

Optical Flow - Testing of the Horn and Schunck Method

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Outline

- Motivation
- Definition of Optical Flow
- Horn & Schunck Algorithm
- Experiments
- Conclusions



Research

- ❑ Computer Vision research under Dr. Hodel
- ❑ Goal → combine vision data with IMU data for navigation
- ❑ Determine orientation, velocity and location through vision
- ❑ Area of CV research → Optical Flow



What is optical flow?

- ❑ Definition: “*The distribution of apparent velocities of movement of brightness patterns in an image.*”[1]
- ❑ Involves projecting 3-D vision onto a 2-D plane
- ❑ Computes motion vectors of objects within an image sequence
- ❑ Vectors can be used to analyze an objects motion
→ navigation
- ❑ Interstate example



Horn and Schucnk Method

- 1981 - Horn and Schucnk publish *Determining Optical Flow*[1]
- One of the first optical flow methods
- Idea: use intensisty derivatives to estimate velocities



H&S Method - Assumptions

- I. Object within image is a flat surface
- II. Illumination on the image is constant and uniform
- III. Reflectance varies smoothly, no spatial discontinuities

→ These assumptions assure that intensity is differentiable



Constraints

- Constrained Brightness Equation
 - ▶ Intensity at a particular point within an object does not change over time:

$$\frac{dE}{dt} = 0$$
$$\frac{\partial E}{\partial x} \frac{dx}{dt} + \frac{\partial E}{\partial y} \frac{dy}{dt} + \frac{\partial E}{\partial t} = 0$$
$$E_x u + E_y v + E_t = 0$$

Constraints

- Constrained Brightness Eq. (cont.) Rearranging the equation:

$$\begin{bmatrix} E_x & E_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -E_t$$

Component of movement in the direction of the brightness gradient (E_x, E_y) is

$$-\frac{E_t}{\sqrt{E_x^2 + E_y^2}}$$



Aperture Effect

- Aperture Effect
- Barber shop pole example
- CBE \Rightarrow motion can only be detected in the direction of the brightness gradient
- H&S method - subject to aperture problem



Constraints

- Smoothness Constraint
 - ▶ Neighboring points on a moving object have similar velocities, therefore the brightness patterns in the image vary smoothly [1].
 - ▶ Occluding objects will give discontinuities in optical flow
 - ▶ Smoothness constraint adds difficulty in areas of occluding edges that are undergoing different movement



Smoothness Constraint (Cont.)

- Expressing the constraint - minimize the square of the gradient magnitude

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 \text{ and } \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2.$$

- The smoothness of the optical flow field can also be measured by the Laplacians of u and v :

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

$$\nabla^2 v = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}$$

Estimations-Derivatives

- E_x , E_y , and E_t must be estimated for the CBE
- Take an average over a 4x4x2 block of back to back images

$$E_x \approx \frac{1}{4} [E_{ii+1,jj+1,kk} - E_{ii+1,jj,kk} + E_{ii,jj+1,kk} - E_{ii,jj,kk} \cdots \\ + E_{ii+1,jj+1,kk+1} - E_{ii+1,jj,kk+1} + E_{ii,jj+1,kk+1} - E_{ii,jj,kk+1}]$$

$$E_y \approx \frac{1}{4} [E_{ii,jj,kk} - E_{ii+1,jj,kk} + E_{ii,jj+1,kk} - E_{ii+1,jj+1,kk} \cdots \\ + E_{ii,jj,kk+1} - E_{ii+1,jj,kk+1} + E_{ii,jj+1,kk+1} - E_{ii+1,jj+1,kk+1}]$$

$$E_t \approx \frac{1}{4} [E_{ii+1,jj,kk+1} - E_{ii+1,jj,kk} + E_{ii,jj,kk+1} - E_{ii,jj,kk} \cdots \\ + E_{ii+1,jj+1,kk+1} - E_{ii+1,jj+1,kk} + E_{ii,jj+1,kk+1} - E_{ii,jj+1,kk}]$$

Estimations-Laplacians

□ Laplacian estimates

$$\nabla^2 u \approx k(\bar{u}_{i,j,k} - u_{i,j,k}) \text{ and } \nabla^2 v \approx k(\bar{v}_{i,j,k} - v_{i,j,k})$$

where \bar{u} and \bar{v} are local averages of the velocity vectors

$$\begin{aligned} \bar{u}_{i,j,k} &= \frac{1}{6} [u_{ii-1,jj} + u_{ii,jj+1} + u_{ii+1,jj} + u_{ii,jj-1}] \cdots \\ &\quad + \frac{1}{12} [u_{ii-1,jj-1} + u_{ii-1,jj+1} + u_{ii+1,jj+1} + u_{ii+1,jj-1}] \\ \bar{v}_{i,j,k} &= \frac{1}{6} [v_{ii-1,jj} + v_{ii,jj+1} + v_{ii+1,jj} + v_{ii,jj-1}] \cdots \\ &\quad + \frac{1}{12} [v_{ii-1,jj-1} + v_{ii-1,jj+1} + v_{ii+1,jj+1} + v_{ii+1,jj-1}] \end{aligned}$$

Minimization

- Algorithm approach → minimize the error in brightness equation and the departure from smoothness.

$$\begin{aligned}\varepsilon_b &= E_x u + E_y v + E_t \\ \varepsilon_c^2 &= \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\end{aligned}$$

The total error to be minimized is then:

$$\varepsilon^2 = \int \int (\alpha^2 \varepsilon_c^2 + \varepsilon_b^2) dx dy$$

- Constrained least squares minimization

Iterative Solution

- Direct solution to minimization → computationally expensive
- Iterative solution is developed
- Velocity estimate based on estimated derivatives and average of previous estimates

$$u^{n+1} = \bar{u}^n - E_x \left(\frac{E_x \bar{u}^n + E_y \bar{v}^n + E_t}{\alpha^2 + E_x^2 + E_y^2} \right)$$
$$v^{n+1} = \bar{v}^n - E_y \left(\frac{E_x \bar{u}^n + E_y \bar{v}^n + E_t}{\alpha^2 + E_x^2 + E_y^2} \right)$$

Algorithm

=====Optical Flow Algorithm=====

obtain image sequence

get size of image sequence, $M(x)$, $N(y)$, number(t)

set alpha; select number of iterations

for $kk = 1:\text{number}$

 for $ii = 1:M$

 for $jj = 1:N$

 compute E_x , E_y and E_t

 end

 end

for $nn = 1:\text{num_its}$

 for $ii = 1:M$

 for $jj = 2:N$

 compute \bar{u}

 compute \bar{v}

 update u

 update v

 end

 end

end

end

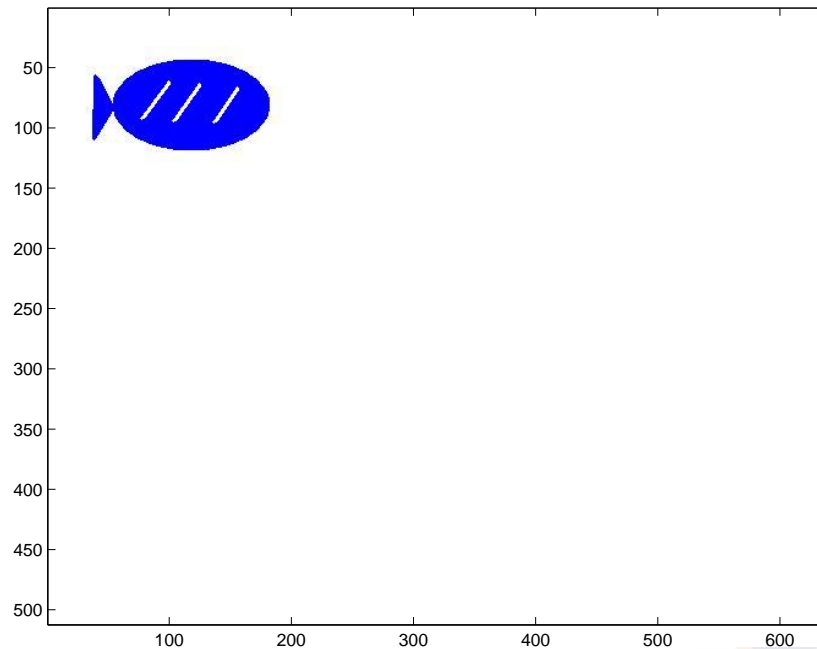


Testing the H&S Algorithm

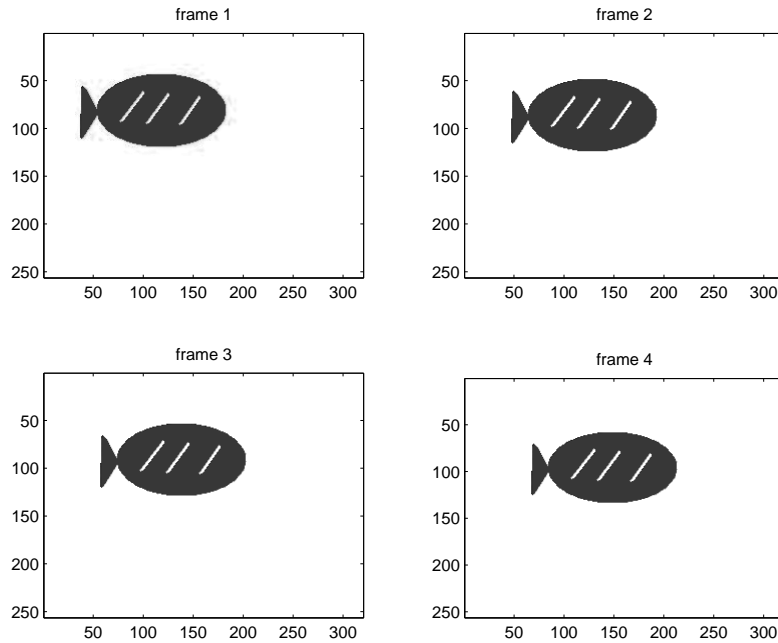
- Horn & Schucnk algorithm implemented in MATLAB
- Tested on:
 - ▶ Synthetic image sequence
 - ▶ Real image sequence
- $\alpha = 20$
- number of iterations = 32
- Computed velocities displayed with a quiver plot

Synthetic Image Sequence

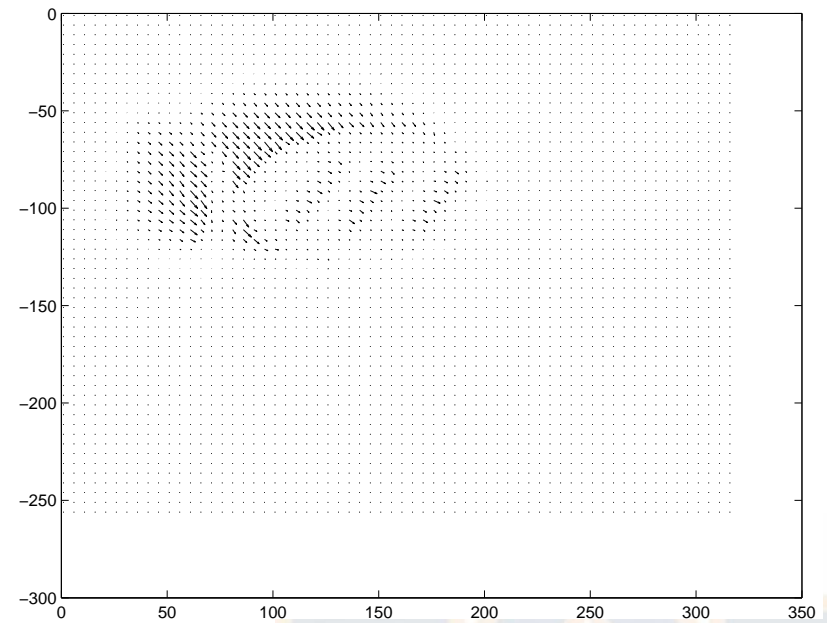
- ❑ Original created in paint
- ❑ Sequence generated in MATLAB



Frames 1-4



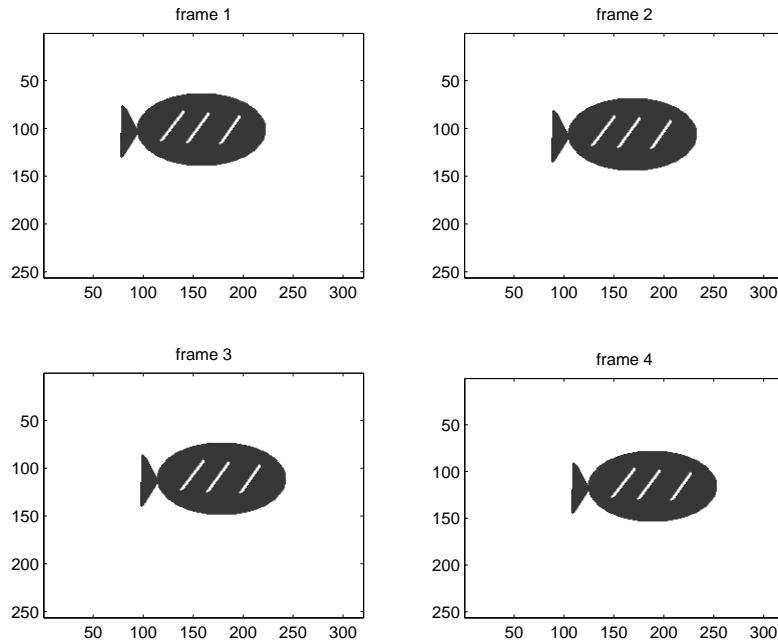
(a) Frames 1 - 4



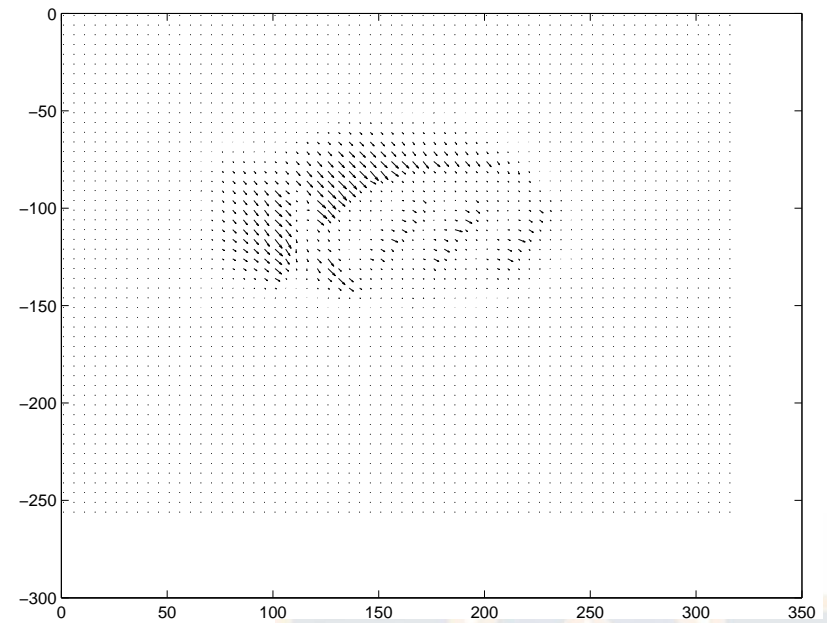
(b) Optical Flow

Figure 1: First four frames of synthetic sequence and the computed optical flow

Frames 5-8



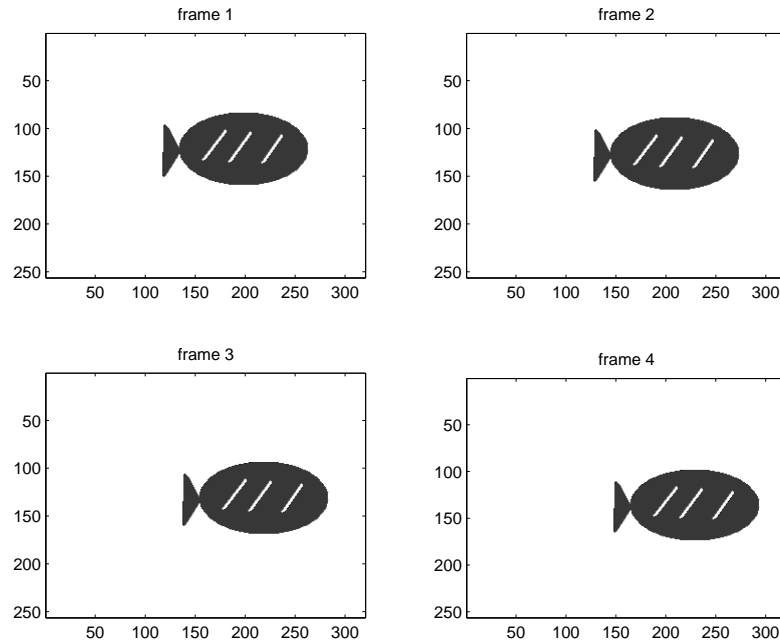
(a) Frames 5 - 8



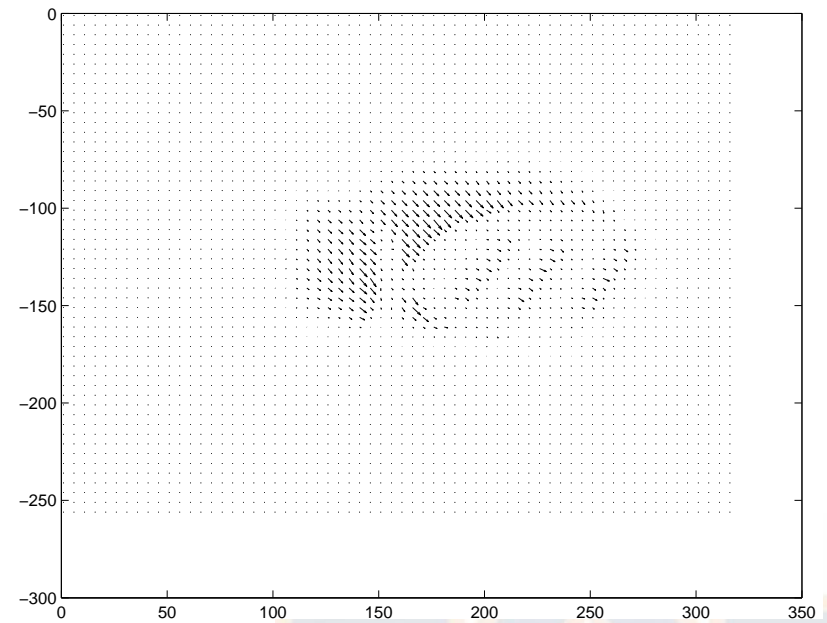
(b) Optical Flow

Figure 2: Next four frames and computed optical flow

Frames 9-12



(a) Frames 9 - 12



(b) Optical Flow

Figure 3: Last four frames and computed optical flow

Real Image Sequence

- Sequence taken from a digital camera
- Images are of a balloon tied to a ceiling fan

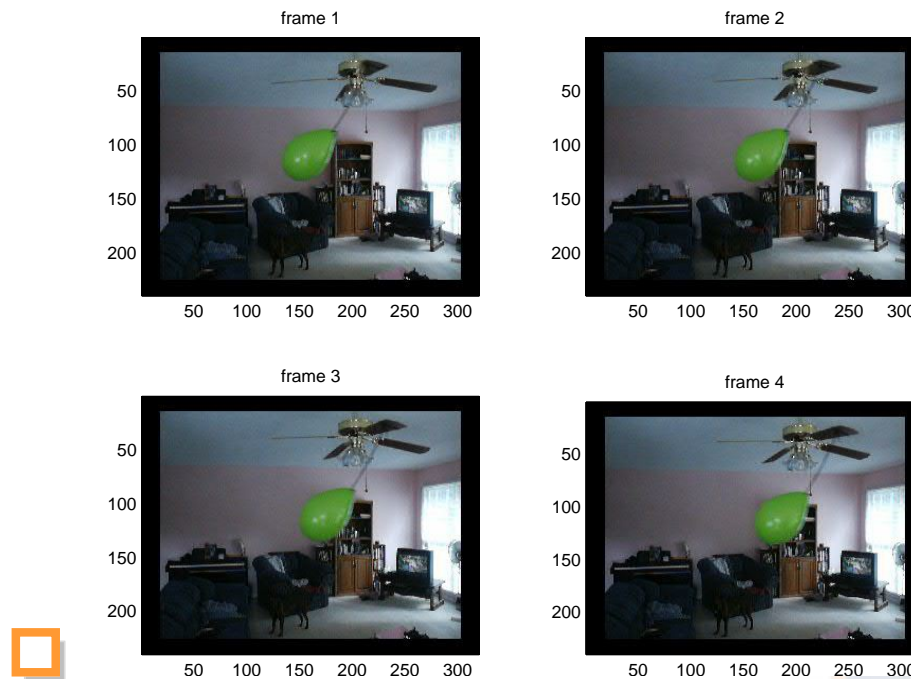
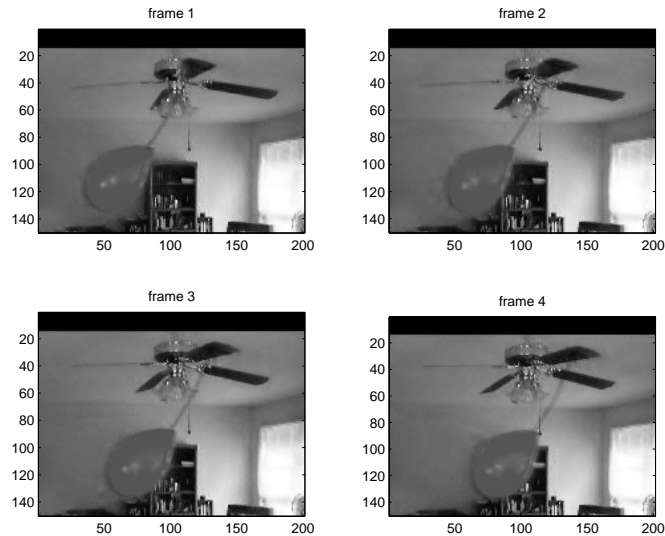
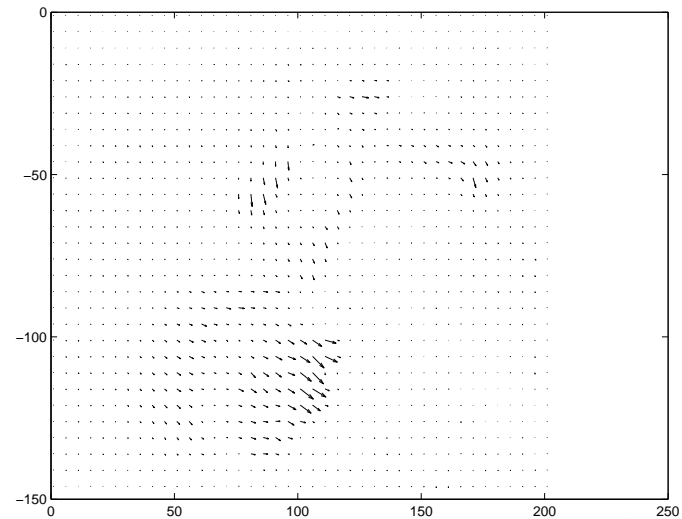


Figure 4: First four images in balloon sequence

Frames 1-4



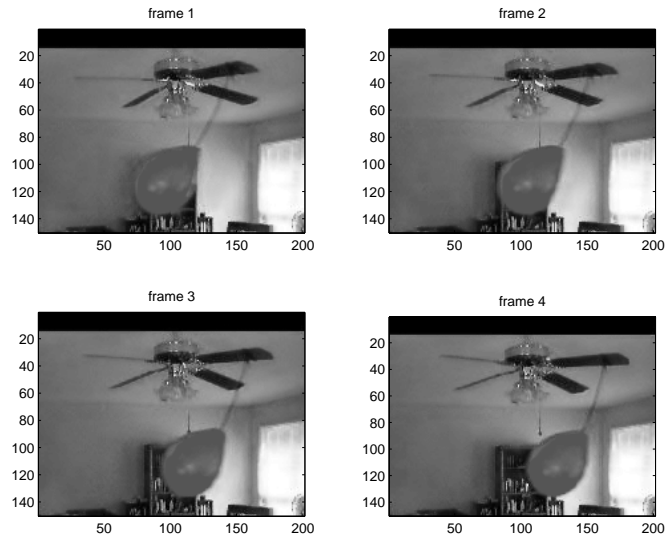
(a) Frames 1 - 4



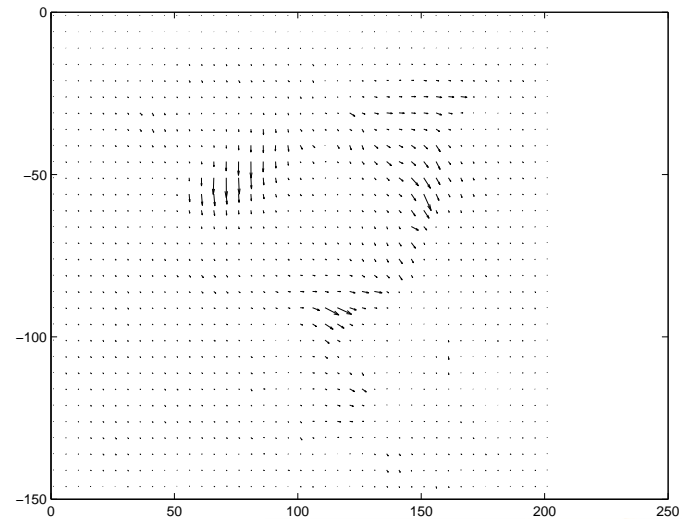
(b) Optical Flow

Figure 5: First four frames of balloon sequence and the computed optical flow

Frames 5-8



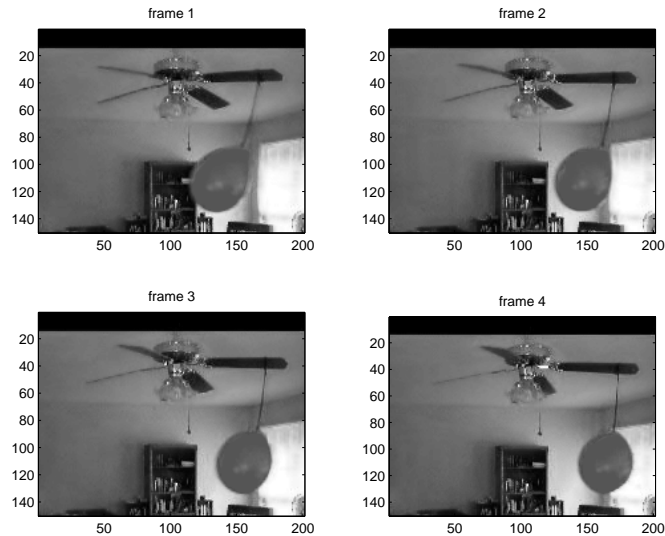
(a) Frames 5 - 8



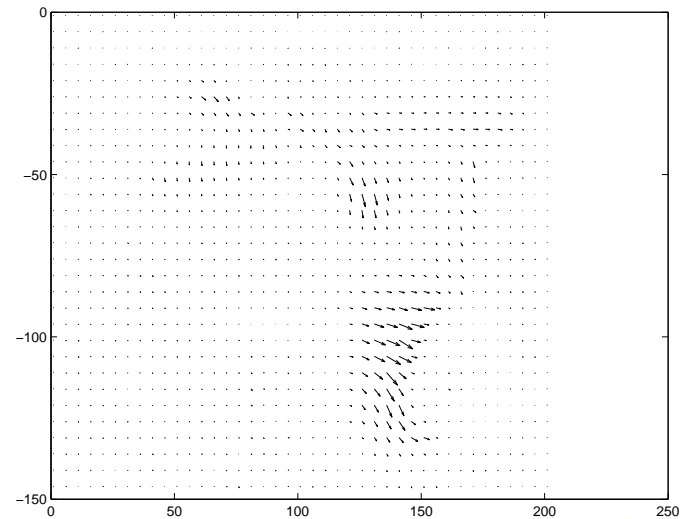
(b) Optical Flow

Figure 6: Next four frames and computed optical flow

Frames 9-12



(a) Frames 9 - 12



(b) Optical Flow

Figure 7: Last four frames and computed optical flow

Animation

Start animation



Conclusions

- Strengths
 - ▶ H&S works well when conditions are suitable for constraints
 - ▶ Easy to implement
 - ▶ Methods are intuitive (What's different between two images?)
- Weaknesses
 - ▶ Subject to aperture effect
 - ▶ Conditions usually aren't suitable for using constraints
- Questions?

References

- [1] B.K.P Horn and B.G. Schunck. Determining optical flow. *Artificial Intelligence*, 17:185–203, 1981.