AMPLIFIER OVERLOAD

AMPLIFIER OVERLOAD

ACHIEVEMENTS: *an introduction to the definition and measurement of distortion in wideband and narrowband systems.*

PREREQUISITES: *completion of the experiment entitled **Spectrum analysis - the WAVE ANALYSER** in this Volume.*

EXTRA MODULES: *SPECTRUM UTILITIES.*

PREPARATION

not too little - not too much

One of the aims of an analog transmission system, such as an audio amplifier or a long-distance telephone circuit, is to present at the output a faithful reproduction of the signal at the input. Analog systems will always introduce some signal degradation, however defined, but it is the aim of the analog design engineer to keep the amount of degradation to a minimum.

As you are probably already aware, if signal levels within a system rise 'too high', then the circuitry will overload; it is no longer operating in a linear manner. As will be seen later in this experiment, extra, unwanted, distortion components will be generated. These distortion, or noise [1], components, are signal-level dependent. In this case the noise components arise due to the presence of the signal itself.

Conversely, if signal levels within a system fall 'too low', then the internal circuit noise, which is independent of signal level, will eventually swamp the small, wanted, output. The background noise of the TIMS system is held below about 10 mV peak - this is a fairly loose statement, since this level is dependent upon the bandwidth over which the noise is measured, and the model being examined at the time. A general statement would be to say that TIMS endeavours to maintain a SNR of better than 40 dB for all models.

Thus analog circuit design includes the need to maintain signal levels at a level 'not to high' and 'not too low', to avoid these two extremes.

The TIMS working level, or ANALOG REFERENCE LEVEL has been set at 4 volts peak-to-peak. Modules will generally overload if this level is exceeded by say a factor of two.

---

1 noise is here considered to be anything that is not wanted.

It is the purpose of this experiment to introduce you to the phenomenon of circuit overload, and to offer some means of defining and measuring its effects.

amplifier 'gain'

ideal amplifier 'gain'.

Consider an amplifier which is said to have a gain of '**g**'. This is understood to mean that, if v_i is the input signal voltage, then the output v_o is given by:

$$v_o = g \cdot v_i \qquad\qquad \text{........ 1}$$

This would be described as an ideal amplifier. Its input/output (i/o) characteristic would be a straight line, with a slope of g volts/volt.

In simple terms, **g** is a dimensionless constant. More generally it can be complex, and frequency dependent; but such complications will be ignored in the work to follow.

real amplifiers

Unfortunately, a real amplifier does not have an ideal straight-line input/output characteristic. It is more likely to look like that of Figure 1.

It is obvious to the eye that the characteristic shown in Figure 1 could not be considered straight, or *linear*, except perhaps for input signal amplitudes in the range say ± ½ volt.

In this range the slope of the characteristic is 10 volts/volt.

The amplifier is said to have a *small signal gain* of 10. Input signal amplitudes in the range say ± ½ volt would be considered *small* signals for *this* amplifier.

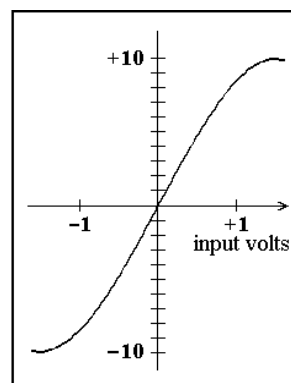A typical amplifier characteristic is likely to flatten off and become parallel to the

Fig 1: typical characteristic

horizontal axis, whereas this characteristic, as defined by eqn. (2) below, will approach and finally cross the input-axis for larger input amplitudes.

So this approximation to an amplifier characteristic should be used for input amplitudes restricted to the range 0 to say ±1½ volts.

Its actual input/output relationship is given by:

$$v_o = g_1 \cdot v_i + g_3 \cdot v_i^3 \qquad\qquad \text{........ 2}$$

where v_i and v_o are the input and output voltages, respectively, and

$$g_1 = 10 \qquad\qquad \text{........ 3}$$

$$g_3 = -1.5 \qquad\qquad \text{........ 4}$$

Note that the range of the so-called linear part of the characteristic is not obvious from a cursory examination of eqn. (2) alone. We shall later obtain a method of defining an acceptable operating input signal range.

harmonic distortion

calculation of harmonic distortion components

Intuition tells us that an amplifier with the input-output characteristic of Figure 1 will introduce distortion, but what sort of distortion ? This can be checked analytically by nominating a test input signal, and then determining the corresponding output.

Let the test signal be a single tone, v_i, where:

$$v_i = V.\cos\mu t \qquad\qquad ....\ 5$$

Substituting this into eqn. (2), and expanding, gives:

$$v_o = g_1 V \cos\mu t + g_3 V^3 (3/4 \cos\mu t + 1/4 \cos3\mu t) \qquad ....\ 6$$

Notice that the cubic term has given rise to two new components; one on the same frequency as the input signal, and the other on its third harmonic.

After combining like harmonic terms the last equation can be rewritten as:

$$v_o = [g_1 V + (3/4) g_3 V^3] \cos\mu t + (1/4) g_3 V^3 \cos3\mu t \qquad ....\ 7$$

Notice that, at the output:

1) the amplitude of the *wanted* term $\cos\mu t$ is no longer simply g_1 times the input amplitude (as suggested by eqn. (1)).

2) there is an extra, *unwanted* term, on the third harmonic of the input.

The original signal has been distorted.

This can be observed in the time domain, using an oscilloscope. For the example under discussion, the output, for an input of amplitude $V = 1$ volt, is shown in Figure 2.
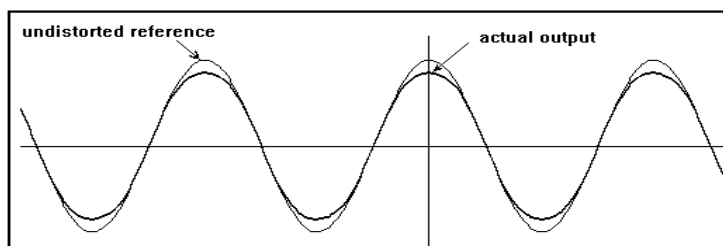

Figure 2: the output voltage waveform of eqn. (7), for an input amplitude of V = 1 volt

definition of harmonic distortion - THD

Notice that the analysis has been performed so as to describe the output in terms of harmonic components of the input. The fundamental, or first harmonic, is the *wanted* term, and all higher harmonic terms (in this example there is only one) are *unwanted*.

The wanted and unwanted harmonic terms can be compared, and some measure defined, to describe the *amount* of *harmonic* distortion.

The comparison is usually made on a power basis, described as the 'total harmonic distortion', or THD, and defined as:

$$THD = 10 \log_{10} \frac{\sum_n H_1^2}{\sum_{j=2} H_j^2} \, dB \qquad \text{........ 8}$$

where H_1 is the amplitude of the output signal on the same frequency as the input test signal, and the H_j are the amplitudes of the 2nd and higher harmonics, or unwanted, terms.
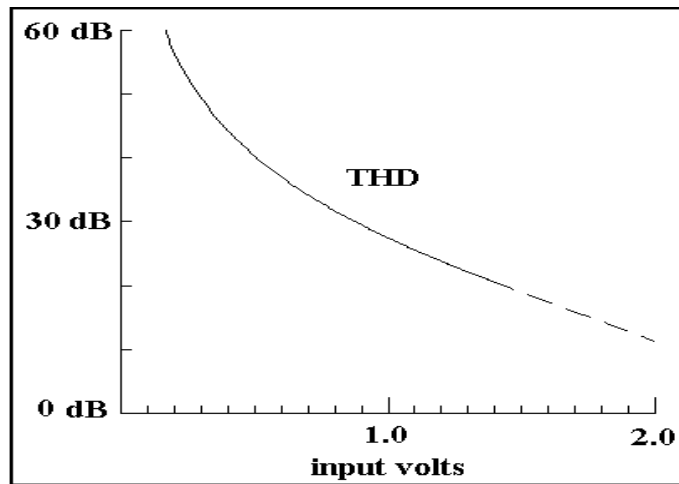

**Figure 3: THD versus input amplitude
for the response of eqn. (2)**

For the example above, where the input amplitude V was 1 volt peak, this evaluates to a THD of 27.48 dB.

Figure 3 shows the THD plotted for input amplitudes in the range 0 to 1½ volts.

Having decided on an acceptable THD for this amplifier (say 40 dB), the user can now specify a maximum input level (½ volt) from Figure 3.

measurement of THD

There are instruments available which measure THD directly. They supply their own test input signal to the device under test, measure the total AC power output, subtract the power due to the wanted signal, and present the THD expressed in decibels.

You will make your own measurements with TIMS by modelling a WAVE ANALYSER [2], measuring the amplitudes of the individual output components, and then applying the THD formula.

Note that this THD is specific to the actual input signal amplitude used for the measurement. There is no subsequent simple step to enable:

1.   prediction of the THD for another input signal amplitude.
2.   determination of the non-linear characteristic of the amplifier.

This is because of the non-linear relationship between the input signal amplitude and the corresponding output THD.

narrow band systems

The previous discussion requires some modification if the system being examined is narrow band [3].

There are many circuits in an analog communications system which are narrowband, since many communications signals themselves are narrow band. A narrowband system is one which has had its frequency response intentionally restricted. This generally simplifies the circuit design, and eliminates out-of-band noise.

To simplify the discussion, suppose there is a bandpass filter at the system output, so that only frequencies over a narrow range either side of the measurement frequency, μ rad/s, will pass to the output. Let the non-linearities be in the circuitry preceding the filter.

If this were the case for the example already discussed, the output waveform would show no sign of distortion, but instead be a pure sinewave ! No distortion would be visible on an oscilloscope connected to the output, because the distorting third harmonic signal would not reach the output. An instrument for measuring THD with a single tone input would register no distortion at all !

Is the system linear or not ? It is definitely non-linear. This can be demonstrated by observing that the relationship between *input amplitude* and *output amplitude* is not linear.

> *Using the methods so far employed, this does not show up as waveform distortion, and would not be revealed by a spectrum analysis of the output*

measurement of a narrow band system

The single-tone test signal we have been using so far is inappropriate for the measurement of THD in a narrow-band system. What is needed is a more demanding test signal, which will reveal the non-linearity, and which is more representative of the signals to be found in most systems. The non-linearity is

---

2 see the experiment entitled *Spectrum Analysis - the Wave Analyser*, or model a spectrum analyser using the TIMS320 module.

3 'wideband' and 'narrowband' signals are defined in the chapter entitled *Introduction to Modelling with TIMS*.

revealed by the transmission of *two* signals simultaneously, namely the *two tone test signal*.

the two-tone test signal

The two-tone test signal consists of two equal amplitude sinusoids, of comparable frequency. Thus:

$$v(t) = V(\cos\mu_1 t + \cos\mu_2 t) \quad \text{where say } \mu_1 < \mu_2 \qquad \text{........ 9}$$

Let us use this as the input to the non-linear amplifier previously examined with a single tone input.

The amplitudes of the various output terms, from trigonometrical expansion of $v(t)$ when substituted into eqn. (2), are shown below:

$$\cos(\mu_1).t \implies g_1.V + (3/4).g_3.V^3 + (3/2).g_3.V^3 \qquad \text{........ 10}$$

$$\cos(\mu_2).t \implies g_1.V + (3/4).g_3.V^3 + (3/2).g_3.V^3 \qquad \text{........ 11}$$

$$\cos(3\mu_1).t \implies (1/4).g_3.V^3 \qquad \text{........ 12}$$

$$\cos(3\mu_2).t \implies (1/4).g_3.V^3 \qquad \text{........ 13}$$

$$\cos(2\mu_1 - \mu_2)t \implies (3/4).g_3.V^3 \qquad \text{........ 14}$$

$$\cos(\mu_1 - 2\mu_2)t \implies (3/4).g_3.V^3 \qquad \text{........ 15}$$

$$\cos(2\mu_1 + \mu_2)t \implies (3/4).g_3.V^3 \qquad \text{........ 16}$$

$$\cos(\mu_1 + 2\mu_2)t \implies (3/4).g_3.V^3 \qquad \text{........ 17}$$

short cuts

The calculation of these amplitude coefficients can be very tedious. But one soon observes certain phenomena involved, and with a narrowband system applies short cuts to avoid unnecessary work. Remember that the two frequencies μ_1 and μ_2 are close together.

Some observations are:

- The trigonometrical expansions can only generate terms on harmonics of the original frequencies, and on sum and difference frequencies of the form $(n.\mu_1 + m.\mu_2)$ and $(n.\mu_1 - m.\mu_2)$, where n and m are positive integers.
- Components with frequencies μ_1 and μ_2 will pass through the system, but none of their higher harmonics.
- Of the sum and difference frequencies $(n.\mu_1 + m.\mu_2)$ and $(n.\mu_1 - m.\mu_2)$, it is agreed that only those of the difference group, where n and m differ by unity, will pass through the system.
- When n is *odd*, the expansion of $(\cos\mu t)^n$ gives rise to all odd harmonics counting down from the nth.

- When n is *even*, the expansion of $(\cos \mu t)^n$ gives rise to all even harmonics counting down from the nth. Note that the count goes down to the zeroeth term, which is DC.

Taking these observations into account when dealing with a narrowband system, the number of necessary calculations can be reduced.

signal-to-distortion ratio - SDR

We have seen that when the test input is a single tone, the distortion components are restricted to being harmonics of this signal. But with a more complex test signal other distortion products are possible. As has just been seen, a two-tone test input gives rise to harmonics of each of these signals, as well as *intermodulation products*. Intermodulation products (IPs) arise from the *products* of two or more signals, and fall on frequencies which are the sums and differences of multiples of their harmonics.

The measure of distortion can no longer be called THD, since the sum and difference frequencies are present in addition to the harmonic terms, so the term *signal-to-distortion ratio*, or SDR, is used. It is evaluated using the same principle as THD, namely:

$$SDR = 10 \log_{10} \frac{\displaystyle\sum_{i=1}^{n} W_i^2}{\displaystyle\sum_{j=1}^{m} U_j^2} \, dB \qquad \text{........ 18}$$

where the amplitude of the *n* wanted terms is W_i and of the *m* unwanted terms is U_j. These are the terms which actually reach the output. In a narrow band system many others will be generated which will not reach the output.

two-tone example

We will now apply eqn. (18) to calculate the SDR for the characteristic of Figure 1. The input signal is defined by eqn. (9), with $V = 0.5$ volts. This makes the peak amplitude of the two-tone signal equal to 1 volt, which is comparable with the amplitude used earlier for the single tone testing. First we will include all unwanted components in the calculation, making this a wideband result.

$$\text{wideband SDR} = 27.01 \text{ dB} \qquad \text{........ 19}$$

For the narrowband case the terms to be neglected, in this example, are those on the third harmonics of μ_1 and μ_2 and the intermodulation products on the sum frequencies. The result is then:

$$\text{narrowband SDR} = 30.25 \text{ dB} \qquad \text{........ 20}$$

Remember that, had a single tone been used for this narrowband system, there would have been no signal-dependent distortion products found at the output, and the amplifier would have appeared ideal.

The noise output would not, in fact, have been zero. We have been dealing with large-signal operation, and so have ignored the existence of random noise.

noise

In the work above we have divided the output signal into wanted and unwanted components. All the unwanted components so far had magnitudes which were directly dependent upon the amplitude of the input signal. Such unwanted components are referred to as *signal dependent* noise.

Also to be considered in any system is random noise, or system noise. This arises naturally in all circuitry, and its magnitude is independent of any input signal [4].

In the work above we have made no mention of such noise. This is because it has been assumed insignificant with respect to signal dependent noise. This was a reasonable assumption, since, in a well designed system, signal dependent noise only occurs for large input signal magnitudes.

two-tone test signal generation

The two-tone signal can be made from any two signals of suitable frequencies; a convenient pair of signals for TIMS is the nominal 2 kHz message at the MASTER SIGNALS module and an AUDIO OSCILLATOR. Refer to the block diagram of Figure 4 below.

A lot can be learned about the two tone signal if it can be displayed in a convenient manner. Recognising it as a form of DSBSC makes this an easy matter.
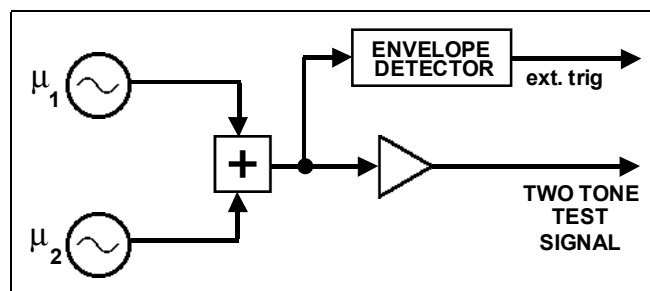

Figure 4: two-tone test signal generation

the two-tone seen as a DSBSC

Recall that a two-tone test signal has been defined earlier as in eqn. (9). The spectrum of this signal is identical with that of a DSBSC, defined as:

$$\text{DSBSC} = 2.V.\cos\mu t.\cos\omega t \qquad \text{........ 21}$$

$$= V\left[\cos(\omega - \mu)t + \cos(\omega + \mu)t\right] \qquad \text{........ 22}$$

To force the signal of eqn. (21) to match that of eqn. (9), it is necessary that:

$$\omega - \mu = \mu_1 \qquad \text{........ 23}$$

---

4 although it *can* be dependent on the presence of an input generator, the output impedance of which can influence the system noise.

$$\omega + \mu = \mu_2 \qquad \text{........ 24}$$

From these two equations the DSBSC frequencies are:

$$\mu = (| \mu_2 - \mu_1 |) / 2 \quad \text{rad/s} \qquad \text{........ 25}$$

$$\omega = (\mu_1 + \mu_2) / 2 \quad \text{rad/s} \qquad \text{........ 26}$$

To display a DSBSC stationary on the screen a triggering signal is required that is related to its *envelope*. The envelope of the DSBSC of eqn. (21) is a full wave rectified version of cosμt, but there is no signal at this frequency [eqn. (25)] if the two-tone signal is made by the addition of two tones as per eqn. (9).

The appropriate triggering signal can be generated with an envelope detector acting on the two-tone signal. Remember this is a rectifier followed by lowpass filter which will pass a few harmonics (ideally the first only) of the difference frequency of eqn. (25), but not the sum frequency eqn. (26).

<div style="text-align:center">

EXPERIMENT

</div>

experimental set-up

For this experiment you will need a WAVE ANALYSER. We suggest you use the model examined in the experiment entitled *Spectrum analysis - the WAVE ANALYSER*.

For the non-linear amplifier you will use the COMPARATOR within the UTILITIES module. Please refer to the **TIMS User Manual**. The COMPARATOR has an analog (YELLOW) output socket, which you will use, and in this application it is called a CLIPPER. The CLIPPER has a non-linear input/output characteristic. That is, it will overload with input signal amplitudes comparable with the TIMS ANALOG REFERENCE LEVEL of 4 volts peak-to-peak. The overload characteristic can be varied by means of two on-board DIP switches. For the present application you will use the 'soft clipping' characteristic; this is set with SW1 switched to 'ON/ON', and SW2 switched to 'OFF/OFF'. The REF input socket, used for the COMPARATOR, is not used for CLIPPER applications.

> *From now on the CLIPPER will be referred to as the 'DUT', namely the 'device-under-test'.*

For the purpose of the experiment you could consider it to be an amplifier in an audio system, where it is required to amplify speech signals. Thus it is a wideband device.

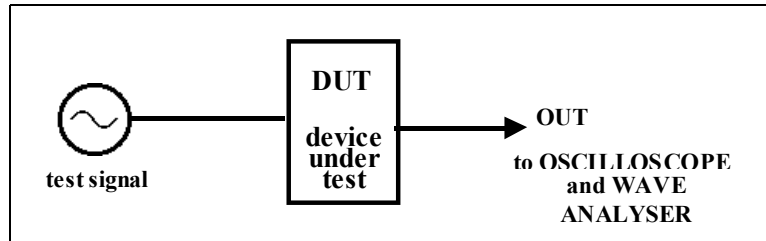You should base your experimental set up on the block diagram of Figure 5.

Figure 5: test setup

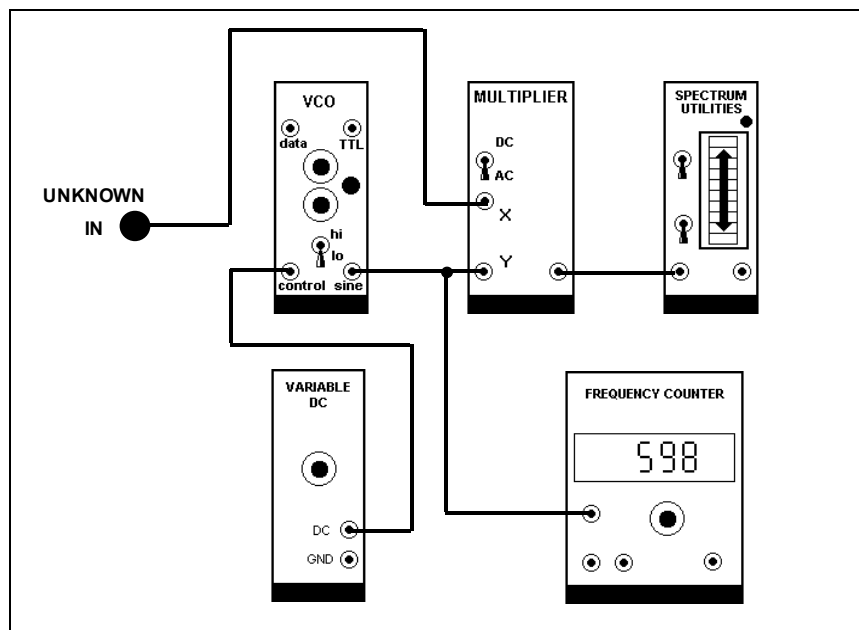The WAVE ANALYSER model is shown in Figure 6

Figure 6: the WAVE ANALYSER model.

single tone testing

measurement of THD

T1 *set the DIP switches on the DUT to the low gain (soft limiting) position, before inserting the module into the TIMS SYSTEM UNIT. This is the device under test ('DUT'). Include the patchings to the SCOPE SELECTOR inputs.*

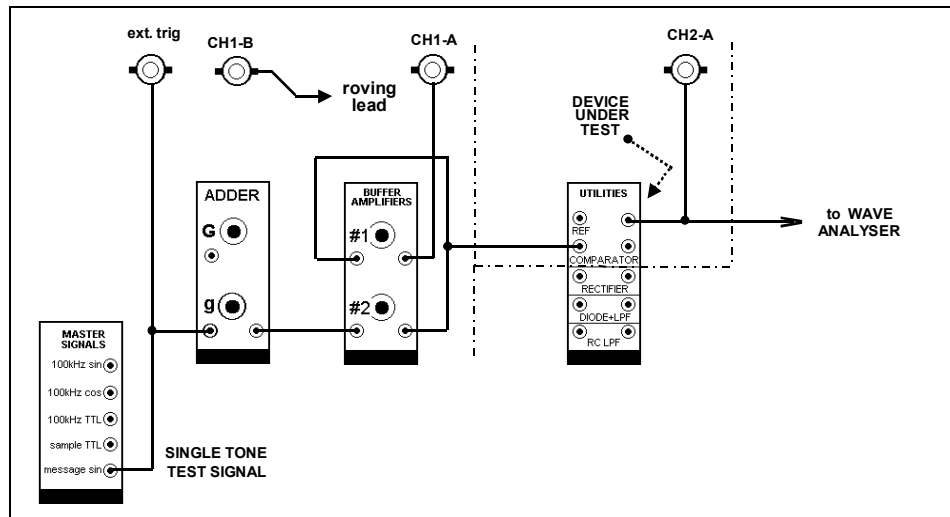**T2** *patch up the model as in Figure 7 below.*

Figure 7: the single tone test model

The ADDER, in cascade with BUFFER #2, will be used later in the two-tone test set up. BUFFER #2 will be used for test signal amplitude control. The other BUFFER, #1, is used for polarity reversal and level adjustment of the oscilloscope display of the input test signal.

T3 set the gain of the ADDER to about $\frac{1}{2}$, and that of BUFFER #2 to about unity.

T4 use the 'ext. trig' from a constant amplitude version of the test source, as shown, to trigger the oscilloscope. Set the sweep speed to show one or two periods of the test signal.

T5 switch to CH1-A and CH2-A. Set both channels to the same gain, say $\frac{1}{2}$ volt/cm. You are looking at both the input and output signals of the DUT.

T6 use the oscilloscope shift controls to superimpose the two traces. Use BUFFER #1 to equalize their amplitudes.

T7 notice that BUFFER #2 varies the amplitude of **both** traces, so they stay superimposed at low input amplitudes, before distortion sets in. Adjust the gain of BUFFER #2 until the output signal indicates the onset of moderate distortion; that is, when its shape is obviously different from the input waveform, which is also being displayed on the oscilloscope. A ratio of about 5:6 for the distorted and undistorted peak-to-peak amplitudes gives a measurable amount of distortion.

You have duplicated Figure 2.

Record the amplitude of the signal at the input to the DUT.

Amplifier overload

You are now set up moderate overload of the DUT. You are ready to measure the distortion components.

T8 *without disturbing the arrangement already patched up, model a WAVE ANALYSER. For example, use the one examined in the experiment entitled **Spectrum analysis - the WAVE ANALYSER**. This is shown in Figure 6.*

T9 *use the WAVE ANALYSER to search for, and record, the presence of all significant components at the output of the DUT for the conditions of the previous Task. Theory suggests that these will be at 2.084 kHz (set by the nominal 2 kHz test signal from the MASTER SIGNALS module) and its **odd** multiples (odd, because of the approximate cubic shape of the transfer function of the DUT).*

T10 *from your measurements of the previous Task calculate the amount of harmonic distortion (THD) under the above conditions.*

T11 *now reduce the level of the input signal to the DUT by (say) 50% (using the gain control of BUFFER #2).*

T12 *measure the amplitude of the largest unwanted component - the third harmonic of the input.*

You should have observed that:

> *whereas the amplitude of the input was reduced by 50%, that of the largest unwanted component fell by more than this.*

This is a phenomenon of non-linear distortion.

Now try a two-tone test, looking for intermodulation products as well as harmonics. You will work on the same amplifier (DUT) as before.

two-tone testing

measurement of SDR

If you built the model of Figure 7, then you are almost ready. The new test set-up is illustrated in Figure 8 below.

The ADDER combines the two signals in equal proportions. The tones should be of 'comparable frequency'; say within 10% or less of each other. BUFFER #2 at the ADDER output is used as a joint level control. As before, BUFFER #1 enables the 'before' and 'after' signals, displayed on CH1-A and CH2-A respectively, to be matched in amplitude.

T13 *patch up the model of Figure 8 below. Include the patchings to the SCOPE*
SELECTOR inputs.

T14 *the two tones are the nominal 2 kHz message from the MASTER SIGNALS*
module, and a second from an AUDIO OSCILLATOR. Set the second
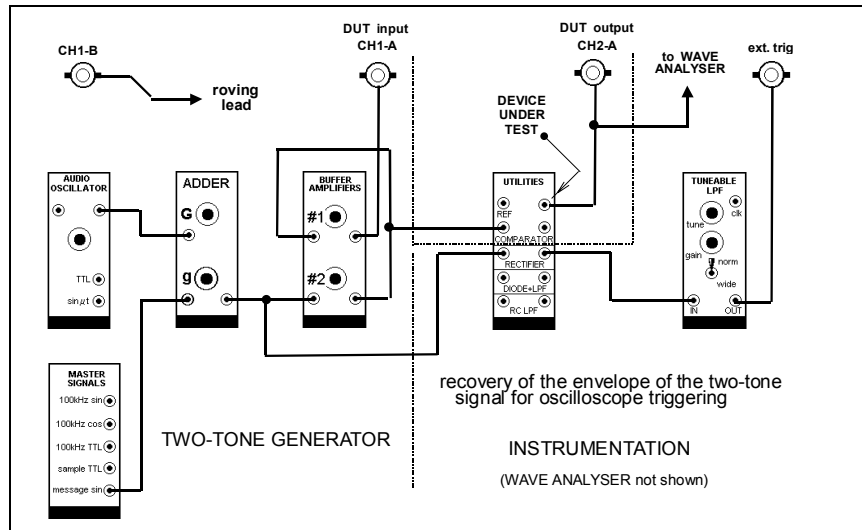*tone close to the first, say 1.8 kHz.*

Figure 8: two-tone test setup

The two-tone signal necessitates new oscilloscope triggering arrangements. As
already explained, an envelope recovery circuit is needed, and this is shown
modelled with a RECTIFIER in the UTILITIES module, and a TUNEABLE LPF.

Three of the four SCOPE SELECTOR positions are shown permanently connected.
The fourth, CH1-B, can be used as a roving lead, for various waveform inspections,
including the next task.

T15 *adjust the two tones at the ADDER output to equal amplitudes (say 2 volt*
peak-to-peak each). Adjust the gain of BUFFER #2 to about unity.

T16 *check the envelope detector output, using CH1-B. What is wanted is a*
periodic signal at envelope frequency, suitable for oscilloscope
triggering. Its shape is not critical. Tune the filter to its lowest
bandwidth; set the front panel passband GAIN *control to its mid*
range.

Amplifier overload

T17 *when satisfied with the previous Task, use the output of the envelope detector to trigger the oscilloscope, and check on CH1-A (with BUFFER #1 set to mid-gain) that the envelope of the two-tone signal is stationary on the screen (showing one or two periods of the envelope). Note that this display will show up any imperfection in the equality of the amplitudes of the two tones (or could have been used to set them equal in the first place).*

T18 *switch to CH1-A and CH2-A. Set both these oscilloscope channels to the same gain. You are now observing the input and output of the DUT. Adjust the gain of BUFFER #2 so that the output (CH2-A) is not distorted - that is, the same waveform as CH1-A.*

T19 *use the oscilloscope shift controls to overlay the two waveforms. Adjust the gain of BUFFER #1 until they are of exactly the same amplitude, and so appear as a single trace on the screen.*

T20 *now slowly increase the gain of BUFFER #2. Both traces will get larger, **but remain overlaid**, until the signal level into the DUT exceeds its linear operating range, and the output begins to show distortion. The two waveforms will no longer be identical, and the difference should be clearly visible.*

T21 *familiarize yourself with the overloading process by covering the full gain range of BUFFER #2. Then set the gain to the point where distortion of the DUT output waveform is 'moderate' [5]. A ratio of output to input signal amplitudes of about 5:6 is suggested as a start. This is the condition under which you will be making some distortion measurements. Record the input amplitude to the DUT; sketch the input and output waveforms.*

Remember the DUT has a cubic non-linearity. This suggests it will generate odd order harmonic and intermodulation products. If the two test signals are f_1 and f_2, then the largest unwanted components are likely to be on frequencies $3f_1$, $3f_2$, $(2f_1 \pm f_2)$ and $(2f_2 \pm f_1)$.

T22 *use the WAVE ANALYSER to search for, and record, the presence of all significant components in the output of the DUT for the conditions of the previous Task. Record clearly the frequency of each component found, and relate it to the frequencies of the two-tone signal.*

T23 *from your measurements of the previous Task calculate the SDR, using eqn. (18).*

---

[5] if you make it too slight the distortion components will be hard to find !

In the above calculation you might have included all components which you measured.

But suppose the output signal DUT had then been transmitted via a channel with a bandpass characteristic. Then many of the distortion components would have been removed. But distortion would still have been measured, since those intermodulation products close to the two wanted tones would have been passed.

T24 *re-calculate the SDR, assuming transmission was via a bandpass filter.*

conclusions

During the experiment you might have taken the opportunity to listen to the signals with and without distortion, to gain a qualitative idea and appreciation of what level of distortion - with these types of signals - is detectable by ear.

This experiment has served as an introduction to the methods of measuring and describing the non-linear performance of an analog circuit. A problem associated with the measurement of a narrowband system has been demonstrated, together with a method of overcoming it.

TUTORIAL QUESTIONS

Q1 *why were the two tones, in the two-tone test signal, set relatively close in frequency to each other ?*

Q2 *equation (19) suggests an alternative method of making a two-tone test signal. It has the particular advantage of providing a synchronizing signal (from the low frequency source), the two tones are automatically of equal amplitude, and the whole signal can be swept across the spectrum with one control - that of the high frequency oscillator. What are some disadvantages of this method of generation ?*

Q3 *explain qualitatively how the display of the two added tones, as a DSBSC signal, can be used to equalize the amplitudes of the two tones. Use a phasor diagram or other method to explain the process quantitatively.*

Q4 *two businesses advertise the same amplifier, one saying it is a 50 watt amplifier, and the other a 60 watt amplifier. There is no dishonesty. How could this be ?*

<div style="border: 1px solid black; text-align: center;">

APPENDIX

</div>

some useful expansions.

In analysing a non-linear system in terms of sinusoidal signals as in the above work, the aim is to convert expressions in terms of *powers* of sinusoidal signals to expressions in terms of *harmonics* of the fundamental frequencies involved.

Some useful expansions are:

$\cos^2 A = 1/2 + 1/2.\cos 2A$

$\cos^3 A = 3/4.\cos A + 1/4.\cos 3A$

$\cos^4 A = 3/8 + 1/2.\cos 2A + 1/8.\cos 4A$

$\cos^5 A = 5/8.\cos A + 5/16.\cos 3A + 1/16.\cos 5A$

$\cos^6 A = 5/16 + 15/32.\cos 2A + 3/16.\cos 4A + 1/32.\cos 6A$

Perhaps you can see the pattern developing ? It is clear that:

- when n is *odd*, the expansion of $(\cos mt)^n$ gives rise to all odd harmonics counting down from the nth.
- when n is *even*, the expansion of $(\cos \mu t)^n$ gives rise to all even harmonics counting down from the nth. Note that the count goes down to the zeroeth term, which is DC.

After an expression has been reduced to the sum of harmonic terms, those of similar frequency must be combined, taking into account their relative phases. Thus:

$$V_1.\cos\mu_1 t + V_2.\cos\mu_1 t = (V_1 + V_2).\cos\mu_1 t$$

but

$$V_1.\cos\mu_1 t + V_2.\sin\mu_1 t = V.\cos(\mu_1 t + \alpha)$$

where

$$V = \sqrt{(V_1^2 + V_2^2)}$$

and

$$\alpha = \tan^{-1}(\frac{V_2}{V_1})$$

As an exercise, develop the above expansions in terms of sin functions. There must be some obvious similarities, but just as importantly there must be differences. Explain !

Further useful expansions may be found in Appendix B to this Text.