i, then a_i can take its limiting values anywhere inside the hypercube enclosing the parameters.

From Theorem 1 and Lemma 1 it is clear that by exhaustively searching the space in the hypercube of each parameter we can get the maximum and minimum values of the polynomial coefficient. Typically this can be formulated as a non-linear optimization problem to find the maximum and minimum values of coefficient with constraints on parameters allowing only a normal drift.

Theorem 2: In polynomial expansion of non-linear analog circuit there exists at least one coefficient that is a monotonic function of all circuit parameters [14].

From Lemma 1 and Theorem 2 we find that circuit parameter deviations have a bearing on coefficients and monotonically varying coefficients can be used to detect parametric faults of the circuit parameters.

Theorem 3: A continuous non-monotonic function $f : \Re \rightarrow \Re$ can be decomposed into piecewise monotonic functions [15] as follows:

$$\begin{gathered} f(x) = f(x)u(x_0 - x) + f(x)\left(u(x-x_0) - u(x-x_1)\right) + \\ f(x)\left(u(x-x_1) - u(x-x_2)\right) + \cdots \\ +f(x)\left(u(x-x_{n-1}) - u(x-x_n)\right) \end{gathered} \tag{4}$$

where $x_0, x_1, \cdots x_n$ are all stationary points of $f(x)$ and $u(x) = \begin{cases} 1 & \forall\ x \ge 0 \\ 0 & \forall\ x < 0 \end{cases}$

Using Theorem 3, we can express every polynomial coefficient as a monotonic function of circuit parameters and thus we can use every coefficient to track the drifts in circuit parameters.

B. Definitions

Definition 1: Minimum size detectable fault (MSDF), (ρ) of a parameter is defined as the minimum fractional deviation of a circuit parameter from its nominal value for it to be detectable with all other parameters being held at their nominal values. The fractional deviation can be positive or negative and is named upside-MSDF (UMSDF) or downside-MSDF (DMSDF), accordingly.

Definition 2: Nearly-minimum size detectable fault (NMSDF), (ρ^*) of a parameter is defined as some fractional deviation of the circuit parameter from its nominal value with all the other parameters being held at their nominal values that is close to its MSDF with an error, ϵ (infinitesimally small). That is to say,

$$\epsilon = |\rho - \rho*| \qquad\qquad \epsilon << 1 \tag{5}$$

NMSDF also has notions of upside and downside as in case of MSDF. In equation (5), ϵ can be perceived as a coefficient of uncertainty about the MSDF of a parameter. Let ψ be the set of all coefficient values spanned by the parameters while varying within their normal drifts, i.e.,

$$\begin{gathered} \psi = \{\upsilon_0, \upsilon_1, \cdots, \upsilon_n \,|\upsilon_0 \in A_0, \upsilon_1 \in A_1, \cdots, \upsilon_n \in A_n\} \\ \forall_k \quad p_{k,nom}(1-\alpha) < p_k < p_{k,nom}(1+\alpha) \end{gathered}$$

Note that by Definitions 1 and 2, ψ includes all possible values of coefficients that are not detectable. Any parametric fault inducing coefficient value outside this set ψ will result in a detectable fault.

III. Problem Description and Sketch of Solution

We shall first give an illustrative example of calculation of limits for polynomial coefficients for a simple circuit using

MOS transistors. We shall follow this up with MSDF values for the circuit parameters.

Example. Two stage amplifier
Consider the cascade amplifier shown in Figure 1. The output voltage $\mathrm{V_{out}}$ in terms of input voltage results in a fourth degree polynomial equation as follows:

$$v_{out} = a_0 + a_1 v_{in} + a_2 v_{in}^2 + a_3 v_{in}^3 + a_4 v_{in}^4 \tag{6}$$

where the constants a_0, a_1, a_2, a_3 are defined symbolically in (7) for M1 and M2 operating in saturation region. Nominal values of $\mathrm{V_{DD}}$=1.2V, $\mathrm{V_T}$= 400mV, $\left(\frac{W}{L}\right)_1 = \frac{1}{2}\left(\frac{W}{L}\right)_2 = 20$, and $\mathrm{K} = 100\mu\mathrm{A/V^2}$ are substituted to get coefficients in terms of parameters $\mathrm{R_1}$ and $\mathrm{R_2}$ as given by (8).

$$\begin{gathered}
a_0 = V_{DD} - R_2 K \left(\tfrac{W}{L}\right)_2 \left\{ \begin{array}{l} (V_{DD}-V_T)^2 + \\ R_1^2 K^2 \left(\tfrac{W}{L}\right)_1^2 V_T^4 - \\ 2(V_{DD}-V_T) R_1 \left(\tfrac{W}{L}\right)_1 V_T^2 \end{array} \right\} \\
a_1 = R_2 K \left(\tfrac{W}{L}\right)_2 \left\{ \begin{array}{l} 4R_1^2 K^2 \left(\tfrac{W}{L}\right)_1^2 V_T^3 \\ +2(V_{DD}-V_T) R_1 K \left(\tfrac{W}{L}\right)_1 V_T \end{array} \right\} \\
a_2 = R_2 K \left(\tfrac{W}{L}\right)_2 \left\{ \begin{array}{l} 2(V_{DD}-V_T) R_1 K \left(\tfrac{W}{L}\right)_1 \\ -6R_1^2 K^2 \left(\tfrac{W}{L}\right)_1^2 V_T^2 \end{array} \right\} \\
a_3 = 4V_T K^3 \left(\tfrac{W}{L}\right)_1^2 \left(\tfrac{W}{L}\right)_2^2 R_1^2 R_2 \\
a_4 = -K^3 \left(\tfrac{W}{L}\right)_1^2 \left(\tfrac{W}{L}\right)_2^2 R_1^2 R_2
\end{gathered} \tag{7}$$

$$\begin{gathered}
a_0 = 1.2 - R_2 \left(\begin{array}{l} 2.56 \times 10^{-3} + 1.024 \times 10^{-7} R_1^2 \\ -5.12 \times 10^{-4} R_1 \end{array} \right) \\
a_1 = 4.096 \times 10^{-9} R_1^2 R_2 + 5.12 \times 10^{-6} R_1 R_2 \\
a_2 = 1.28 \times 10^{-5} R_1 R_2 - 1.536 \times 10^{-8} R_1^2 R_2 \\
a_3 = 2.56 \times 10^{-8} R_1^2 R_2 \\
a_4 = 1.6 \times 10^{-8} R_1^2 R_2
\end{gathered} \tag{8}$$

To find the limiting values of the coefficient a_0 we assume the parameters $\mathrm{R_1}$ and $\mathrm{R_2}$ deviate by fractions x and y from their nominal values, respectively. Maximizing a_0 we have the objective function as given by (9), subject to constraints in (10–14). Note that here we have set out to find MSDF of $\mathrm{R_1}$. Similar approach can be used to find the MSDF of $\mathrm{R_2}$.

$$1.2 - R_{2,nom}(1+y) \left\{ \begin{array}{l} 2.56 \times 10^{-3} + \\ 1.024 \times 10^{-7} R_{1,nom}^2 (1+x)^2 \\ -5.12 \times 10^{-4} R_{1,nom}(1+x) \end{array} \right\} \tag{9}$$

$$\begin{aligned}
&4.096 \times 10^{-9} R_{1,nom}^2 (1+x)^2 R_{2,nom}\ (1+y) \\
&+5.12 \times 10^{-6} R_{1,nom}(1+x) R_{2,nom}(1+y) \\
&\quad = 4.096 \times 10^{-9} R_{1,nom}^2 (1+\rho)^2 R_{2,nom} \\
&\quad + 5.12 \times 10^{-6} R_{1,nom}(1+\rho) R_{2,nom}
\end{aligned} \tag{10}$$

TABLE I

MSDF FOR CASCADE AMPLIFIER OF FIGURE 1 WITH α = 0.05.

Circuit parameter	%upside MSDF	%downside MSDF
Resistor $\mathrm{R_1}$	10.3	7.4
Resistor $\mathrm{R_2}$	12.3	8.5

$$\begin{aligned}
&1.28 \times 10^{-5} R_{1,nom}(1+x) R_{2,nom}(1+y) \\
&-1.536 \times 10^{-8} R_{1,nom}^2 (1+x)^2 R_{2,nom}\ (1+y) \\
&\quad = 1.28 \times 10^{-5} R_{1,nom}(1+\rho) R_{2,nom} \\
&\quad - 1.536 \times 10^{-8} R_{1,nom}^2 (1+\rho)^2 R_{2,nom}
\end{aligned} \tag{11}$$

$$\begin{aligned}
&2.56 \times 10^{-8} R_{1,nom}^2 (1+x)^2 R_{2,nom}(1+y) \\
&\quad = 2.56 \times 10^{-8} R_{1,nom}^2 (1+\rho)^2 R_{2,nom}
\end{aligned} \tag{12}$$

$$\begin{aligned}
&1.6 \times 10^{-8} R_{1,nom}^2 (1+x)^2 R_{2,nom}(1+y) \\
&\quad = 1.6 \times 10^{-8} R_{1,nom}^2 (1+\rho)^2 R_{2,nom}
\end{aligned} \tag{13}$$

$$-\alpha \le x, y \le \alpha \tag{14}$$

The extreme values for x and y on solving the set of equations (10–14) are obtained as, x = $-\alpha$ and y = $-\alpha$, this gives us the MSDF value for $\mathrm{R_1}$, as ρ in (15).

$$\rho = (1-\alpha)^{1.5} - 1 \approx 1.5\alpha - 0.375\alpha^2 \tag{15}$$

Table I gives the MSDF for $\mathrm{R_1}$ and $\mathrm{R_2}$ based on above calculation.

IV. GENERALIZATION

In general, the calculation as described above cannot be done for an arbitrarily large circuit. Such circuits are handled by obtaining a nominal numeric polynomial expansion of the fault free circuit. This is done by sweeping the input voltage across all possible values and noting the corresponding output voltages using any of the standard circuit simulators like SPICE. Now, the output voltage is plotted against the input voltage. A polynomial is fitted to this curve and the coefficients of this polynomial are taken to be the nominal coefficients of the desired polynomial. The circuit is simulated for different drifts in the parameter values at equally spaced points from inside the hypercube enclosing each circuit parameter, spaced at a suitably chosen resolution(=ϵ). Polynomial coefficients are obtained for each of these simulations. The maximum and the minimum values of a coefficient in this search are taken as the limiting values on that coefficient. This process of modeling the circuit as a polynomial expansion and obtaining limit values on coefficients is repeated at "key" frequencies of interest. For example, the cut-off frequency in case of a non-linear filter can be a good candidate for such characterization. Once the limit values on all coefficients have been determined the CUT is subjected to full range of input at DC and each of the "key" frequencies. Its response to input sweep is curve fitted to a polynomial of order same as the fault free circuit. If there are any coefficients that lay outside the limit values of corresponding coefficients of the fault free circuit, we can conclude the CUT is faulty. The converse is also true with a high probability that is inversely proportional to coefficient of uncertainty ϵ. Flow chart in Figure 2 summarizes the process

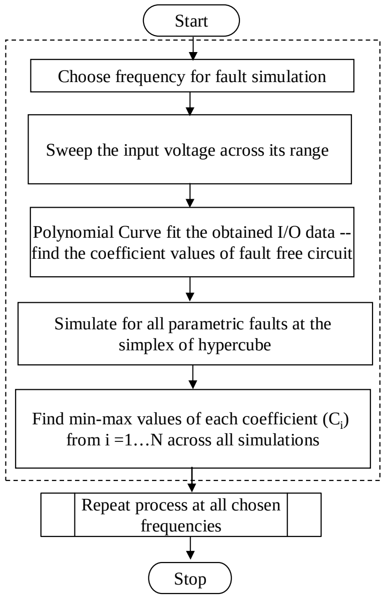

Fig. 2. Flow chart showing fault simulation process and bounding of coefficients.

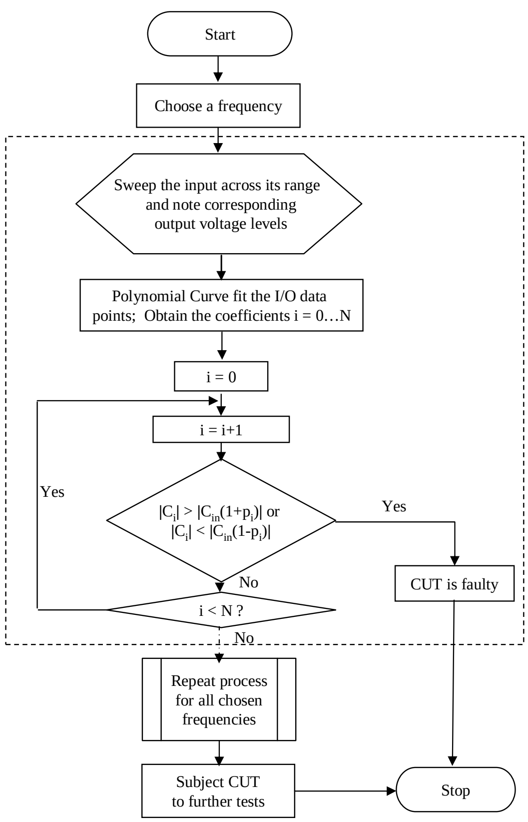

Fig. 3. Flow chart outlining test procedure for CUT.

of numerically finding the polynomial and finding the bounds on coefficients. Flow chart in Figure 3 outlines the procedure to test CUT using the described method.

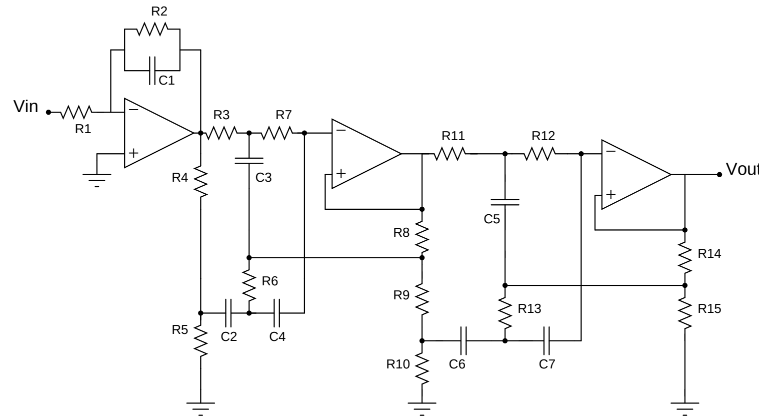

Fig. 4. Elliptic filter.

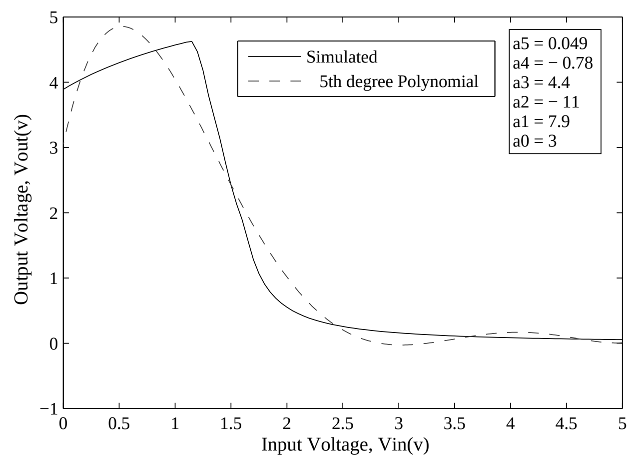

Fig. 5. Curve-fitting polynomial with coefficients at frequency = 100Hz.

V. EXPERIMENTAL RESULTS

We subjected an elliptic filter shown in Figure 4 to Polynomial Coefficient based test. The circuit parameter values are as in the benchmark circuit maintained by Stroud et al. [16]. We simulated the circuit at four different frequencies. Two of them were chosen close to its 3-dB cut-off frequency (f_c), which is 1000Hz. The estimated polynomial expansion obtained by curve fitting the I/O data at DC and the frequencies f=100Hz, 900Hz, 1000Hz, 1100Hz are given by equations 16 through 20. The plots tracing I/O response with polynomial at frequencies from 100Hz through 1100Hz is shown in Figures 5 through 8. The combinations of parameter values leading to limits on the coefficients for the tone at 1000Hz are shown in Table II. Further, the pass/fail detectabilities of several injected faults are tabulated in Table III.

$$v_{out} = 4.5341 - 3.498v_{in} - 2.5487v_{in}^2 + 2.1309v_{in}^3 - 0.50514v_{in}^4 + 0.039463v_{in}^5 \quad (16)$$

$$v_{out} = 3 + 7.9v_{in} - 11v_{in}^2 + 4.4v_{in}^3 - 0.78v_{in}^4 + 0.049v_{in}^5 \quad (17)$$

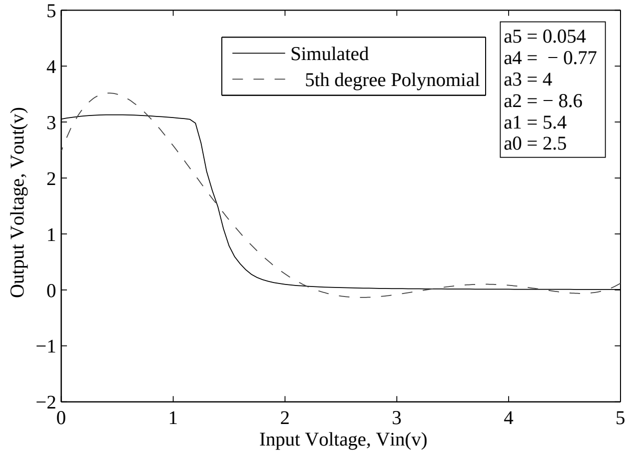

Fig. 6. Curve-fitting polynomial with coefficients at frequency = 900Hz.

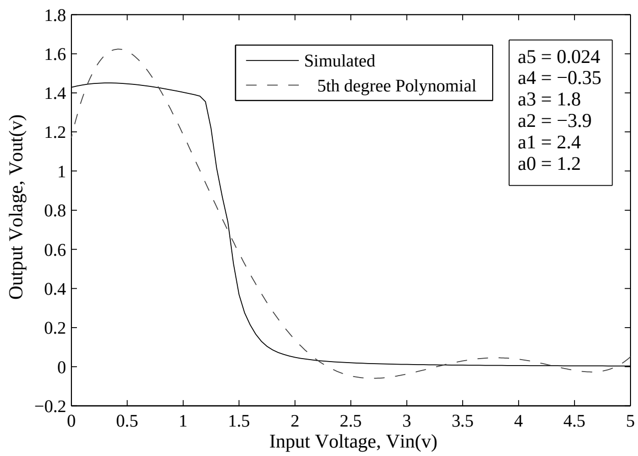

Fig. 7. Curve-fitting polynomial with coefficients at frequency = 1000Hz.

$$v_{out} = 2.5 + 5.4v_{in} - 8.6v_{in}^2 + 4v_{in}^3 - 0.77v_{in}^4 + 0.054v_{in}^5 \quad (18)$$

$$v_{out} = 1.1707 + 2.4132v_{in} - 3.8777v_{in}^2 + 1.8035v_{in}^3 - 0.3465v_{in}^4 + 0.023962v_{in}^5 \quad (19)$$

$$v_{out} = 0.23 + 0.48v_{in} - 0.74v_{in}^2 + 0.34v_{in}^3 - 0.063v_{in}^4 + 0.0043v_{in}^5 \quad (20)$$

VI. Conclusion

A new approach for testing non-linear circuits based on polynomial expansion of the circuit function has been proposed. By expanding polynomial coefficients at critical frequencies the fault coverage is significantly improved, yielding a minimum size of detectable faults in some parameters as low as 5%. The method can be extended to sensitivity based fault diagnosis with probabilistic confidence levels in parameter drifts. In our ongoing work we are trying to enhance the sensitivity of polynomial coefficients further to the circuit parameters so that parametric faults of smaller sizes can be uncovered.

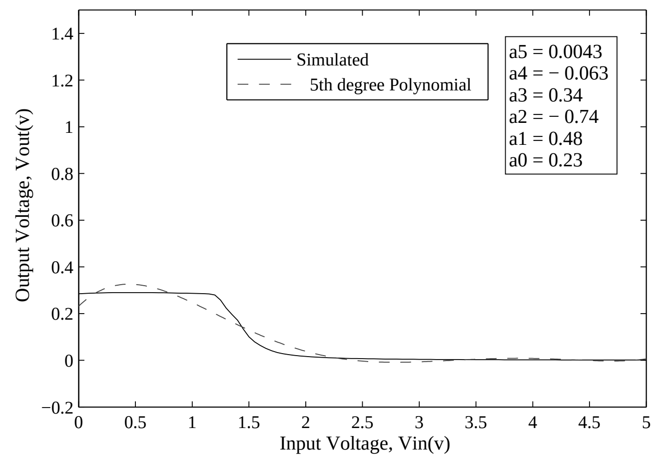

Fig. 8. Curve-fitting polynomial with coefficients at frequency = 1100Hz.

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TABLE II

PARAMETER COMBINATIONS LEADING TO MAX AND MIN VALUES OF COEFFICIENTS WITH α = 0.05 AT 1000HZ.

Circuit Parameters (Resistance in Ω,Capacitance in Farad)												
Nominal Values	$a_{0,max}$	$a_{1,max}$	$a_{2,max}$	$a_{3,max}$	$a_{4,max}$	$a_{5,max}$	$a_{0,min}$	$a_{1,min}$	$a_{2,min}$	$a_{3,min}$	$a_{4,min}$	$a_{5,min}$
$R_1 = 19.6k$	18.6k	18.6k	20.5k	20.5k	20.5k	18.6k	18.6k	18.6k	18.6k	18.6k	20.5k	20.5k
$R_2 = 196k$	205k	205k	205k	205k	186k	186k	205k	186k	186k	205k	205k	205k
$R_3 = 147k$	139k	139k	154k	139k	139k	139k	139k	139k	154k	139k	139k	139k
$R_4 = 1k$	950	950	1.05k	1.05k	1.05k	1.05k	1.05k	950	1.05k	950	950	1.05k
$R_5 = 71.5$	75	67	75	67	67	75	75	75	67	67	75	67
$R_6 = 37.4k$	35k	39k	39k	35k	35k	39k	39k	39k	35k	35k	35k	35k
$R_7 = 154k$	146k	146k	161k	161k	146k	146k	146k	146k	161k	161k	146k	146k
$R_8 = 260$	247	273	273	247	247	273	273	247	273	247	273	247
$R_9 = 740$	703	777	703	703	777	703	703	703	777	703	703	703
$R_{10} = 500$	475	525	525	475	525	525	475	525	475	475	525	475
$R_{11} = 110k$	115k	115k	115k	104k	104k	104k	115k	115k	104k	115k	104k	104k
$R_{12} = 110k$	104k	104k	115k	115k	115k	115k	115k	115k	104k	104k	115k	104k
$R_{13} = 27.4k$	28.7k	26k	26k	26k	28.7k	28.7k	26k	26k	28.7k	26k	28.7k	26k
$R_{14} = 40$	42	38	42	38	38	42	42	38	42	42	38	42
$R_{15} = 960$	912	912	912	912	912	1k	1k	1k	912	1k	912	912
$C_1 = 2.67n$	2.5n	2.5n	2.5n	2.5n	2.5n	2.5n	2.8n	2.5n	2.8n	2.8n	2.8n	2.5n
$C_2 = 2.67n$	2.5n	2.8n	2.8n	2.5n	2.8n	2.8n	2.8n	2.8n	2.5n	2.8n	2.5n	2.8n
$C_3 = 2.67n$	2.8n	2.8n	2.8n	2.5n	2.8n	2.8n	2.8n	2.8n	2.8n	2.5n	2.8n	2.8n
$C_4 = 2.67n$	2.5n	2.8n	2.5n	2.5n	2.5n	2.5n	2.5n	2.5n	2.8n	2.5n	2.5n	2.8n
$C_5 = 2.67n$	2.5n	2.5n	2.5n	2.5n	2.5n	2.8n	2.8n	2.8n	2.8n	2.8n	2.8n	2.8n
$C_6 = 2.67n$	2.5n	2.8n	2.5n	2.8n	2.5n	2.8n	2.5n	2.5n	2.8n	2.8n	2.8n	2.5n
$C_7 = 2.67n$	2.5n	2.8n	2.8n	2.8n	2.8n	2.5n	2.8n	2.5n	2.5n	2.5n	2.5n	2.8n

TABLE III

RESULTS OF SOME INJECTED FAULTS AT DIFFERENT FREQUENCIES.

Injected fault	Coefficients out of Bounds at					Detected
	DC	f_1=100Hz	f_2=900Hz	f_3=1000Hz	f_4=1100Hz	
R_1 down 15%	$a_0 - a_4$	$a_1 - a_4$	a_3, a_5	a_2, a_4	a_1, a_2	Yes
R_2 down 5%	a_2, a_5	a_1, a_3	a_1, a_5	a_1, a_2, a_5	a_1, a_2	Yes
R_3 up 10%	a_1, a_2, a_3	a_3, a_5	a_0, a_3, a_4	a_1, a_3, a_4	a_1, a_5	Yes
R_4 down 20%	$a_0 - a_3$	$a_1 - a_2$	a_2, a_3	a_1, a_2, a_3	a_2, a_3	Yes
R_5 up 15%	a_0, a_5	a_1	a_0, a_2	a_0, a_2, a_3	a_3	Yes
R_6 up 5%	–	a_1, a_2	a_2, a_3, a_5	a_1, a_3	a_1	Yes
R_7 down 10%	a_2, a_4	a_3, a_5	a_0, a_1, a_2	a_1, a_4, a_5	a_2, a_3	Yes
R_8 up 10%	–	a_2	a_0, a_4	a_0, a_2, a_5	a_3, a_4	Yes
R_9 down 5%	–	a_3, a_2	a_1, a_2, a_4	a_2, a_3, a_5	a_1, a_3	Yes
R_{10} up 15%	–	a_1, a_4	a_1, a_3, a_4	a_0, a_1, a_4	a_1, a_2	Yes
R_{11} down 10%	a_0, a_2	a_3, a_4	a_0, a_1	a_1, a_2, a_4	a_1, a_2	Yes
R_{12} down 15%	a_0, a_4	a_1, a_3	a_1, a_2, a_3	a_1, a_2	a_2, a_5	Yes
R_{13} up 5%	–	a_3, a_5	a_1, a_2	a_1, a_2, a_4	a_0, a_2	Yes
R_{14} up 20%	–	a_1, a_3	a_0, a_3, a_4	a_0, a_1, a_2	a_3, a_4	Yes
R_{15} up 5%	–	a_4	a_3, a_5	a_0, a_1, a_3	a_0, a_5	Yes
C_1 down 10%	–	a_4, a_5	a_4, a_5	a_1, a_2, a_3	a_1, a_4	Yes
C_2 up 10%	–	a_2, a_3	a_1, a_2	a_2, a_3, a_4	a_0, a_4	Yes
C_3 down 15%	–	a_1, a_3	a_0, a_1, a_2	a_4, a_5	a_0, a_1	Yes
C_4 down 10%	–	a_0, a_1	a_1, a_2	a_2, a_3	a_2, a_5	Yes
C_5 up 5%	–	a_0, a_1	a_1, a_5	a_1, a_2	a_3, a_4	Yes
C_6 up 15%	–	a_3, a_4	a_1, a_2, a_4	a_3, a_4, a_5	a_1, a_2	Yes
C_7 up 15%	–	a_1, a_4	a_1, a_3, a_4	a_1, a_3, a_5	a_3, a_4	Yes