## Lecture 4 - Finite State Machines

## Modeling Finite State Machines (FSMs)

- "Manual" FSM design \& synthesis process:

1. Design state diagram (behavior)
2. Derive state table
3. Reduce state table
4. Choose a state assignment
5. Derive output equations
6. Derive flip-flop excitation equations

- Steps 2-6 can be automated, given a state diagram

1. Model states as enumerated type
2. Model output function (Mealy or Moore model)
3. Model state transitions (functions of current state and inputs)
4. Consider how initial state will be forced

## FSM structure



## Mealy Machine and Moore Machine



Moore Machine


## FSM example - Mealy model


entity seqckt is
port ( $x$ : in std_logic; -- FSM input
z: out std_logic; -- FSM output clk: in std_logic ); -- clock
end seqckt;

## FSM example - behavioral model

architecture behave of seqckt is
type states is (A,B,C); -- symbolic state names (enumerate)
signal state: states; --state variable
begin
-- Output function (combinational logic)
z <= ' 1 ' when ((state $=\mathrm{B}$ ) and ( $\mathrm{x}=$ = '1')) --all conditions
or ((state = C) and ( $x=$ ' 1 ') ) $\quad$--for which $z=1$. else ' 0 ';
--otherwise $\mathrm{z}=0$
-- State transitions on next slide

## FSM example - state transitions

```
process (clk) - trigger state change on clock transition
    begin
    if rising_edge(clk) then -- change state on rising clock edge
        case state is -- change state according to x
        when A => if ( }\textrm{x}=\mp@code{\prime}0\mathrm{ ') then
                                state <= A;
            else -- if ( }\textrm{x}=\mp@subsup{=}{}{\prime}1\mathrm{ ')
                state <= B;
            end if;
        when B => if ( }x=\mp@subsup{}{}{\prime}0\mathrm{ ') then
                                state <= A;
            else -- if ( }\textrm{x}=\mp@subsup{}{}{\prime}1\mathrm{ ')
                        state <= C;
            end if;
        when C => if ( }x=`0'0') the
                        state <= C;
            else -- if (x = '1')
                        state <= A;
                            end if;
    end case;
    end if;
end process;
```


## FSM example - alternative model

architecture behave of seqckt is
type states is (A,B,C); -- symbolic state names
(enumerate)
signal pres_state, next_state: states;
begin
-- Model the memory elements of the FSM
process (clk)
begin
if (clk'event and clk=‘1') then pres_state <= next_state;
end if;
end process;
(continue on next slide)


## FSM example (alternate model, continued)

-- Model next-state and output functions of the FSM
-- as combinational logic
process (x, pres_state) -- function inputs begin
case pres_state is -- describe each state when $A=>$ if ( $x=$ ' 0 ') then z <= '0'; next_state <=A;
else -- if ( $x=$ ' 1 ')
z <= '0';
next_state <= B;
end if;
(continue on next slide for pres_state = B and C)

## FSM example (alternate model, continued)

```
when \(B=>\) if ( \(x={ }^{\prime} 0^{\prime}\) ) then
                        z <= '0';
    next_state <=A;
    else
    z <= ' 1 ';
    next_state <= C;
    end if;
when \(\mathrm{C}=>\) if ( \(\mathrm{x}={ }^{\prime} 0^{\prime}\) ) then
    z <= '0';
    next_state <= C;
else
    z <= ' 1 ';
    next_state <=A;
end if;
end case;
end process;
```


## Alternative form for output and next state

 functions (combinational logic)-- Next state function (combinational logic) next_state < = A when ((curr_state $=A$ ) and $(x=$ ' 0 ' $)$ ) or ((curr_state $=B)$ and $\left(x={ }^{\prime} 0^{\prime}\right)$ ) or ((curr_state $=C$ ) and ( $x=$ ' 1 ')) else B when ((curr_state $=1$ ) and $(x=' 1$ ')) else C;
-- Output function (combinational logic)
$z<=$ ' 1 ' when ((curr_state $=B$ ) and $(x=$ '1')) --all conditions or ((curr_state $=C)$ and $(x=' 1$ ')) --for which $z=1$. else ' 0 ';

## Moore model FSM



Write a VHDL code using three process blocks!

## How Verilog Explicit FSM Works

- The nonblocking and blocked assignments are scheduled in the same time step of the simulation in a particular order

1. The nonblocking assignments in the edge-sensitive behavior are sampled first at the beginning of the time step (i.e. before any assignments are made)
2. The blocked assignments in level-sensitive behavior are then executed (with the previous register value because there is no assignment done in Step 1)
3. After Step 2, the nonblocking assignments are completed by assigning LHS variables with the values that were sampled at Step 1


## Verilog Explicit FSM Design and Synthesis Tips

- Use 2 cyclic behaviors for an explicit state machine
- One level-sensitive behavior for combinational logic to describe the next state and output logic
- One edge-sensitive behavior for state flip-flops to synchronize state transition
- In the level-sensitive behavior for N/S and O/P
- Use blocked assignments/procedural assignments "="
- Completely specify all outputs
$>$ Can be achieved by initializing all outputs in the beginning
- In the edge-sensitive behavior for state transition
- Use nonblocking assignments "<="
> For state transition
$>$ For register transfer of a data path
- Always decode all possible states in the level sensitive behavior
- To avoid unnecessary latches


## Decode All Possible States!

- Matching simulation results between behavioral model and a synthesized circuit does NOT guarantee that an implementation is correct!
- Unless exercising all possible input sequences
$>$ Which is almost impossible to do
- Because, if the testbench exercises the circuit only allowable input sequences, then it is not sufficient to verify the circuit's behaviors that are not covered by the exercise of the testbench


## Verilog: Mealy Machine



## Verilog: Mealy Machine- Cont.

```
module mealy_2processes(input clk,
input reset, input x, output reg
parity);
reg state, nextstate;
parameter S0=0, S1=1;
always @(posedge clk or posedge
reset)
if (reset)
    state <= S0;
else
    state <= nextstate;
```



```
```

always @(state or x)

```
```

always @(state or x)

```
```

always @(state or x)
begin
begin
begin
parity = 1'b0;
parity = 1'b0;
parity = 1'b0;
case(state)
case(state)
case(state)
S0: if(x)
S0: if(x)
S0: if(x)
begin
begin
begin
parity = 1; nextstate = S1;
parity = 1; nextstate = S1;
parity = 1; nextstate = S1;
end
end
end
else
else
else
nextstate = S0;
nextstate = S0;
nextstate = S0;
S1: if(x)
S1: if(x)
S1: if(x)
nextstate = S0;
nextstate = S0;
nextstate = S0;
else
else
else
begin
begin
begin
parity = 1; nextstate = S1;
parity = 1; nextstate = S1;
parity = 1; nextstate = S1;
end
end
end
default:
default:
default:
nextstate = S0;
nextstate = S0;
nextstate = S0;
endcase
endcase
endcase
end
end
end
endmodule

```
```

endmodule

```
```

endmodule

```
```

[^0]
## Verilog: Mealy Machine- Cont.

```
module mealy_3processes(input clk, input
reset, input x, output reg parity);
reg state, nextstate;
parameter S0=0, S1=1;
always @(posedge clk or posedge reset)
if (reset)
    state <= S0;
else state <= nextstate;
```

```
always @(state or x) //Output Logic
begin
        parity = 1'b0;
        case(state)
        S0: if(x)
        parity = 1;
        S1: if(!x)
        parity = 1;
        endcase
    end
```

always @(state or x) // Nextstate Logic begin
nextstate = s0;
case(state)
S0: if(x) nextstate = S1;
S1: if(!x) nextstate = S1;
endcase
end
endmodule

[^1]
## Verilog: Moore Machine

```
module mealy_3processes(input clk, input
reset, input x, output reg parity);
reg state, nextstate;
parameter S0=0, S1=1;
always @(posedge clk or posedge reset)
```

```
always @(state) // Output Logic
```

always @(state) // Output Logic
begin
begin
case(state)
case(state)
S0: parity = 0;
S0: parity = 0;
S1: parity = 1;
S1: parity = 1;
endcase
endcase
end
end
if (reset)
state <= S0;
else state <= nextstate;

```

always @(state or x) // Nextstate Logic begin
```

nextstate = S0;
case(state)
S0: if(x) nextstate = S1;
S1: if(!x) nextstate = S1;
endcase

```
end
endmodule

\footnotetext{
*Xilinx Documentation
}

\section*{FSM Example: BCD-to-Excess-3 Code Converter (Mealy)}
- BCD-to-Excess-3 Code Converter for manual design
- A serially-transmitted BCD (8421 code) word is to be converted into an Excess-3 code
\(>\mathrm{B}_{\text {in }}\) transmitted in sequence, LSB first
- An Excess-3 code word is obtained by adding 3 to the decimal value and taking the binary equivalent.
> Excess-3 code is self-complementing
\begin{tabular}{|c|c|c|}
\hline Decimal Digit & \begin{tabular}{l}
8-4-2-1 \\
Code \\
(BCD)
\end{tabular} & Excess-3 Code \\
\hline 0 & 0000 & 0011 \\
\hline 1 & 0001 & 0100 \\
\hline 2 & 0010 & 0101 \\
\hline 3 & 0011 & 0110 \\
\hline 4 & 0100 & 0111 \\
\hline 5 & 0101 & 1000 \\
\hline 6 & 0110 & 1001 \\
\hline 7 & 0111 & 1010 \\
\hline 8 & 1000 & 1011 \\
\hline 9 & 1001 & 1100 \\
\hline
\end{tabular}

\section*{BCD-to-Excess-3 Code Converter (cont.)}


\section*{BCD-to-Excess-3 Code Converter (cont.)}
```

module BCD_to_Excess_3b (B_out, B_in, clk, reset_b);

```
output input parameter
```

                    B_out;
    B_in, clk, reset_b;
S_0 = 3'b000, // State assignment, which may be omitted
S_1 = 3'b001, // If omitted, allow synthesis tool to assign
S_2 = 3'b101,
S_3 = 3'b111,
S_4 = 3'b011,
S_5 = 3'b110,
S_6 = 3'b010,
dont_care_state = 3'bx,
dont_care_out = 1'bx;
reg[2: 0] state, next_state;
reg B_out;

```

\section*{BCD-to-Excess-3 Code Converter (cont.)}
always @ (posedge clk or negedge reset_b) // edge-sensitive behavior with NBAs
if (reset_b == 0) state <= S_0; else state <= next_state;
always @ (state or B_in) begin // level-sensitive behavior with blocked assignments
B_out = 0; // initialize all outputs here
case (state) // explicit states
S_0: if (B_in == 0) begin next_state = S_1; B_out = 1 ; end else if (B_in == 1) begin next_state = S_2; end // Mealy machine
S_1: if (B_in ==0) begin next_state = S_3; B_out = 1 ; end else if ( \(B\) _in \(==1\) ) begin next_state \(=S \_4\); end
S_2: begin next_state = S_4; B_out = B_in; end
S_3: begin next_state = S_5; B_out = B_in; end
S_4: if (B_in == 0) begin next_state = S_5; B_out = 1; end else if (B_in == 1) begin next_state = S_6; end
S_5: begin next_state = S_0; B_out = B_in; end
S_6: begin next_state = S_0; B_out = 1; end
/* default: begin next_state = dont_care_state;
B_out = dont_care_out; end */
endcase
end
endmodule

\section*{Zero Detector}
- Asserting its output when a 0 is detected in a stream of 1 s .
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{Present State} & Input & \multicolumn{2}{|l|}{\begin{tabular}{l}
Next \\
State
\end{tabular}} & Output \\
\hline A & B & \(\boldsymbol{x}\) & A & B & \(\boldsymbol{r}\) \\
\hline 0 & 0 & 0 & 0 & 0 & 0 \\
\hline 0 & 0 & 1 & 0 & 1 & 0 \\
\hline 0 & 1 & 0 & 0 & 0 & 1 \\
\hline 0 & 1 & 1 & 1 & 1 & 0 \\
\hline 1 & 0 & 0 & 0 & 0 & 1 \\
\hline 1 & 0 & 1 & 1 & 0 & 0 \\
\hline 1 & 1 & 0 & 0 & 0 & 1 \\
\hline 1 & 1 & 1 & 1 & 0 & 0 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{\multirow[b]{2}{*}{Present State}} & \multicolumn{4}{|c|}{Next State} & \multicolumn{2}{|r|}{Output} \\
\hline & & \multicolumn{2}{|l|}{\(\boldsymbol{x}=0\)} & \multicolumn{2}{|l|}{\[
x=1
\]} & \(x=0\) & \(x=1\) \\
\hline A & B & \(\boldsymbol{A}\) & B & A & B & \(y\) & \(\boldsymbol{V}\) \\
\hline 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\hline 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
\hline 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
\hline 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
\hline
\end{tabular}

\section*{Zero Detector: Mealy Machine}


\section*{Zero Detector: Mealy Machine}
//Verilog 2001, 2005 syntax module Mealy_Zero_Detector ( output reg y_out, input x_in, clock, reset
);
reg [1: 0] state, next_state;
parameter S0 = 2'b00, S1 = 2'b01,
S2 = 2'b10, S3 = 2'b11;
always @ (state, x_in) // Mealy output case (state)
S0: y_out = 0;
S1, S2, S3: y_out \(=\sim x\) in; endcase
endmodule
always @ ( posedge clock, negedge reset)
if (reset == 0) state <= S0; else state <= next_state;
always @ (state, x_in) // Next state
case (state)
S0: if (x_in) next_state = S1; else next_state = S0;
S1: if (x_in) next_state = S3; else next_state = S0;
S2: if ( \(\sim x\) _in) next_state \(=\) S0; else next_state \(=\) S2; S3: if (x_in) next_state \(=\) S2; else next_state \(=\) S0; endcase

\section*{Binary Counter: Moore Machine}


\section*{Binary Counter: Moore Machine}

Write a Verilog code for Binary Counter (Moore Machine).```


[^0]:    *Xilinx Documentation

[^1]:    *Xilinx Documentation

