Chapter 8: Baseband Digital Communication Systems

ELEC 3400
Introduction

Transmission of digital signal

- Over a baseband channel (Chapter 8)
  - Local communications
  - Transmission of digital signal
- Over a bandpass channel using modulation (Chapter 9)
  - Inter-symbol interference (ISI)
  - Channel-induced transmission impairments

ISI:
- Solutions to be studied in this chapter
  - Major source of bit errors in many cases
  - Each received pulse is affected by neighboring pulses
  - Many channels are bandwidth limited: dispersive, unlike low-pass filters
  - Digital data has a broad bandwidth with a significant low-frequency content

Noise:
- Matched filter
  - Maximize the signal-to-noise level at the receiver

Pulse shaping:
- Minimize the ISI at the sampling points

Equalization:
- Compensate the residual distortion for ISI

Interference:
- Sometimes treated as noise
  - Channel noise, or receiver noise
Transmission Impairment: Noise

- Other sources of noise: interference
- Always exists

If bandwidth is B Hz, then the noise power is \( N = BK T \) (273.15 + [°C])

Receiver system noise temperature in Kelvins \( (K) = [°C] \) and \( T \) is the receiver system temperature.

Noise spectral density: \( N = KT \) (watts per hertz), where \( K \) is the Boltzmann constant \( 1.38 \times 10^{-23} \), and \( T \) is the temperature in kelvins.

Modelled as an Additive White Gaussian Noise (AWGN)

Due to the random thermal motion of the electrons inside the receiver circuit

Thermal noise: generated by the equilibrium fluctuations of the electric current inside the receiver circuit

Transmission Impairment: Noise
Additive Noise
Transmission Impairment: ISI

- Line codes
- Mapping 1's and 0's to symbols
- Random process, since 1's and 0's are random
- Power spectrum (Section 5.8)
- Mismatch between signal bandwidth $B_s$ and channel bandwidth $B_c$
- The nominal bandwidth of the signal is the same order of magnitude as $1/T_b$ and is centered around the origin
- If $B_c > B_s$, the channel is dispersive, the pulse shape will be changed and there will be ISI
- If $B_c \geq B_s$, no problem
- If $B_c < B_s$, the channel is dispersive, the pulse shape will be changed and there will be ISI

Mismatch between signal bandwidth $B_s$ and channel bandwidth $B_c$

Representation in the frequency domain

Baseband

$6/8/20106$
Power Spectra of Several Line Codes

- Average power is normalized to unity.
- Frequency axis normalized with $T_b$.
- Average power is normalized to unity.
Transmission Impairments due to Limited Channel Bandwidth

- Each received symbol may be wider than the transmitted one, due to
  - Loss of high frequency components
  - Overlap between adjacent symbols
  - Inter-symbol interference
- Or need to shape the pulses to cancel ISI at sampling points
- Limit on data rate: use guard time between adjacent symbols

From Data Communications and Networking, Behrouz A. Forouzan

Transmission Impairments due to Limited Channel Bandwidth
Basic Problem

- Transmitted pulse $g(t)$ for each bit is unaffected by the transmission except for the additive white noise at the receiver front.
- No problem of ISI.
- E.g., low data rate over a short range cable.
- First assume an ideal channel and only consider noise.
- Cope with the two types of impairments separately.

Methodology:

Matched Filter – The Problem
Matched Filter

\[ (t)u + (t)b = (t)x = (t)h \otimes (t)x = (t) \delta \]

- Output signal: \( (t)u + (t)b = (t)x = (t)h \otimes (t)x = (t) \delta \)
- Spectral density: \( N_0/2 \)
- \( w(t) \): White Gaussian noise process of zero mean and power
- \( g(t) \): Represents a binary symbol 1 or 0
- \( T \): An arbitrary observation interval
- Received (or input) signal: \( y(t) \)

\[ w(t) + (t)b = (t)x \]

- Time \( t \)
- Sample at \( t \)
- \( \delta \): Impulse response
- Invariant filter of linear time-
- \( (t)h \): Impulse response

White noise

Signal

6/8/2010
Detection of Received Signal

$\int_{0}^{T} s_0(t) \, dt$

Optimal detection time

$\int_{0}^{T} s_1(t) \, dt$

Optimal detection time
Matched Filter (contd.)

Problem

Find $h(t)$ to maximize the peak pulse signal-to-noise ratio at the sampling instant $t = T$.

If $G(f) \leftrightarrow g(t)$, $H(f) \leftrightarrow h(t)$, then $g_0(t) \leftrightarrow H(f)G(f)$.

We can derive $g_0$ by inverse Fourier transform:

$$g_0(t) = \mathcal{F}^{-1}[G(f)H(f)]$$

The instantaneous signal power at $t = T$ is:

$$\int_{-\infty}^{\infty} |\int_{-\infty}^{\infty} g_0(t) \exp(j2\pi f t) df|^2 df = \mathcal{E}$$

The average output noise power is:

$$\mathcal{E} = \int_{-\infty}^{\infty} |h(t)|^2 \mathcal{E}$$

Why square? Why square? Why square?
Find $u$ that maximizes

$$ fp \left| (f) H \right| \int_{\infty}^{0} \frac{\zeta}{N} = u $$

The peak pulse signal-to-noise ratio is

$$ fp \left| (f) H \right| \int_{\infty}^{0} = fp(f)^N S \int = [(1) \mu] E $$

The average noise power is

$$ \left| (f) H \right| \frac{\zeta}{0} = (f)^N S $$

The power spectral density of the output noise $n(t)$ is

Matched Filter (cont'd.)
Therefore, we have:

\[
\int (L, \mu \no \bar{\gamma}) \cdot \text{exp}(f) \cdot \partial \gamma = (f) \text{id}_H
\]

when

\[
\lim_{\int \frac{\text{max} u}{\text{max}} \gamma} \int \frac{\text{max} u}{\text{max}} \gamma = i
\]

\[
\frac{\gamma}{\text{max}} \int \frac{\text{max} u}{\text{max}} \gamma \Rightarrow \frac{\gamma}{\text{max}} \int \frac{\text{max} u}{\text{max}} \gamma = i
\]

The equality holds if and only if:

\[
\int \gamma = (x) \phi \gamma = (x)^{\dagger} \phi
\]

\[
\int \gamma > \int \gamma \phi \Rightarrow \int \gamma \phi \phi
\]

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\]
A time-inversed and delayed version of the input signal $g(t)$

**Matched filter:** matched to the signal

$$
(i - L) \delta_k = 
\int_{-\infty}^{\infty} \gamma = (i)^{100}\eta 
$$

The optimal filter is found by inverse Fourier transform

For real signal $g(t)$, we have $G(f) = G^*(f)$: time inversed

$\exp(j2\pi f T)$: time shift

$k$: scales the amplitude

$\mathcal{F}$: spectrum of the input signal

Optimal filter is the same as the complex conjugate of the transfer function of the

Except for the factor $k \cdot \exp(-j2\pi f T)$, the transfer function of the
Properties of Matched Filters

- Matched filter:
- Received signal:
- Noise power:
  \[ \mathcal{F} \gamma = (L)^{\circ} \delta \]

\[
\mathcal{F} \gamma = \mathcal{F} (\mathcal{F} (f) \delta \ast \gamma)
\]

\[
\mathcal{F} \gamma = \mathcal{F} (\mathcal{F} (f) \delta \ast \gamma) = (f) \delta (f) \delta (f) \delta \ast \gamma = (f) \delta (f) \delta (f) \delta (f) \delta \ast \gamma = (L)^{\circ} \delta \gamma = (L)^{\circ} \delta \gamma
\]

Matched filter:
Received signal:

\[
(i - L) \delta \gamma = (1)^{\circ} \delta \gamma
\]
Properties (contd.)

- Maximum peak pulse signal-to-noise ratio

\[
\frac{E^0}{N^0} = \frac{(k)N_0E^0/2}{(\sigma^2)E} = u_{\text{max}}
\]

Observations

- Signal energy (or, transmit power) matters
- For combating additive white Gaussian noise, all signals that have the same energy are equally effective
- Not true for ISI, where the signal waveform matters
- Independent of \( g(t) \): removed by the matched filter
- \( E/N_0 \): signal energy-to-noise spectral density ratio
Example 8.1: Matched Filter for Rectangular Pulse

\[
\begin{align*}
\gamma_{\text{rect}}(t) &= \left(\frac{T}{t} - \frac{1}{T}\right) \gamma_A \\
\gamma_{\text{rect}}(t) &= \left(\frac{1}{t} - \frac{T}{t}\right) \gamma_A \\
\gamma_{\text{rect}}(t) &= (1 - T) \delta(t) \\
\gamma_{\text{rect}}(t) &= (1) \delta(t)
\end{align*}
\]
Matched Filter for Rectangular Pulse (contd.)

- Output \( g(t) \) occurs at \( t = T \).
- Optimal sampling instance implemented using the integrate-and-dump circuit.
- Max output \( KA_2T \) occurs
  \[
  (1) \eta \ast (1) \delta = (1)^0 \delta
  \]
  \( (1)^0 g \) output

Example 8.1 Matched Filter for Rectangular Pulse
Bit error rate (BER) = \( \frac{2}{15} \approx 13.3\% \)

Line coding:
“1”: -5 volts
“0”: +5 volts

Properly chosen decision threshold
Sampling instance

Figure 3.16: Effect of Noise on a Digital Signal
Probability of Error due to Noise

Assume polar nonreturn-to-zero (NRZ) signaling

The receiver has prior knowledge of the pulse shape, need

\[ \begin{cases} \text{Symbol 0 was sent} & (t)w + a^- \\ \text{Symbol 1 was sent} & (t)w + a^+ \end{cases} = (t)x \]

Received signal is

Additive white Gaussian Noise \( w(t) \) of zero mean and

power spectral density \( \frac{N_0}{2} \)

Interval \( 0 \leq t \leq t_b \)

Decide 1 or 0 for a received amplitude in each signaling

0: Negative rectangular pulse, \( -a \)

1: Positive rectangular pulse, \( +a \)

Assume polar nonreturn-to-zero (NRZ) signaling
Probability of Error due to Noise (contd.)

- Sampled value $y$, with threshold $\lambda$,
  \[
  \begin{cases}
  y > \lambda, \text{ symbol 1 received} \\
  y \leq \lambda, \text{ symbol 0 received}
  \end{cases}
  \]

- Two kinds of errors

- How to quantify residual BER?
- How to choose threshold to minimize BER?
Consider the Case When Symbol 0 Was Transmitted

Receiver gets \( x(t) = -A + w(t) \), for \( 0 \leq t \leq T_b \)

The matched filter output, sampled at \( t = T_b \), is the sampled value of a random variable \( Y \) is also Gaussian, since \( w(t) \) is white and Gaussian.

\[ E[Y] = -A, \quad \text{and} \quad \sigma^2 = \frac{\lambda}{2} \]

Sampled value of a random variable \( Y \) follows a Gaussian distribution, which is completely determined by its mean and variance.
\[ \theta > 0 \]

Because \( m \rightarrow 1 \) if \( y > 0 \)

**Threshold Rule (on the Receiver)**

\[ f_{N}(n) = \frac{1}{N!} \frac{2^{-\frac{n}{2}}}{\pi^{\frac{n}{2}}} \exp \left[ -\frac{2^{n}}{\pi^{2}} \right] \]

\[ \text{when} \quad n \to \infty \]

\[ \frac{0}{0} \]

\[ \text{Solution:} \quad Y \]

\[ N + A \]

\[ A + Y \]

\[ \text{Simplified version} \]
$P = P(\emptyset \cup \{\{\{\{1, 1\}, \emptyset\}, 0\}, \emptyset\})$

$P(A \cup B) = P(\emptyset) + P(B)$

$P(A \lor B) = P(\emptyset) \lor P(B)$

\[
\begin{array}{c|c|c}
\lor & 0 & 1 \\
\hline
0 & 0 & 1 \\
1 & 1 & 1 \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\lor & 0 & 1 \\
\hline
0 & 0 & 0 \\
1 & 1 & 1 \\
\end{array}
\]
\[
\begin{align*}
\overline{P(N \geq A)} &= P(A + N > 0) \cdot P(T_n^1) \\
\overline{P(Y > 0 | T_n^1)} &= P(T_n^1, Y > 0) \\
\{0 \leq y\} &= \{\text{f.c. even}\} \\
\{y > 0\} &= \{\text{f.c. odd}\} \\
\text{Conditional push of A given} B &= P(A|B)P(B) = P(B|A)P(A) \\
P(A | B = p(A \cup B) = P(AB)
\[ p \leq \frac{1}{n} \sum \frac{x_n^2}{n^{\frac{3}{2}}} \Rightarrow p = \frac{1}{n} \sum \frac{x_n^2}{n^{\frac{3}{2}}} \]

For \( \mu = \frac{1}{n} \sum \frac{x_n^2}{n^{\frac{3}{2}}} \), we have

\[ \frac{1}{n} \sum \frac{x_n^2}{n^{\frac{3}{2}}} = \mu \]

So \( x_n = \frac{1}{n} \sum \frac{x_n^2}{n^{\frac{3}{2}}} \)

Let

\[ n = A \]

\[ p(N > A) = \int_{x=A}^{\infty} \frac{1}{x^{\frac{3}{2}}} \exp\left(-\frac{2x}{n}\right) dx \]

\[ \frac{1}{n} \sum \frac{x_n^2}{n^{\frac{3}{2}}} \]

By symmetry, \( f(n) \) about \( n = 0 \), we have

\[ p(N > A) = p(N < -A) \]

Similarly, \( f(n) \) about \( n = 0 \), we have

\[ p(-\infty < N < -A) \]

\[ p \leq \frac{1}{n} \sum \frac{x_n^2}{n^{\frac{3}{2}}} \Rightarrow p = \frac{1}{n} \sum \frac{x_n^2}{n^{\frac{3}{2}}} \]
\[
\begin{align*}
\Pr(N \geq A) &= 0 \Pr(A) \\
\Pr(A + N \geq 0) &= \Pr(A) \Pr(N = 0) \\
\Pr(Y \geq 0 | Y \leq 0) &= \Pr(N = 0, Y \leq 0) \\
= & \Pr(\text{"next", } 1) = \Pr(N = 0, Y = 0) \\
& \Pr(A) \Pr(N = 0)
\end{align*}
\]
\[ q = \frac{\binom{n}{k}}{\frac{4^n}{3^n}} \]

\[ p = \frac{1}{\binom{n}{k}} + (1-p) \cdot q \cdot \binom{n}{k+1} \]
When $0$ was transmitted (contd.)

Since $w(t)$ is white Gaussian, $P(t)$, the power spectral density of $w(t)$, is $\frac{N_0}{2T_b}$.

The conditional probability density function (PDF) of $Y$, conditioned on that symbol $0$ was transmitted, is

$$P(Y|0) = \frac{1}{\sqrt{2\pi \sigma_Y^2}} \exp\left(-\frac{(Y - \mu_Y)^2}{2\sigma_Y^2}\right)$$

where $\mu_Y = -A$ and $\sigma_Y^2 = \frac{N_0}{2T_b}$.

The variance

$$\sigma_Y^2 = \frac{N_0}{2T_b}$$

The $q_{LT}$

$$q_{LT} = npq(n-t) q_{0N} \int_{-\infty}^{0} \exp\left(-\frac{qL}{I}\right) I = q_{0N} \int_{-\infty}^{0} \exp\left(-\frac{qL}{I}\right) I$$

The power spectral density $S(f)$

$$S(f) = \frac{N_0}{2f} \frac{\pi}{T}$$
When no noise, \( Y = -A \)

With noise, drifts away from

When 0 was transmitted

(\text{contd.})
When 0 was transmitted (contd.)

Assume symbol 1 and 0 are equally likely to be transmitted, we choose Z = 0 due to symmetry.

\[ E_b: \text{the transmitted signal energy per bit} \]

\[ q_L \cdot \mathbb{E} \cdot \mathbb{E}^q \]

Define

\[ \frac{q_L / N^L}{A + \lambda} = Z \]

\[ \lambda p (\frac{q_L / N^L}{Z (A + \lambda)} - ) \exp \left( \frac{0}{\infty} \right) \mathbb{E} \cdot \mathbb{E}^q \]

\[ \frac{q_L / N^L}{A + \lambda} = Z \]

\[ p (\frac{q_L / N^L}{Z (A + \lambda)} - ) \exp \left( \frac{0}{\infty} \right) \mathbb{E} \cdot \mathbb{E}^q \]

\[ \frac{q_L / N^L}{A + \lambda} = Z \]
When 0 Was Transmitted (contd.)

Define $Q$-Function:

The conditional bit error probability when 0 was transmitted is

$$
\Pr(Z/n - Z/n = 0) \exp \left( \int_{1}^{\infty} \frac{\nu Z^n}{1} \right) = 0^d
$$

The conditional bit error probability when 0 was transmitted is

$$
\Pr(Z/n - Z/n = 0) \exp \left( \int_{1}^{\infty} \frac{\nu Z^n}{1} \right) = (n)q
$$

Q-Function: see Page 401
When I Was Transmitted

Receives: $x(t) = A + w(t)$, $0 \leq t \leq T_b$

$Y$ is Gaussian with $\mu_Y = A$, $\sigma_Y^2 = N_0/(2T_b)$

The conditional bit error rate is

$$\alpha_p \left( \frac{q\varnothing / 0N}{\varnothing (\varnothing - \xi)} - \right) \exp \left( \frac{-q\varnothing / 0N \xi^\wedge}{I} \right) = \beta$$

$\beta$ is Gaussian with $\mu = \lambda$, $\sigma = \sqrt{q\varnothing / 0N} + \phi$
What are the limiting factors?

- Bit Error Rate (BER)
- Energy plays the crucial role in transmit power
- Noise is usually fixed for a given temperature
- Energy per bit to the noise spectral density depends only on $E_b/N_0$, the ratio of the transmitted signal's energy per bit to the noise spectral density

$$\bar{\sigma} = \left( \frac{0}{E_b} \right) \bar{\sigma} \times \frac{\tau}{1} + \left( \frac{0}{E_b} \right) \bar{\sigma} \times \frac{\tau}{1} = i_1 d \times i_1 d + i_0 d \times i_0 d = \quad \frac{i_1 d \times \{Pr\} \text{ is transmitted} + i_0 d \times \{Pr\} \text{ is transmitted}}{Pr} = \frac{i_1 d \times \{Pr\} \text{ is transmitted} + i_0 d \times \{Pr\} \text{ is transmitted}}{Pr}$$

Battery life, interference to others, data rate requirement

- Higher amplitude
- Wider pulse

$N \Rightarrow q \frac{E_b}{q}$
Then consider the more general case of \(M\)-ary data.

First examine binary data.

Example

Use discrete pulse-amplitude modulation (PAM) as

\[ |f| < \sqrt{M} \quad \text{blocks all frequencies} \]

\[ |f| > \sqrt{M} \quad \text{passes all frequencies without distortion} \]

E.g., band-limited channel.

The channel has a frequency-dependent (or, frequency-selective) amplitude spectrum.

Happens when the channel is dispersive.

The next source of bit errors to be addressed.

Inter-symbol interference
8.2: The Dispersive Nature of a Telephone Channel

- Band-limited and dispersive

Block dc
cut-off at 3.5 kHz

Block high frequencies:

Insertion loss (dB)

Frequency (kHz)

Envelope delay (ms)

Frequency (kHz)

Phase shift (Cycles)
Example 8.2: The Dispersive nature of a Telephone Channel

- Conflicting requirements for line coding
- High frequencies blocked → need a line code with a narrow spectrum
- Low frequencies blocked → need a line code that has no dc
  - But Manchester code
  - But Manchester code has high frequency
  - High frequencies blocked → need a line code with a narrow spectrum
  - Low frequencies blocked → need a line code that has no dc
  - But polar NRZ has dc
Example 8.2: The Dispersive nature of a Telephone Channel (contd.)

This page: data rate at 3200 bps

Previous page: data rate at 1600 bps

- 6/8/2010
Eye Pattern

Synchronized superposition of all possible realizations of the signal viewed within a particular signaling interval.

An operational tool for evaluating the effects of ISI.
Eye pattern (cont'd.)

Eye opening: the interior region of the eye pattern
Interpreting Eye Pattern

- Impossible to avoid errors due to ISI
- Under severe ISI: the eye may be completely closed
- Noise margin of the system
- The height of the eye opening
- The slope
- The sensitivity of the system to timing errors
- The best sampling time: when the eye is open the widest
- Sampled without error from ISI
- Defines the time interval over which the received signal can be
- The width of the eye opening

The width of the eye opening
The channel has no bandwidth limitation: the eyes are open. Band-limited channel: blurred.

Example 8.3
Eye Pattern on Oscilloscope

http://www.myprius.co.za/pcmx2_processor1.htm
Baseband Binary PAM System

Input sequence: \( \{ b_k \mid b_k = 0 \text{ or } 1 \} \)

Transmitted signal:

\[
(\mu) c \ast (\mu) h \ast (\mu) \delta = (\mu) d \ast n'
\]

where:

\[
(\mu) d = \sum_{k} (q_{L} - 1) d_{k} b_{k} n = (\mu) x
\]

The receiver filter output:

\[
(\mu) y = \sum_{k} (q_{L} - 1) \delta_{k} b_{k} n = (\mu) s
\]

Input sequence: \( \{ 0, 1 \} \)
In the absence of both ISI and noise:

Sampling instant $t_i$: the ISI

Before and after the occurrence of pulses

Transmitted bit

Contribution of the $i$-th transmitted bit

Residual effect due to the effect of noise,

For the $i$-th received symbol, sample the output $y(t)$ at

$$y(t) = \sum_{n=\text{-}\infty}^{\infty} (r_{i-n})^{(t)}$$

$$y(t) = \sum_{n=\text{-}\infty}^{\infty} (r_{i-n})^{(t)}$$

Yielding

$$(r_{i-n})^{(t)} = y(t)$$

Baseband Binary PAM System
Nyquist's Criterion for Distortionless Transmission

Recall that:

- Usually the transfer functions of the channel $h(t)$ and the transmitted pulse shape are specified (e.g., P32 telephone channel) to determine the transfer functions of the transmit and receive filters so as to reconstruct the input binary алгі.

The receiver performs:

1. Extraction: sampling $y(t)$ at time $t = iT_b$
2. Decoding:
   - Requires the ISI to be zero at the sampling instance.

The problem:

$$\{^q p\} \text{ performs: } (1)c * (1)y * (1)\delta = (1)d \times n'$$

Recall that:

For $m = 1, 2, 3, \ldots$

$$\hat{p}(\pm mT_b) = 0$$
Nyquist’s Criterion for Distortionless Transmission (contd.)

\[ I = (0)d = ip(1j\nu \tau - \omega)d \exp(i\nu)g(0)d \int_{-\infty}^{\infty} = (f)g \]

If the condition is satisfied

\[ ip(1j\nu \tau - \omega)d \exp[(qL-u-1)g(qL)u] \int_{-\infty}^{\infty} = (f)g \]

Its Fourier transform is

\[ (qL-u-1)g(qL)u \int_{-\infty}^{\infty} = (1)g \]

On the other hand, the sampled signal is

\[ (qL-u-f)d \int_{-\infty}^{\infty} = (f)g \]

On the frequency domain, we have

Sampling in the time domain produces periodicity in

\[ \{(qLu)d\} \leftarrow \ldots, u \neq 0, u \neq \tau, u = \tau \]

Sampling \( p(t) \) at \( t \) produces

Nyquist’s Criterion for Distortionless Transmission (contd.)
Nyquist's Criterion for Distortionless Transmission (contd.)

Finally we have

\[ q_L = \left( q_R - f \right)_p \sum_{-\infty}^{\infty} \]

\[ q_L = q_R / 1 = (q_R u - f)_p \sum_{-\infty}^{\infty} \]

\[ 1 = (q_R u - f)_p \sum_{-\infty}^{\infty} q_R = (f)^g p \]

\[ p(t) = \left\{ 0 \right\} \quad t = 0 \]

Nyquist's Criterion for Distortionless Transmission
\[
\begin{align*}
\mathbb{P}(\mathcal{H}^0) & = 0 \quad \text{for} \quad |f| > 3 \\
\mathbb{P}(\mathcal{H}^0) = 0 & \quad \text{if} \quad \mathcal{P}(f) \neq 0 \\
\mathcal{P}(f) & = \frac{c(f)}{p(f)} \\
\mathcal{P}(f) & = \mathcal{G}(f) \\
\mathcal{P}(f) & = \mathcal{G}(f) \\
\mathcal{P}(f) & = \mathcal{G}(f) \\
\mathcal{P}(f) & = \mathcal{G}(f)
\end{align*}
\]
\( p(t) = \{ 0, c, -\frac{7}{8} t \} \)

\( R^c_t > \frac{7}{8} t \)

\( R^c_t < \frac{7}{8} t \)

Unique solution: \( R^c_t = 2B \)

No solution: \( R^c_t > 2B \)

\( \frac{2}{7} p(t) \) is bounded below \( p(t) \) is bounded below
All functions are uniformly continuous if \( R^2 \rightarrow R^2 \) and \( R^2 \rightarrow R^2 \).
Ideal Nyquist Channel

**Nyquist Criterion**

The simplest way of satisfying the Nyquist Criterion is the sinc function.

**Nyquist Rate:**

\[ R = 2W \]

**Nyquist Bandwidth:**

\[ W = \frac{1}{2T} \]

The signal that produces zero ISI is the sinc function.

Where \( W = R/2 = 1/(2T) \).

\[
M < |f| < \frac{2W}{1} \quad \text{for} \quad 0 < M < \frac{W}{1} \quad \Rightarrow \quad \left( \frac{M2}{f} \right) = \left( \frac{2W}{f} \right) = \frac{M2}{1} = (f)P
\]
Ideal Nyquist Channel (contd.)
Raised Cosine Spectrum

\[
P(f) = \left( \frac{1 - 16W^2}{(1 + 16W^2) \cos(2\pi W)} \right) \left( \text{sinc}(2W) \right) = (f)P
\]

1. If \( f - MZ \leq |f| \)
2. If \( f - MZ > |f| \equiv f \)
3. If \( f > |f| \geq 0 \)

Use Raised cosine spectrum: a flat top + a rolloff portion

- Error (see Fig. 8.16)
- \( d(t)\) decays at rate \( 1/|f|\): too slow, no margin for sampling time
- No filter can have the abrupt transitions at \( W \)
- \( p(f) \) is physically unrealizable

\[ \frac{M^2}{2} = \frac{M}{W} \]
Rolloff factor: \( \alpha = 1 - f_1/W \)

Indicates the excess bandwidth over the ideal solution, \( W \)

\[
\frac{W}{f} - 1 + 1 \quad M = 1 - f - M2 = \frac{B}{f}
\]

Same property as \( \text{sinc}(2Wt) \), but now it is practical

\[
R = \frac{R_0/2}{f_1/2T_f}
\]

\( \alpha = 1 \Rightarrow R \leq 1 \)

Raised Cosine Spectrum (contd.)

\( 0 \leq \alpha \leq 1 \) = Roll-Off Factor
Mann-Whitney U:

\[ U = \frac{n_1 \cdot n_2 + n_1 \cdot (n_1 + 1) - R_1}{2} \]

where:

- \( n_1 \) is the number of observations in Group 1
- \( n_2 \) is the number of observations in Group 2
- \( R_1 \) is the rank sum of Group 1

Endurance Benchmark: (\( t_0 = 0 \))

- \( m = 1, 2, 3, \ldots \)
- \( t = t_0, t_1, t_2, \ldots \)

Time Benchmark:

\[ \text{Pure Point: } \text{NYOIST First-Criterion Zero} \]

Necessary Condition: (\( t_0 = 0 \))
\[ B \leq B_c = \text{Channel BW} \]

where \[ x = \text{roll-off factor} \quad (0 \leq x \leq 1) \]

\[
B_T = \begin{cases} 
\frac{R_s}{2} \frac{1 + x}{2} & \text{Raised-cosine family of pulses} \\
\frac{R_s}{2} \frac{1}{2} & \text{Binary} 
\end{cases}
\]

\[ B_w = B_c t \]
How to Design the Transceiver

1. Nyquist Criterion
   \[ \text{raised cosine spectrum} \]

2. Study the channel and find \( h(t) \)
3. Matched filter (to cope with noise) \( c(t) \) and \( g(t) \) are symmetric
4. Solve for \( c(t) \) and \( g(t) \)
5. Nyquist Criterion
   \[ \text{raised cosine spectrum} \]

Mathematical Equations:

- \( (f)C \times (f)H \times (f)G = (f)p \times n' \)
- \( (t)c \ast (t)h \ast (t)\delta = (t)d \times n' \)
- \( \sum_{k=-\infty}^{\infty} \exp(-j2\pi f \tau) g(t) = (f)G \)
- \( (t)c \) and \( (t)g \) are

Symbols,

- \( C \): Coefficient
- \( H \): Channel
- \( G \): Gain
- \( p \): Power
- \( n' \): Noise
- \( h(t) \): Channel Impulse Response
- \( c(t) \): Matched Filter Impulse Response
- \( g(t) \): Interference Filter Impulse Response
- \( \delta \): Dirac Delta Function
Example 8.4 Bandwidth Requirement of the T1 System

- In Chapter 3, if we SSB and FDM, the bandwidth is

\[ B = 24 \times 4 = 96 \text{ KHz} \]

- \( W = M = \frac{1}{2} B \)

- \( \alpha = 1 \). The minimum transmission bandwidth is

\( \omega = \frac{8}{24} \text{ MHz} \)

- In practice, a full-cosine rolloff spectrum is used with

\( B = 3 \text{ MHz} \)

- The required bandwidth is

\[ B = 0.647 \mu s \]

- Assuming an ideal Nyquist channel, the minimum

\[ B = \frac{6 \times 773}{24} = 773 \text{ KHz} \]

- Based on 8-bit PCM word, \( T = 0.647 \mu s \)

- T1 system: multiplexing 24 voice calls, each 4 KHz,
Baseband M-ary PAM Transmission

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Digit</th>
</tr>
</thead>
<tbody>
<tr>
<td>+3</td>
<td>10</td>
</tr>
<tr>
<td>+1</td>
<td>11</td>
</tr>
<tr>
<td>-1</td>
<td>10</td>
</tr>
<tr>
<td>-3</td>
<td>00</td>
</tr>
</tbody>
</table>

Possible amplitude levels, with $M > 2$

$\log_2(M)$ bits are mapped to one of the levels

Baseband M-ary PAM system

# 1.6.1.5/Spreads
in the binary data case

Similar procedure used for the design of the filters as

\[ W \log_2 R = q \]

\[ W \log_2 q L = \frac{5}{7} \]

Binary data rate: \( R = \frac{q}{L} \) in symbols per second, or bauds

Binary symbol duration: \( T_b \)

Signaling rate: \( R = \frac{q}{L} \) in symbols per second, or bauds

Symbol duration: \( T \)

Baseband M-ary PAM Transmission (cont'd.)
Eye Pattern for M-ary Data

Contains (M-1) eye openings stacked up vertically.
Equalizer: the filter used for such process

Intrinsic residual distortion

Use a process, equalization, to compensate for the

A limiting factor for data rates

Causes residual distortion

However, in practice, \( h(t) \) may not be known, or be known with errors (i.e., time-varying channels)

Find \( P(f) \rightarrow \) find \( G(f) \rightarrow \) find \( C(f) \)

Find \( P(f) \) \( \rightarrow \) find \( G(f) \) \( \rightarrow \) find \( C(f) \)

Transmit and receive to make ISI arbitrarily small

If channel \( h(t) \) or \( H(f) \) is known precisely, one can design transmissions

ISI is the major cause of bit error in baseband

Tapped-Delay-Line Equalization
\[
\left\{ \begin{array}{l}
(f^2 H(f))(x) \ni \frac{1}{p(x)} C(r)
\end{array} \right. 
\]
Tapped-Delay-Line Filter

Tapped Delay-Line Filter

\[ T = T_b \]

Symbol duration

\[ \sum_{i=0}^{N} w_i \]

Totally \((2N+1)\) taps, with weights \(w_i\)

\[
\frac{1}{T} (L - 1) \sum_{i=0}^{N} w_i = 1
\]
We have

\[ (I - L) e^{\sum_{N=1}^{N=N} (\gamma - u)} = (I - L) \gamma \times (I \gamma)^{-1} \geq \sum_{N=1}^{N=N} (\gamma - u) \]
The Nyquist criterion must be satisfied. We have:

\[
\begin{bmatrix}
0 & \cdots & N \vspace{1.5em} \\
\vdots & \ddots & \vdots \\
0 & \cdots & \vspace{1.5em} \\
1 & \cdots & \vspace{1.5em} \\
0 & \cdots & \vspace{1.5em} \\
1 & \cdots & \vspace{1.5em} \\
0 & \cdots & \vspace{1.5em} \\
\vdots & \ddots & \vdots \\
0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
0 \\
\vdots \\
0 \\
1 \\
0 \\
1 \\
0 \\
\vdots \\
0
\end{bmatrix} = \begin{bmatrix}
0 \\
\vdots \\
0 \\
1 \\
0 \\
1 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

Denote \( c^n = c(nT) \), we have:

\[
\int_{-N}^{N} \left( \frac{N}{N}\right) dt = (Lu)d
\]
\[
\begin{align*}
\text{Example: Recurrence Equation} & \quad 2N + 1 = 3 \implies N = 1 \\
\text{Design 3-Step Zero-Force Equations:} & \\
\text{For } n \text{ for which odd } \ n \\
& \quad c^n = 0 \quad \text{for even values of } n \\
& \quad c^0, c^1, c^{2}, c^{1} = \frac{1}{2}, c^{-1} = \frac{1}{4} \\
& \quad \text{Sampled Command Inputs} \\
\end{align*}
\]
\[ p = \frac{2}{3} \quad m' = -0.2 \quad 0 + \frac{5}{2} = -1 \quad 0 + 0 \neq 0 \]

\[ p = \frac{4}{3} \quad m - \frac{1}{2} = -1 \quad 0 \neq 0 \]

\[ p^{-1} = p' = 0, \quad p = 1 \]

\[ p^{-1} = p' = 0, \quad p = 1 \]

Actual Output: \[ p = 0 \text{ for } n < 3 \quad \text{and} \quad a_n \geq 3 \]

\[ h = 0 = \frac{s}{2} = \frac{m}{1} = \frac{1}{2} \]

\[ \frac{8}{s} = \frac{8}{a_1} = \frac{8}{1 + 8 + 1} \]

\[ 8 \cdot 0 = \frac{s}{h} = 0 \quad m \iff 1 = m \left[ \frac{8}{1 + 1 + \frac{8}{1}} \right] \iff \]

\[ 1 = \left( m \frac{2}{1} \right)^{\frac{h}{1}} + m + \left( \frac{m \frac{2}{1}}{-} \right)^{\frac{1}{1}} \]

\[ \frac{2}{1} = \frac{1}{m} \]

\[ \frac{h}{1} = \frac{1}{-m} \]
Tapped-Delay-Line Filter (contd.)

Remarks

Referred to as a zero-forcing equalizer

Optimum in the sense that it minimizes peak distortion

The longer, the better, i.e., the closer to the ideal condition as specified by the Nyquist criterion

Simple to implement

For time-varying channels

Training

Adaptive equalization: adjusts the weights

(ISI) peak distortion
Transmission of 100 Mbps over Twisted Pair

Fast Ethernet: 100BASE-TX

Using two pairs of twisted copper wires Category 5

Up to 100 Mbps

One pair for each direction

Maximum distance: 100 meters

First stage: NRZ 4B5B → provide clocking information

Second stage: NRZI

Third stage: three-level signaling MLT-3

With tapped-delays-line equalization

Twisted Pair Example – 100Base-TX – 100 Mbps over
Summary

- Binary transmissions and M-ary transmissions
- Tapped-delay line equalization
- Nyquist criterion for distortionless criterion
- Eye pattern
- Evaluating the BER
- Matched filter
- How to mitigate the effects of transmission impairments
- Impact of the two transmission impairments
- ISI
- Noise
- Two transmission impairments