EFFICIENT OPTIMIZATION OF ALL-TERMINAL RELIABLE NETWORKS USING AN EVOLUTIONARY APPROACH

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Summary & Conclusions - The use of computer communication networks has been rapidly increasing to 1) share expensive hardware and software resources, and 2) provide access to main systems from distant locations. The reliability and the cost of these systems are important considerations that are largely determined by network topology. Network topology consists of nodes and the links between nodes. The selection of optimal network topology is an NP-hard combinatorial problem so that the classical enumeration based methods grow exponentially with network size. In this study, a heuristic search algorithm inspired by evolutionary methods is presented to solve the all-terminal network design problem when considering cost and reliability. The genetic algorithm heuristic is considerably enhanced over conventional implementations to improve effectiveness and efficiency. This general optimization approach is shown to be computationally efficient and highly effective on a large suite of test problems with search spaces up to 2×10^{90}.

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1. INTRODUCTION

Acronyms

GA genetic algorithm

GAKBS GA with knowledge-based steps

SGA simple GA

An important stage of the design of communication networks is to find the best layout of components to minimize cost while meeting a performance criterion, such as transmission delay, throughput or reliability [1]. This paper focuses on large scale backbone communication network [2] design where the relevant reliability metric is all-terminal network reliability (also called overall network reliability); defined as the probability that every pair of nodes can communicate with each other [1, 3]. The problem of optimal design of link topology can be formulated as a combinatorial problem where the selection of components either maximizes reliability, or minimizes cost. This problem is NP-hard [4], and further compounding this is the computational effort required to calculate or estimate network reliability.

This problem has been studied with both enumerative based methods and heuristic methods. Jan et al. [1] developed an algorithm using decomposition based on branch-and-bound to minimize link costs with a minimum network reliability constraint; this is computationally tractable for fully connected networks up to 12 nodes. Chopra et al. [6] and Aggarwal et al. [5] both used greedy heuristic approaches. Venetsanopoulos and Singh [7] used a two step heuristic for minimizing cost subject to a reliability constraint. GA has recently been used in combinatorial optimization approaches to reliable design, mainly for series-parallel systems [8-10]. For network design, Kumar et al. [11, 12] developed a GA considering diameter, distance and reliability to design and expand computer networks. Deeter and Smith present a GA for a generalized network design problem with alternative link reliabilities [13]. This paper also uses a GA, but significantly customizes it to the all-terminal design problem to result in an effective and efficient optimization methodology. Furthermore, this approach is demonstrated on a large test suite of problems, including networks with up to 300 possible links. Previous papers have demonstrated their optimization procedures on small networks (usually less than 10 nodes), thus the important issue of scale-up is left unanswered.

A significant issue is the calculation of the reliability of candidate network topologies. When minimizing cost subject to a minimum reliability constraint, the network reliability calculation is necessary to determine feasibility of the candidate design. When maximizing reliability subject to a maximum cost constraint, the network reliability calculation is the objective function. It either case, this calculation is a critical part of evaluating each candidate network topology, however it is also problematic since it involves considerable computational effort. All known analytic methods for all-terminal network reliability calculation have worst case computation times which grow exponentially in the size of the network considered. Monte Carlo simulation methods, for which computation time grows only slightly faster than linear with network size, are suitable for large networks [14]. An efficient Monte Carlo simulation technique [15] is used to estimate all-terminal reliability in this paper. To further reduce the computational effort required, candidate networks must meet a 2-connectivity measure [16], after verifying that a spanning tree exists using the method of [17]. Then an upper bound on reliability by [18] is used to screen candidate networks prior to the estimation of reliability using simulation. However, any method of calculating or estimating network reliability could be incorporated into the evolutionary optimization methodology presented in this paper.

Section 2 gives the mathematical formulation of the optimization problem and its assumptions. Section 3 gives the algorithm used in this paper. Section 4 provides the results and discussion of applying the optimization methodology to a suite of test problems.

Notation

G	a probabilistic graph
N	set of nodes (terminals)
L	set of links (edges, arcs)

(i,j)	a link between nodes i and j
p,q	link reliability, link unreliability for all links; p+q = 1
x_{ij}	decision variable, $x_{ij} \in \{0,1\}$
$\mathbf{x}$	a link topology of $\{x_{11}, x_{12}, \ldots, x_{ij}, \ldots, x_{N,N-1}\}$
$R(\mathbf{x})$	all-terminal reliability of $\mathbf{x}$
R_o	network reliability requirement
Z	objective function
c_{ij}	cost of (i,j)
c_{MAX}	the maximum value of c_{ij}
δ	0, if $Rel(\mathbf{x}) \geq R_o$; 1, if $Rel(\mathbf{x}) < R_o$
g_{MAX}	maximum number of generations in genetic algorithm
n	population size of genetic algorithm
r_c	crossover rate of genetic algorithm
r_m	mutation rate of genetic algorithm

2. STATEMENT OF THE PROBLEM

A communication network can be modeled by a probabilistic graph G = (N, L, p), in which N and L correspond to computer sites and communication links, respectively. Any graph G = (N, L) is connected if there is at least one path between every pair of nodes $N_i, N_j \in N$ edges, which minimally requires a spanning tree with N-1 edges.

Assumptions

1. The location of each network node is given.
2. Nodes are perfectly reliable.
3. Each c_{ij} and p are fixed and known.
4. Each link is bi-directional.
5. There are no redundant links in the network.

6. Links are either operational or failed.

7. The failure of links are *s*-independent.

8. No repair is considered.

The optimization problem is:

Minimize $$Z = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_{ij} x_{ij} \tag{1}$$

Subject to : $R(\mathbf{x}) \geq R_o$

where $x_{ij} \in \{0,1\}$ is the decision variable and $R(\mathbf{x})$ is the system reliability.

At any instant of time, only some edges of G may be operational. A state of G is a sub-graph (N,L′) with L′∈L, where L′ is the set of operational edges. An operational state is generally called a pathset, and a minimal operational state is termed a min-path. A failed state L′ is called L \ L′ (a cutset) and when L′ is a maximal failed state L \ L′ is a mincut [19]. The network reliability of state L′ ⊆ L is shown below :

$$\sum_{\substack{\text{all operational}\\ \text{states}}} \prod_{l \in L'} p_l \prod_{l \in (L \setminus L')} q_l \tag{2}$$

Summing this state occurrence probability over all operational states gives the network system reliability. However, the exponential number of states makes such a computation infeasible, even for networks of moderate sizes [19]. All current exact algorithms of reliability computation for general networks are based on the enumeration of states, minpaths or mincuts [20-24]. But few of them give a little practical improvement over complete state enumeration, while others still examine exponentially many states, minpaths or mincuts [19]. Therefore, estimation of network reliability is commonly used for non-trivial sized networks, and this paper uses an efficient Monte Carlo based simulation as described in section 3.

3. SOLUTION ALGORITHM

A GA was selected as the heuristic optimization vehicle because of its flexibility and robustness as demonstrated on many NP-hard problems, including those of reliability design [8-

13]. GA is a meta-heuristic inspired by the biological paradigm of evolution. They were pioneered by Holland [25], De Jong [26], and Goldberg [27] in the context of continuous non-linear optimization, and later extended by various authors [e.g., 28-30] to combinatorial problems. In GA, the search space is composed of candidate solutions to the problem, each represented by a string, termed a chromosome. Each chromosome has an objective function value, called the fitness. A set of chromosomes together with their associated fitness is called the population. This population, at a given iteration of the GA, is referred to as a generation.

There are three main steps in the repeat loop for GA:

1) The process of selecting strings from the current generation to be parents of the next generation with preference for fitter strings. This is the **selection** process for reproduction.
2) The process of combining two selected strings to generate new children strings, which is called **crossover**. Probabilistically, components of a chromosome are perturbed while generating a child. This process is called **mutation**. Together, crossover and mutation comprise **reproduction**.
3) Computation of the **fitness** value using the objective function of each new solution.

The steps in the GA approach in this research are discussed below, followed by a flowchart of the algorithm.

3.1. Coding Structure

Each **x** represents a candidate network design with the size of the string equal to N(N-1)/2, the number of possible links in a fully connected network.[2] For example, Figure 1 shows a simple network whose base graph consists of 5 nodes and 10 possible links, but with only 7 links present.

Figure 1 here.

The string representation of this network is:

[2] This is reduced for networks where not all possible links are permitted, as demonstrated on test problems 18 - 20.

$$\left[\begin{array}{cccccccccc} 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$
$$\left[\begin{array}{cccccccccc} x_{12} & x_{13} & x_{14} & x_{15} & x_{23} & x_{24} & x_{25} & x_{34} & x_{35} & x_{45} \end{array}\right]$$

3.2. Initial Population

The initial population in GA can be obtained randomly or by using a heuristic method ("seeding" the population). In this paper, the initial population consists of a set of connected networks which are also 2-connected but is otherwise generated in a random fashion with preference to combinations yielding high reliability. The 2-connectivity measure is used as a preliminary screening, since it is usually a property of highly reliable networks. The selection of the probability values which are used in deciding whether a link exists or not is an important stage for the efficient generation of such an initial population. Table 1 shows the probability intervals used in this research, established by exploratory research.

Insert Table 1 Here

3.3. Genetic Algorithm Parameters

The choice of parameters for GA can affect performance of the algorithm. These parameters include n, r_c and r_m. There have been many different studies to find the optimal control parameters for GA, however this is little in the way of useful guidance. Instead, a set of experiments are usually run to establish parameter values which work well and to gauge the sensitivity of the GA to alterations in those values. For this study, the best results were: n = 20, r_c = 0.95, and r_m = 0.05.

3.4. Selection Mechanism, Genetic Operators, Replacement Strategy

The approach in this paper uses the conventional GA operators of roulette wheel selection, single point crossover and bit flip mutation. See Goldberg [27] for a definition of these. Below is an example of the single point crossover strategy with the splice point after x_{15}. and bit flip mutation used in this research:

Parent 1

$$\left[\begin{array}{cccccccccc} 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$
$$\left[\begin{array}{cccccccccc} x_{12} & x_{13} & x_{14} & x_{15} & x_{23} & x_{24} & x_{25} & x_{34} & x_{35} & x_{45} \end{array}\right]$$

Parent 2

[1 0 1 1 0 0 1 1 1 1]

[x_{12} x_{13} x_{14} x_{15} x_{23} x_{24} x_{25} x_{34} x_{35} x_{45}]

Child 1

[1 1 0 1 0 0 1 1 1 1]

[x_{12} x_{13} x_{14} x_{15} x_{23} x_{24} x_{25} x_{34} x_{35} x_{45}]

Mutated Child 1

[1 0 0 1 0 0 1 1 1 1]

[x_{12} x_{13} x_{14} x_{15} x_{23} x_{24} x_{25} x_{34} x_{35} x_{45}]

Child 2

[1 0 1 1 1 0 1 1 0 1]

[x_{12} x_{13} x_{14} x_{15} x_{23} x_{24} x_{25} x_{34} x_{35} x_{45}]

Mutated Child 2

[0 0 1 1 1 0 1 1 1 1]

[x_{12} x_{13} x_{14} x_{15} x_{23} x_{24} x_{25} x_{34} x_{35} x_{45}]

3.5. Objective & Fitness Functions

The objective function is the sum of the total cost for all links in the network plus a quadratic penalty function for networks which fail to meet the minimum reliability requirement. The objective of the penalty function is to lead the optimization algorithm to near-optimal feasible solutions. It is important to allow infeasible solutions into the population because good solutions are often the result of breeding between a feasible and an infeasible solution and the genetic algorithm reproduction procedure does not ensure feasible children, even if both parents are feasible [31], especially in highly constrained problems where the constraint is likely to be active. The fitness function considering possible infeasible solutions is:

$$Z(\mathbf{x}) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_{ij}\, x_{ij} + \delta \left(c_{MAX}(R(\mathbf{x}) - R_o)\right)^2 \qquad (3)$$

In GA, the fitness, $F(\mathbf{x})$, traditionally improves with an improved objective function, creating a maximization problem. Therefore, fitness is:

$$F(\mathbf{x}) \equiv (Z_{MAX} - Z(\mathbf{x})) \qquad (4)$$

where Z_{MAX} is the largest (worst) value of (3) for the current population.

3.6. Network Reliability

The GA methodology in this paper uses three reliability estimations to tradeoff accuracy with computational effort. An ideal strategy employs only the computationally intensive method of Monte Carlo simulation (or exact network reliability calculation) on the optimal network design. Since GA is an iterative algorithm, this ideal cannot be attained because many candidate networks must be evaluated during the search. Therefore, screening of candidate network designs is done:

- A connectivity check for a spanning tree is made on all new network designs using the method of [17].
- For networks which pass this check, the 2-connectivity measure [16] is made by counting the node degrees.
- For networks which pass both of these preliminary checks, Jan's upper bound [18] is used to compute the upper bound of reliability of a candidate network, $R_U(\mathbf{x})$.

This upper bound in used in the calculation of the objective function (3) for all networks except those which are the best found so far ($\mathbf{x}_{BEST}$). Networks which have $R_U(\mathbf{x}) \geq R_o$ and the lowest cost so far are sent to the Monte Carlo subroutine for more precise estimation of network reliability using an efficient Monte Carlo technique by Yeh et al. [15]. The simulation is done for 3000 iterations for each candidate network for all problems studied in this paper.

3.7. Termination Condition

The criterion is g_{MAX}, which varies according to the size of the network, N, under study.

Algorithm

Step 1: Generate the initial population, $k = 1$ to n, randomly according to the link probabilities in Table 1, discarding any solutions which fail to meet the 2-connectivity requirement. Calculate the fitness of each candidate network in the population using (3) and (4) and Jan's upper bound as $R(\mathbf{x})$, except for the lowest cost network with $R_U(\mathbf{x}) \geq R_o$. For this network, $\mathbf{x}_{BEST}$, use the Monte Carlo estimation of $R(\mathbf{x})$ in (4). $g = 1$.

Step 2: Select two candidate networks from current population by the selection mechanism.

Step 3: To obtain two children candidate networks, apply reproduction to the selected networks using r_c and r_m.

Step 4: Determine the 2-connectivity of each new child. Discard any that do not satisfy 2-connectivity.

Step 5: Calculate $R_U(\mathbf{x})$ for each child and compute its objective function using (3).

Step 6: If the number of new children < n-1 go to Step 2.

Step 7: Replace parents with children, retaining $\mathbf{x}_{BEST}$ from the previous generation.

Step 8: Sort the new generation in increasing order of Z with k the indices of a candidate network. $k = 1$ to n.

a) If $Z(\mathbf{x}_k) < Z(\mathbf{x}_{BEST})$, then estimate the system reliability of this network using Monte Carlo simulation, else go to Step 9.

b) $\mathbf{x}_{BEST} = \mathbf{x}_k$. Go to Step 9.

Step 9 : Calculate the fitness value, $F(\mathbf{x})$, using (4) for each network in the new population.

Step 10 : If $g = g_{MAX}$ stop, else go to Step 2 and $g = g+1$.

4. RESULTS & DISCUSSION

4.1 Results

Several comparisons are used to judge the effectiveness and efficiency of the GA described in section 3, which will be termed GAKBS. The first comparison is against a GA which does not include the problem specific structure, termed SGA. In SGA, the initial population consists of connected networks, generated using a constant probability value of 0.5 to generate a link. These networks are not subject to the 2-connectivity screening calculation. System reliability is then estimated on all networks using the improved Monte Carlo procedure. The second comparison is against the branch-and-bound method of Jan et al. [1].

The test problems are summarized in Table 2 and detailed in the Appendix. These problems are both fully connected and non-fully connected networks (*viz.*, only a subset of L is possible for selection). N of the connected networks ranges from 5 to 25. The available links of the non-fully connected networks were randomly generated and were 1.5 times N. The link costs for all networks were randomly generated over [1,100] except for problems 3 through 5 which used costs over [1,150]. Each problem for the GA was run 10 times, each time with a different random number seed. Optimal solutions, as obtained by the method of Jan et al. [1], are also given in Table 2 with topologies shown in the Appendix. Jan's method cannot be practically used on the larger problems (numbers 15-17) because of the computational effort of the branch-and-bound procedure. As shown, GAKBS gives the optimal value for the all replications of problems 1 - 3 and finds optimal for all but two of the problems for at least one run of the 10. The two with suboptimal results (12 and 13) are very close to optimal.

Insert Table 2 Here

The performance of SGA and GAKBS were compared for all networks with the enhanced GA converging significantly faster to a value closer to optimum. Figure 2 is a typical convergence plot showing the best cost network for a single problem averaged over ten runs with different seeds. Speed of convergence is important because the reliability calculation, especially for larger problems, precludes extensive search of the problem space. Table 3 lists the search space for each problem along with the proportion actually searched by the GAKBS during a single run ($n \times g_{\text{MAX}}$). g_{MAX} ranged from 30 to 20000, depending on problem size. This proportion is an upper bound because GA's can (and often do) revisit solutions already considered earlier in the evolutionary search. It can be seen that the GA approach examines only a very tiny fraction of the possible solutions for the larger problems, yet still yields optimal or near-optimal solutions. Table 3 also compares the efficacy of the Monte Carlo estimation of network reliability. The exact network reliability is calculated using a backtracking algorithm also used by Jan et al. [1] and compared to the estimated counterpart for the final network for those problems where the GA found optimal. The reliability estimation of the Monte Carlo method is unbiased and is always within 1% of the exact network reliability. Since the computation time for the Monte Carlo method is constant with network size, and estimation accuracy does not degrade with an increase in L, this simulation estimation is a very effective surrogate for an exact network reliability calculation.

Figure 2 here.

Insert Table 3 Here

4.2 Discussion

In this study, a stochastic search algorithm inspired by evolution was developed to solve network topology design with minimum cost subject to a reliability constraint. The strengths of this evolutionary approach are almost non-increasing computational effort, effective optimization and flexibility. Since GA is an iterative algorithm and improvement is typically diminishing (as in Figure 2), it may be terminated at any time and still return good results. The computational effort over the test problems studied did not vary significantly although network size increased by many orders of magnitude. The GA returned optimal or near-optimal solutions on every run regardless of problem instance, problem size or random number seed. The methodology is very flexible and alternative objectives (e.g., maximize reliability subject to a cost constraint) and alternative methods of reliability calculation (e.g., backtracking or another Monte Carlo method) could be easily substituted for those used in this research.

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TABLE 1: Probability values used to generate the initial population.

Number of Nodes (N)	Probability Values
10	(0.15-0.60)
20	(0.15-0.50)
30	(0.10-0.30)

TABLE 2: Comparison of results.

Problem	N	L	p	R_o	Optimum Cost*	Results of Genetic Algorithms[@] Best Cost	Mean Cost	Coef. of Variation
FULLY CONNECTED NETWORKS								
1	5	10	0.80	0.90	255	255	255.0	0
2	5	10	0.90	0.95	201	201	201.0	0
3	7	21	0.90	0.90	720	720	720.0	0
4	7	21	0.90	0.95	845	845	857.0	0.0185
5	7	21	0.95	0.95	630	630	656.0	0.0344
6	8	28	0.90	0.90	203	203	205.4	0.0198
7	8	28	0.90	0.95	247	247	249.5	0.0183
8	8	28	0.95	0.95	179	179	180.3	0.0228
9	9	36	0.90	0.90	239	239	245.1	0.0497
10	9	36	0.90	0.95	286	286	298.2	0.0340
11	9	36	0.95	0.95	209	209	227.2	0.0839
12	10	45	0.90	0.90	154	156	169.8	0.0618
13	10	45	0.90	0.95	197	205	206.6	0.0095
14	10	45	0.95	0.95	136	136	150.4	0.0802
15	15	105	0.90	0.95	---	317	344.6	0.0703
16	20	190	0.95	0.95	---	926	956.0	0.0304
17	25	300	0.95	0.90	---	1606	1651.3	0.0243
NON FULLY CONNECTED NETWORKS								
18	14	21	0.90	0.90	1063	1063	1076.1	0.0129
19	16	24	0.90	0.95	1022	1022	1032.0	0.0204
20	20	30	0.95	0.90	596	596	598.6	0.0052

* Found by the method of Jan et al. [1].

@ Over ten runs.

TABLE 3: Comparison of search effort and reliability estimation.

Problem	Search Space	Solutions Searched	Fraction Searched	R_o	Actual $R(\mathbf{x})$	Estimated $R(\mathbf{x})$	Percent Difference
1	1.02 E3	6.00 E2	5.86 E-1	0.90	0.9170	0.9170	0.000
2	1.02 E3	6.00 E2	5.86 E-1	0.95	0.9579	0.9604	0.261
3	2.10 E6	1.50 E4	7.14 E-3	0.90	0.9034	0.9031	-0.033
4	2.10 E6	1.50 E4	7.14 E-3	0.95	0.9513	0.9580	0.704
5	2.10 E6	1.50 E4	7.14 E-3	0.95	0.9556	0.9569	0.136
6	2.68 E8	2.00 E4	7.46 E-5	0.90	0.9078	0.9078	0.000
7	2.68 E8	2.00 E4	7.46 E-5	0.95	0.9614	0.9628	0.001
8	2.68 E8	2.00 E4	7.46 E-5	0.95	0.9637	0.9645	0.083
9	6.87 E10	4.00 E4	5.82 E-7	0.90	0.9066	0.9069	0.033
10	6.87 E10	4.00 E4	5.82 E-7	0.95	0.9567	0.9545	-0.230
11	6.87 E10	4.00 E4	5.82 E-7	0.95	0.9669	0.9668	-0.010
12	3.52 E13	8.00 E4	2.27 E-9	0.90	0.9050	*	
13	3.52 E13	8.00 E4	2.27 E-9	0.95	0.9516	*	
14	3.52 E13	8.00 E4	2.27 E-9	0.95	0.9611	0.9591	-0.208
15	4.06 E31	1.40 E5	3.45 E-27	0.95	@	0.9509	
16	1.57 E57	2.00 E5	1.27 E-52	0.95	@	0.9925	
17	2.04 E90	4.00 E5	1.96 E-85	0.90	@	0.9618	
18	2.10 E6	1.50 E4	7.14 E-3	0.90	0.9035	0.9035	0.000
19	1.68 E7	2.00 E4	1.19 E-3	0.95	0.9538	0.9550	0.126
20	1.07 E9	3.00 E4	2.80 E-5	0.90	0.9032	0.9027	-0.055

* Optimal not found by GA.

@ Network is too large to exactly calculate reliability.

APPENDIX

COST MATRICES OF FULLY CONNECTED NETWORKS

Problems 1 and 2: 5 nodes, 10 arcs

	1	2	3	4	5
1	-	32	54	62	25
2		-	34	58	45
3			-	36	52
4				-	29
5					-

Problem 1 Optimum : {1,2}{1,3}{1,5}{2,3}{2,5}{3,4}{4,5}
Problem 2 Optimum : {1,2}{1,5}{2,3}{2,5}{3,4}{4,5}

Problems 3, 4 and 5: 7 nodes, 21 arcs

	1	2	3	4	5	6	7
1	-	125	150	125	150	150	130
2		-	75	100	150	200	250
3			-	75	90	250	200
4				-	75	100	150
5					-	75	100
6						-	75
7							-

Problem 3 Optimum : {1,2}{1,7}{2,3}{2,4}{3,5}{4,5}{5,6}{6,7}
Problem 4 Optimum : {1,2}{1,7}{2,3}{2,4}{3,5}{4,5}{5,6}{5,7}{6,7}
Problem 5 Optimum : {1,2}{1,7}{2,3}{3,4}{4,5}{5,6}{6,7}

Problems 6, 7, and 8: 8 nodes, 28 arcs

	1	2	3	4	5	6	7	8
1	-	59	19	98	77	35	40	93
2		-	68	39	16	48	12	81
3			-	17	41	24	89	41
4				-	60	23	72	45
5					-	23	51	84
6						-	54	1
7							-	33
8								-

Problem 6 Optimum : {1,2}{1,5}{1,6}{1,7}{1,8}{2,4}{3,5}{3,8}{4,7}{6,8}
Problem 7 Optimum : {1,3}{1,7}{2,4}{2,5}{2,7}{3,4}{3,6}{4,6}{5,6}{6,8}{7,8}
Problem 8 Optimum : {1,3}{1,6}{2,5}{2,7}{3,4}{4,6}{5,6}{6,8}{7,8}

Problems 9, 10 and 11: 9 nodes, 36 arcs

	1	2	3	4	5	6	7	8	9
1	-	37	77	61	97	58	41	63	3
2		-	40	30	4	53	61	37	63
3			-	56	63	71	13	90	34
4				-	33	70	39	7	35
5					-	89	55	97	65
6						-	23	57	88
7							-	2	70
8								-	77
9									-

Problem 9 Optimum : {1,2}{1,9}{2,4}{2,5}{2,6}{3,7}{3,9}{4,5}{4,8}{6,7}{7,8}
Problem 10 Optimum : {1,2}{1,6}{1,9}{2,5}{2,8}{3,7}{3,9}{4,5}{4,8}{4,9}{6,7}{7,8}
Problem 11 Optimum : {1,2}{1,9}{2,5}{2,6}{3,7}{3,9}{4,5}{4,8}{6,7}{7,8}

Problems 12, 13 and 14: 10 nodes, 45 arcs

	1	2	3	4	5	6	7	8	9	10
1	-	24	26	69	25	48	3	82	45	98
2		-	12	75	22	33	82	54	4	82
3			-	30	8	75	38	21	79	23
4				-	67	18	64	50	78	12
5					-	72	92	94	21	96
6						-	5	81	18	84
7							-	19	37	34
8								-	7	98
9									-	50
10										-

Problem 12 Optimum : {1,5}{1,7}{2,3}{2,9}{3,5}{3,10}{4,6}{4,10}{6,7}{6,9}{7,8}{8,9}
Problem 13 Optimum : {1,5}{1,7}{2,3}{2,5}{2,9}{3,5}{3,8}{3,10}{4,6}{4,10}{6,7}{6,9}{7,8}{8,9}
Problem 14 Optimum : {1,5}{1,7}{2,3}{2,9}{3,5}{3,10}{4,6}{4,10}{6,7}{7,8}{8,9}

Problem 15: 15 nodes, 105 arcs

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	-	70	97	75	2	12	66	66	2	70	20	36	53	17	48
2		-	14	75	27	39	93	24	59	3	41	43	20	9	33
3			-	34	52	99	90	39	50	66	44	29	27	20	31
4				-	38	38	78	38	87	38	13	7	95	60	66
5					-	84	57	10	38	93	24	55	11	35	32
6						-	91	38	61	66	20	44	73	49	21
7							-	90	43	2	25	16	55	23	6
8								-	95	11	61	81	44	63	14
9									-	62	40	53	16	72	51
10										-	69	17	90	96	65
11											-	51	29	69	94
12												-	98	13	100
13													-	43	88
14														-	35
15															-

Problem 16: 20 nodes, 190 arcs

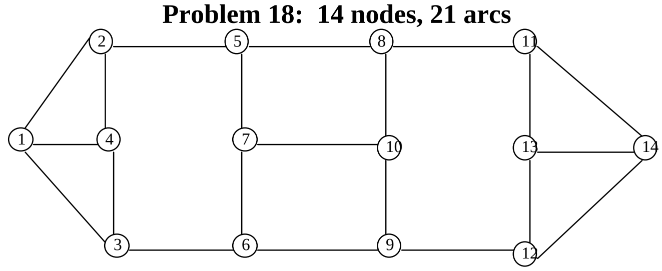
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	-	67	95	11	13	62	67	69	2	93	49	28	48	63	42	78	82	86	46	77
2		-	74	83	84	51	79	24	41	42	12	14	68	32	27	10	70	13	7	95
3			-	71	86	97	61	2	2	36	62	94	61	50	56	55	75	49	39	46
4				-	31	10	33	13	89	39	96	20	66	76	50	5	25	42	59	43
5					-	100	72	96	82	38	75	78	15	5	14	64	65	13	95	47
6						-	24	67	17	74	71	19	3	87	41	84	33	60	37	79
7							-	57	81	3	52	18	6	4	39	50	66	68	62	30
8								-	38	98	45	81	77	61	16	61	84	90	31	89
9									-	61	93	31	97	74	25	7	98	27	67	59
10										-	13	7	47	53	2	70	61	16	91	69
11											-	1	29	81	42	95	9	85	14	12
12												-	77	46	82	81	72	2	90	48
13													-	40	18	1	47	16	14	34
14														-	65	84	30	24	48	72
15															-	20	14	36	24	76
16																-	89	63	51	16
17																	-	28	49	60
18																		-	75	3
19																			-	58
20																				-

Problem 17: 25 nodes, 300 arcs

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	-	34	62	7	46	26	19	31	29	26	15	16	68	37	15	100	58	10	49	86	100	36	31	49	78
2		-	100	22	98	80	8	98	15	7	80	35	54	85	22	71	81	5	29	46	37	29	79	17	20
3			-	39	25	84	5	36	22	12	86	96	9	79	15	54	42	27	25	39	52	90	80	35	58
4				-	19	2	34	4	43	51	64	19	36	26	36	16	71	41	51	52	50	13	34	58	73
5					-	66	76	84	81	20	45	10	61	34	86	50	18	21	94	25	27	50	61	81	33
6						-	54	18	93	7	62	18	75	28	12	37	73	62	34	89	44	85	96	78	7
7							-	57	50	43	48	78	25	53	16	45	55	71	11	69	50	93	86	62	18
8								-	23	9	73	22	44	24	32	31	3	50	47	76	11	92	63	44	24
9									-	30	63	46	66	44	70	23	10	72	7	63	9	17	87	41	64
10										-	42	70	42	78	23	92	6	89	53	55	91	34	12	42	16
11											-	59	45	83	19	79	21	5	41	56	50	11	6	50	2
12												-	7	46	59	34	51	26	55	32	83	38	85	24	99
13													-	80	78	6	32	45	97	13	73	69	25	5	72
14														-	31	55	52	47	93	42	54	24	32	34	16
15															-	1	88	79	57	71	40	92	53	73	89
16																-	13	10	48	98	98	23	21	27	28
17																	-	98	29	46	14	3	83	52	50
18																		-	82	17	80	33	85	57	99
19																			-	42	48	45	57	74	52
20																				-	67	72	47	54	42
21																					-	23	36	35	61
22																						-	64	98	51
23																							-	53	74
24																								-	4
25																									-

COST MATRICES OF NON-FULLY CONNECTED NETWORKS

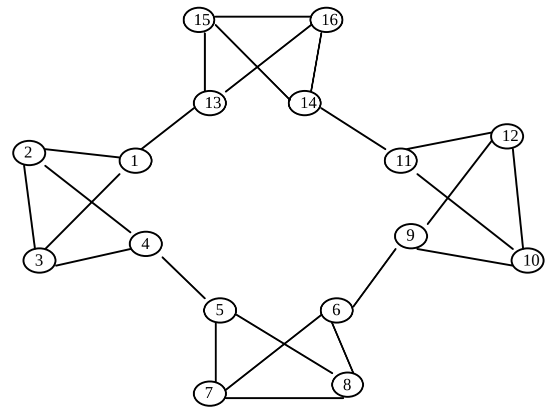
Problem 18: 14 nodes, 21 arcs



	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	-	47	61	20	-	-	-	-	-	-	-	-	-	-
2		-	-	46	95	-	-	-	-	-	-	-	-	-
3			-	25	-	51	-	-	-	-	-	-	-	-
4				-	-	-	-	-	-	-	-	-	-	-
5					-	-	98	45	-	-	-	-	-	-
6						-	42	-	98	-	-	-	-	-
7							-	-	-	50	-	-	-	-
8								-	-	78	46	-	-	-
9									-	22	-	87	-	-
10										-	-	-	-	-
11											-	-	60	95
12												-	77	66
13													-	10
14														-

Problem 18 Optimum :
{1,2}{1,4}{2,4}{2,5}{3,4}{3,6}{5,7}{5,8}{6,7}{6,9}{7,10}{8,10}{8,11}{9,10}{9,12}{11,13}{12,13}{12,14}{13,14}

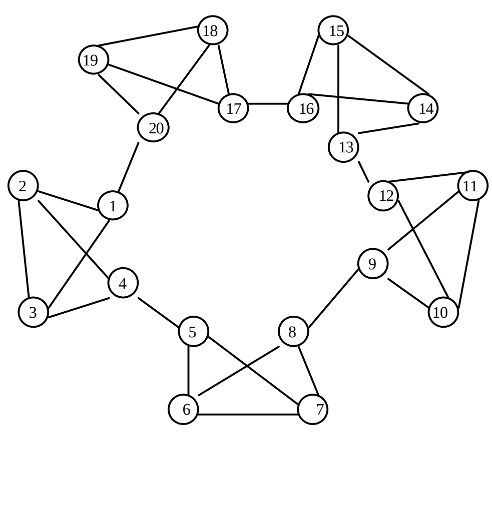
Problem 19: 16 nodes, 24 arcs



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	-	13	99	-	-	-	-	-	-	-	-	-	-	-	-	-
2		-	12	32	-	-	-	-	-	-	-	-	-	-	-	-
3			-	28	-	-	-	-	-	-	-	-	-	-	-	-
4				-	64	-	-	-	-	-	-	-	-	-	-	-
5					-	-	68	7	-	-	-	-	-	-	-	-
6						-	39	38	79	-	-	-	-	-	-	-
7							-	61	-	-	-	-	-	-	-	-
8								-	-	-	-	-	-	-	-	-
9									-	92	-	87	-	-	-	-
10										-	80	81	-	-	-	-
11											-	55	-	89	-	-
12												-	-	-	-	-
13													-	-	50	36
14														-	86	51
15															-	98
16																-

Problem 19 Optimum : {1,2}{1,13}{2,3}{2,4}{3,4}{4,5}{5,7}{5,8}{6,7}{6,8}{6,9}{9,10}
{9,12}{10,11}{11,12}{11,14}{13,15}{13,16}{14,15}{14,16}

Problem 20: 20 nodes, 30 arcs



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	-	7	40	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	10
2		-	46	51	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3			-	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4				-	8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5					-	34	19	-	-	-	-	-	-	-	-	-	-	-	-	-
6						-	84	24	-	-	-	-	-	-	-	-	-	-	-	-
7							-	66	-	-	-	-	-	-	-	-	-	-	-	-
8								-	27	-	-	-	-	-	-	-	-	-	-	-
9									-	60	25	-	-	-	-	-	-	-	-	-
10										-	11	9	-	-	-	-	-	-	-	-
11											-	76	-	-	-	-	-	-	-	-
12												-	19	-	-	-	-	-	-	-
13													-	21	11	-	-	-	-	-
14														-	100	9	-	-	-	-
15															-	22	-	-	-	-
16																-	13	-	-	-
17																	-	33	65	-
18																		-	4	59
19																			-	44
20																				-

Problem 20 Optimum : {1,2}{1,3}{1,20}{2,4}{3,4}{4,5}{5,6}{5,7}{6,8}{7,8}{8,9}
{9,11}{10,11}{10,12}{12,13}{13,14}{13,15}{14,16}
{16,17}{15,16}{1718}{18,19}{18,20}{19,20}

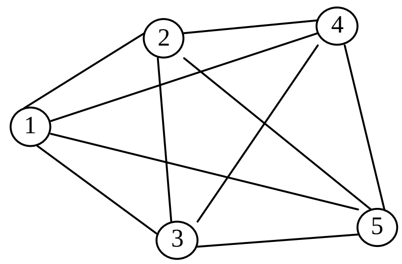


Figure 1. Typical network.

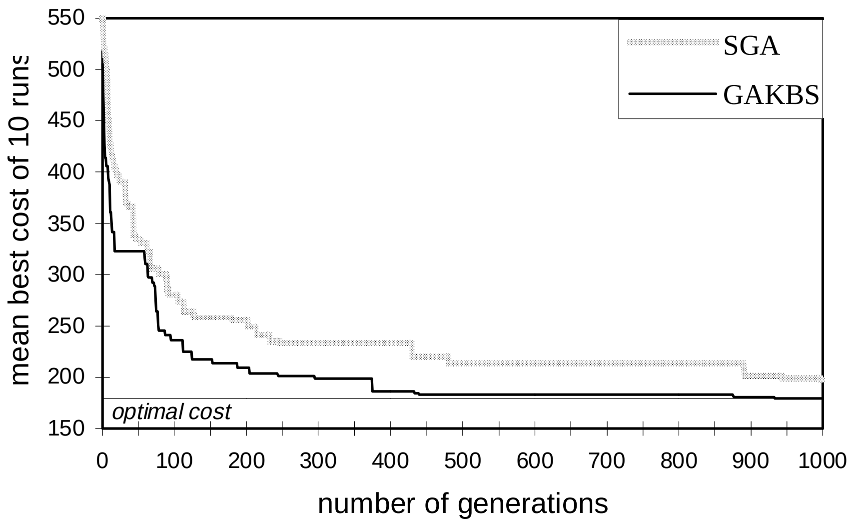


Figure 2. Typical convergence plot of SGA and GAKBS, averaged over ten seeds.