A Metamodel Approach to Sensitivity Analysis of Capital Project Valuation

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Abstract

A method is presented for using statistical experiment design and multiple regression in the sensitivity analysis of engineering economy or capital budgeting studies where multiple factors are subject to uncertainty. Rather than an enumeration of all combinations of possible changes, a metamodel of the behavior of the project is constructed. Application of this methodology can lead to more specific and useful understanding of the effects of various factors on the project value than is practical with Monte Carlo simulation. In addition, the metamodel can be combined with simulation to provide a distribution of outcomes upon which the project's overall risk can be assessed. The method is illustrated using two actual case studies - a municipal building project and an industrial equipment project.

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1. Introduction

Sensitivity analysis is an important aspect of any engineering economy study. Simply computing traditional project valuation metrics such as net present value (NPV), equivalent annuity (EA), or internal rate of return (IRR) based on a single set of assumptions can be misleading and imprudent. Monte Carlo simulation methods can be helpful in this regard. One can assume that factors such as interest rates, project life, up-front investment or first costs, and salvage value at the end of the project's life are random variates. A probability distribution and its parameters can be specified and the project value under a variety of conditions can be calculated. A study of the distribution of resulting values leads to an understanding of the uncertainty (moments, distribution, outliers) of the project valuation. This method is most straightforward when the net cash flows of a project can be assumed constant over the life of the project and can therefore be treated as an annuity. Simulation is still approachable when the cash flows are not constant, but is more cumbersome, especially for longer study periods and when many factors are uncertain. In addition, simulation does not provide any explicit information on the sensitivity of the results as a function of the factors.

This paper illustrates the development of a method for performing sensitivity analysis based on a design of experiments and metamodel approach. Metamodels have been used in engineering and business contexts where direct evaluation of the system, or problem, under various conditions is impractical. A metamodel uses data gathered directly from the system or problem, then constructs a statistically valid model that is used for analysis and decision making. The method in this paper allows for an estimate of risk even when cash flows are not constant over a variety of uncertainty scenarios and where the number of uncertain factors is large. The metamodel also allows for identification of the factors to which the results are most sensitive, and it can be used to find minimum conditions needed for a project to "break-even."

1.1 Previous Work in Sensitivity Analysis

Many researchers and practitioners in engineering economy have studied the effects of uncertainty on project valuation as a matter of course during their evaluation of proposed projects. These sensitivity analyses are usually done by assuming a variety of possible forms that the uncertainty can take, such as probability distributions for continuous and discrete random variates, or likely ranges or values for individual factors. Project valuation metrics are recalculated for a selected (randomly or strategically chosen) set of factor values. The results can be presented in a tabular format, a graphic format or summarized, as in break-even analysis.

There has been less methodological work done explicitly on this sensitivity analysis step. Eschenbach has published papers emphasizing graphic portrayal of sensitivity analysis and urged careful selection of the graphic technique that conveys the most information to the decision maker [7, 8]. Other papers have more narrowly focused on the issue of assumed probability distributions and their effect on valuation. These include a Bayesian approach by Harpaz and Thomadakis [9], a Bayesian approach using PERT probability mass functions as prior distributions by Prueitt and Park [17], an empirical comparison by Chandra and Guild [5], and approximating a uniform distribution using a normal distribution in the use of chance-constrained programming [19]. Choobineh and Behrens examined the use of interval mathematics and fuzzy sets to deal with imprecision in engineering economy studies [6], while Mustafa and Al-Bahar used the Analytic Hierarchy Process with the same objective [14].

There is a need to explore more fundamental understanding of the sensitivity of project valuation under a general set of possibilities. It is this objective that is the center of the research presented. The sensitivity analysis is undertaken by fitting a metamodel to the results of project factor values chosen by experimental design so that a closed form may be discerned. While this form will not be a perfect representation of the relationship between factor change and project valuation, an appropriate form facilitates both analysis and understanding, as will be shown in sections 2 through 4.

1.2 Description of Metamodeling

To more explicitly define a metamodel, let X_j denote the value of a factor j influencing the evaluation, Y, of the project under study where $\{X_j \mid j = 1, 2, ..., n\}$. The relationship between the valuation variable Y and the factors X_j (this can be generalized to a study with more than a single valuation variable) is:

$$Y = f_1\,(X_1, X_2, \ldots, X_n) \tag{1}$$

A metamodel is a simplification of the project under study, which is deterministic and employs a subset of the factors $\{X_j \mid j = 1, 2, \ldots, m\}$, where $m \leq n$:

$$Y' = f_2\,(X_1, X_2, \ldots, X_m) + \varepsilon_m \tag{2}$$

Y' is the response of the metamodel and ε_m is the composition of the error of the effects of any excluded factors and the metamodeling error of fitting the metamodel to the underlying relationship.

The main issues in metamodeling are (1) the choice of underlying the functional form, (2) the choice of factors and their corresponding valuation variable to be used in the metamodel, (3) selection of the sample from the project under study to construct the metamodel, and (4) validation of the metamodel [2]. The most popular metamodeling approach involves the use of parametric polynomial regression models in response surface methods [2]. Following equation 2 above, a polynomial of order p is created using the summation of the power functions of each X_m:

$$Y' = \sum_{j=1}^{m}\sum_{k=1}^{p}\beta_{kj}\,p_k\left(X_j\right) \tag{3}$$

While straightforward to implement, the performance of a polynomial metamodel depends on the appropriateness of the polynomial functional forms.

Metamodeling research has been published since 1970 [e.g., 3], although the descriptor *metamodel* was developed more recently by Kleijnen [11-13]. A current overview of published research on metamodels for industrial and production applications can be found in Yu and Popplewell [20]. Applications cited include production planning and control, work in progress inventory management, flexible manufacturing system design, order level inventory control, facilities storage design and shop floor control. Other uses of metamodeling have included such diverse venues as studying environmental change [18] and the operations of a hospital emergency room [10]. Work on metamodels for engineering economic analysis has been limited to a single paper by Badiru and Sieger [1]. Badiru and Sieger studied uncertainty in first costs, annual benefits and salvage values using Monte Carlo simulation, then postulated that a neural network model could learn the relationship to project future value using data from the simulation. They tested several versions of the popular backpropagation neural network and found that most performed satisfactorily in predicting future value given first costs, annual benefits and salvage values. They also speculated in their conclusions that this approach could be used to identify the critical factors influencing the future value.

This paper takes the metamodel theme of Badiru and Sieger and couples it with statistical experimental design. The choice of metamodel is not important to this paper, though the common polynomial regression is used, but the iterative methodology of experimental design, metamodel functional choice, lack of fit tests and validation is put forth as a general technique. It is also shown how a completed metamodel can be used in analysis of the project under consideration. This paper offers a practical, rigorous and statistically correct method for systematic and thorough investigation of uncertain factors when evaluating a capital project, or making relative comparisons between multiple projects. The metamodel technique used herein is polynomial regression, however other models can be used such as non-linear regression, splines or artificial neural networks, if the project warrants a more complex approach. This is the first known explicit application of the metamodel coupled with design of experiments to the field of engineering economy.

2. Methodology

The methodology is summarized below. Two case studies will follow that illustrate the implementation and usefulness of the method; a municipal construction project and an industrial equipment project.

1. Identify components of the cash flow analysis that may have a significant impact on the project value. These components are termed “factors”.
2. Consider the range of values that the factors may take. One approach is to identify the upper and lower bounds as the most optimistic and most pessimistic estimates. One might also have in mind an expected value for the factor and even a variance. Upper and lower bounds might then be computed based on some percentage deviation or number of standard deviations about this expected value.
3. Set up a sampling scheme to cover the factor ranges identified in Step Two. As will be shown, an efficient starting sample takes the form of a three-level central composite experimental design.
4. Compute the project value at each of the factor combinations called for by the experimental design.
5. Use the resulting project values to develop a metamodel. The form of the metamodel will depend on what can be supported by the experimental design employed. In developing the model, one should consider inclusion of only significant terms and interactions to keep the model as simple as possible and a lack of fit assessment should be conducted. If a metamodel of a chosen form does not pass the lack of fit test, a different form should be tried. It is a good policy to begin with the simplest, reasonable metamodel and add

complexity as indicated by lack of fit tests and reduce terms as indicated by analysis of variance.

6. Carry out the sensitivity analysis on the project evaluation using the metamodel. This analysis can include the following elements:

    a. Inspection of the analysis of variance for the metamodel. Terms with large F values have a large influence on the project value.

    b. Definition of an objective function for optimization approaches. For example, if one is interested in finding the conditions under which the project "breaks even" (i.e. has a value of 0), use the model to minimize the squared project value.

    c. Generation of contour or surface plots of predicted project values. This allows the analyst to visualize sets of conditions under which constraints on factor and project values are satisfied.

    d. Monte-Carlo simulation. One can randomly vary inputs to the metamodel and compute the project values for a large set of inputs. A traditional distributional analysis can then be done with the simulation results.

3. A Municipal Construction Study

This is a project that has been under consideration at a north Pittsburgh suburb and is a new municipal building. Table 1 shows the cash flows for the municipal construction project. The elements of the flows are:

- **Principle Balance:** This is the amount borrowed to finance the project less any payments on principle. The original amount borrowed is referred to as the **First Cost**, ***c*.**
- **Debt Service**: The dollar value of interest that must be paid on the principle balance. This is based on the principle balance and interest rate, ***i*.**

- **Principle Payment**: The dollar value paid on the principle in a given year.
- **Operating Savings**: The dollar value saved each year due to investment in the project. The initial value, ***s***, is assumed to increase yearly at the **rate of inflation**, ***r***.
- **Other Revenue**: Positive cash inflows resulting from sources such as government funds for the project or costs avoided due to investment in the current project.
- **Net Cash Flow**: The sum of all cash flows for a given year. Specifically, Debt Service + Principle Payment + Operating Savings + Other Revenue.
- **Present Value**: The current value of the net cash flow, discounted back to present value based on the number of years forward and the weighed average cost of capital (WACC) of the municipality funding the project. The WACC is a function of the cost of debt of the project in question, the amount financed, and the interest rate and amount of other obligations that the municipality is carrying.

The values shown in Table 1 (all money amounts are in units of $1,000) are arrived at using c=7,010, i=6%, s=300, and r=3%. The project value of interest is the Net Present Value, NPV. This is the sum of the present values in Table 1. For the factor settings just listed, the NPV is (2,004)[1]. As can be seen, the timing of the various elements in the analysis lead to non-uniform cash flows. As such, the flows cannot be treated as an annuity, which would have greatly simplified the application of simulation to this problem.

Four factors impacting the final project value are subject to uncertainty: c, i, s, and r. The expected values of these quantities that were used to obtain the NPV are used as the midpoints of the range to be investigated in the sensitivity analysis. Reasonable upper and lower

[1] All negative amounts are shown in parentheses.

bounds relative to these midpoints were chosen based on knowledge of the project as summarized in Table 2.

3.1 Selection of an Experimental Design and Preliminary Metamodel

With four factors, the following experimental designs might be considered:

1. A two-level full factorial, the 2^4 design. This requires 16 NPV calculations.
2. A three-level central composite design. This requires 25 NPV calculations.
3. A three-level full factorial, the 3^4 design. This requires 81 NPV calculations.

The half fraction of the 2^4 was not considered because the computational cost of obtaining the NPV is small and it is expected that many of the factor-factor interactions will be important. An excellent overview of the properties of two-level full and fractional factorial designs is given by Box, Hunter and Hunter [4]. The central composite design mentioned above does not include the centerpoint replicates that are usually a feature of this design because the data is calculated, rather than measured. Thus, experimental error is not an issue - replicated values will produce exactly the same results and are therefore of no value. In addition, the choice of central composites is restricted to those with axial distance from the center, α, of ± 1. This leads to a cubodial design space and avoids the possibility that the design will call for nonsense inputs such as negative interest rates. For an extensive review of the central composite and other competing second-order designs, see Myers and Montgomery [15].

One might argue that since the computational cost of computing NPV is low, it would be best to select the largest design, the three-level full factorial. While it is true that computing 81 NPV values would be manageable in this case, the inclusion of additional factors and other complicating features would make this an uninviting approach for general consideration. Therefore an iterative approach is adopted as being a more practical general approach. First, the 2^4 sampling scheme is performed fitting a linear regression metamodel with all first order terms and all interaction terms, then an assessment of lack of fit is made. Since the NPV values are without measurement error, any residuals from the model will be due to metamodel bias (imperfect fit to the population modeled) and round-off errors. All models have bias, therefore, the goal of the model validation stage of the analysis is to assess metamodel bias and its potential impact on usefulness. This will be accomplished by computing NPV at a number of randomly selected factor settings that were not part of the experimental design and were therefore not used in developing the metamodel. A comparison of the calculated values to those predicted by the metamodel assesses model bias. This is known as cross-validating the metamodel where a sample from the same population as used to construct the metamodel is independently drawn for validation.

3.2 Data Analysis

The results of the 2^4 experiment shown in Table 3 were subject to analysis of variance assuming that the four-way interaction between *c*, *i*, *s* and *r* was negligible. The metamodel was trimmed by removing terms that contributed little to explaining the variation in project values and the coefficients were based on an "orthogonal" scaling of the factor settings, a good practice when second order and higher effects are included in the model. That is, the regression model was developed with the factor settings scaled such that the lower bounds translate to -1 and the upper bounds translate to +1. This not only allows for a "clean" variance-covariance matrix, it renders the magnitude of the coefficients comparable.

Before doing any sensitivity analysis based on this metamodel, its validity must be tested. Ten points where selected at random from the sample space covered by the experiment. In selecting this set of points, if a point happened to be part of the experimental design, it was discarded and a new point selected. While there was generally good agreement between calculated NPV values and those predicted by the metamodel (see R^2 in Table 4), a runs test of NPV suggested that there was some curvature in the NPV response not accounted for by the model ($p = 0.13$ for the NPV^2 term). In addition, the RMS error of 140 for the validation set does not compare favorably to the RMS error for the model of 10.7 (Table 4). All of this suggested that it was worth investigating a more complicated metamodel.

A second-order central composite design requires just nine more runs than the 2^4. As it turns out, the 2^4 is a subset of the central composite, so the current experiment can be augmented with nine additional design points to arrive at the central composite. Table 5 lists the results of a backwards step-wise regression metamodel that adds pure quadratic terms and third order interaction terms (as opposed to just the factor-factor type second order effects in the analysis of the 2^4 experiment). Terms with $p \leq 0.10$ were kept in the model. As suggested by the analysis of the 2^4 experiment, there is significant curvature in the NPV response and this follow-up experiment identified *r* as the culprit.

This metamodel was validated by again randomly resampling the design space and comparing calculated and predicted NPVs. No significant curvature was present, and the RMS error of 34.9 for the validation set compared favorably with the 39.7 value given for the model in Table 5. Therefore, this metamodel is adequate to continue the analysis.

3.3 Using the Metamodel

One of the main advantages of using a metamodel approach in the evaluation of capital projects is that is it facilitates an understanding of the sensitivity of the project value to variation in the relevant factors. For example, the analysis of variance table of the metamodel can be inspected (Table 5). Since all terms in the model have one degree of freedom, the larger a term's sum of squares, the more sensitive the NPV is to variation in the associated factor. Figure 1 offers a graphical approach to seeing the factor effects. The effect of a factor is defined here as the average NPV at the high value of a factor minus the average NPV at the low end of the range covered by the experiment. It is clear from Figure 1 that *s* has the largest influence on the NPV. This is followed in influence by *i*, *c* and *r*. The *s*-by-*r* interaction is the most important second order effect. The other second and third order effects in the model are small by comparison. This information might lead the project engineers to spend more time and effort to firm-up their estimate of the savings that can be expected from the project.

One can also define an objective function that can be optimized according to a specific objective of the project. Suppose for example that the conditions required for the project to break-even (return a NPV of zero) need to be identified. This can be accomplished by defining the objective function:

$$\min Z = \text{NPV}^2 \tag{4}$$

Table 6 shows the result of a minimization using the approach suggested by Nelder and Mead [16]. The Nelder and Mead (NM) method manipulates a simplex (a polyhedron with n+1 vertices, where n is the number of the factors). The search moves towards the optimum by reflecting the worst vertex through the centroid of the other vertices. The NM algorithm adapts the size and shape of the simplex based on the search space topology. Using the procedure, the project is expected to break-even if c=7,064, i=5.8, s=388 and r=3.8.

One shortcoming of the objective function approach is that it leads to a single-point optimum. There may well be a variety of conditions under which the project may return a zero or even positive value. Another useful approach is to use the metamodel to produce contour plots of the NPV surface. Figures 2a through 2d show contour plots based on the regression model with *i* and *r* as the off-axis variables. The "0" contour line on each plot represents the combinations of *s* and *c* that will lead to an expected NPV of zero for the stated *i* and *r*. NPVs above the zero line are positive, while those below the zero line are negative. A disadvantage of the graphic approach is that only a few variables at a time can be considered while others must be held constant. This may lose usefulness as the dimensionality of the factors increases.

The metamodel can also be used to get an overall assessment of the risk in the investment. This is accomplished by making distributional assumptions about possible variation in the factor settings, using these assumptions to produce random inputs to the model, and then simulating these inputs for a large number of computations. For example, assume that each of the four factors under consideration in this study are independent random normal variates with

$$\mu=\tfrac{1}{2}(U-L) \text{ and } \sigma=1/6(U-L) \tag{5}$$

where U is the upper bound of the range and L is the lower bound of the range for that factor as found in Table 2. This will produce random inputs that will be centered over the expected value for that factor and should only produce values outside the range of the study 0.3% of the time.

A simulation consisting of 5000 NPV calculations was performed. A histogram of the results is shown in Figure 3; assuming that NPV ~ N with μ=(1258) and σ=1584 (the sample average and standard deviation for the simulation sample data), the risk of a negative NPV for this project is about 79%.

4. An Industrial Equipment Study

An investment in an automated environmental control system is being contemplated at a production facility of a large Pittsburgh manufacturer. It is believed that this system will save energy costs by providing optimized air turnover in the facility. A cost estimate of 2,200[2] has been provided by the contractor that would be selected to install the system should the decision be made to go ahead with the project. It is expected that the system will have a ten-year lifetime. Since the system is customized for the facility in question, no salvage value is considered. The costs of the project are split into two years, with 1,230 (55.9%) to be paid in year 0 and 970 (44.1%) to be paid in year 1. The effective income tax rate is 40%. Benefits in Year 1 are estimated at 159, increasing to 405 in Year 2, and then are expected to plateau at 492 for the balance of the project's life. A minimum acceptable after-tax rate of return (MARR) of 10% has been set. Table 7 shows the cash flows under the above conditions lead to an after-tax internal rate of return (IRR) of 11%. Thus the project, under the assumptions listed above, compares favorably to the MARR.

To illustrate the application of the metamodel approach to this project, three factors are considered to be uncertain: first cost (c), maximum savings (s), and lifetime (l). Table 8 shows the ranges for these variables that were investigated. c varies from a low value of 2,200 (the contractor supplied estimate) to this low value plus 20%. s varies from a low value of one-half the estimated savings to a high value equal to the estimate used in the original project value computation. Note that these values are in terms of the maximum annual savings expected to be realized in Year 3 and beyond. For the purposes of computing IRRs for various values of savings, note that the estimated savings in Year 2 is $492 \div 1.21 = 405$ and that the savings in Year 1 is $405 \div 2.55 = 159$. These proportions were maintained when alternative values for s were used. This is also true for variations in the cost of the project. The 55.9%/44.1% split in investment over years 0 and 1 was maintained regardless of the cost. Finally, l between 5 and

[2] All money amounts for this study are in thousands of dollars.

15 years was considered. The IRRs were computed based on the assumption that the standard ACRS depreciation factors for the 10-year class life was used irrespective of the point at which the project is terminated.

Based on the experience from the study in section 3, only experimental designs capable of supporting second-order or higher models were considered. Two three-level designs are available: the 27-run 3^3 factorial and the 15 run central composite. Believing that it would provide sufficient data to find a useful metamodel for this situation, the central composite was used. Using the data, a backwards step-wise regression was performed using the full second order set of terms and all two and three way interactions with $p \leq 0.10$. As in the previous example, the project values (IRR) predicted from the metamodel compared favorably with the calculated values (Table 9). Ten random validation points were selected from a grid of 125 possible points over the design space. The regression predicted values and the actual calculated values are plotted against each other in Figure 4. Lack of fit is more evident in this case than in the previous example, particularly for the seven year data . The model is biased downwards, and further indication of lack of fit can be seen in comparing the RMS error of the validation set to that of the model as fit to the design data. The RMS error for the validation set was 0.65 while Table 9 gives an RMS error based on the design data of about 0.19.

While it could be argued that the discrepancies are not of a magnitude that warrants rejection of the utility of this metamodel, they are perhaps an indication that some higher order effects are missing from the model or that non-linear (in the factors) effects are present. It is not hard to justify the presence of non-linear effects in this problem on the basis of intuition. Even though the decline in depreciation rates over the ten year period is a smooth function, the abrupt truncation of the project at $l < 10$ introduces a discontinuity in the data. This can not be accounted for in the metamodel used. This is not a problem for project lives beyond ten years as the depreciation has been exhausted.

Nevertheless, using the second degree polynomial metamodel, Figure 5 is a Pareto chart of effects in the experiment. Clearly, the life of the project is the most important variable. This is verified in the contourplot of the IRR response surface in Figure 6 when c=2200. Drastic changes in IRR are noted as one moves along the l axis from 5 to 15 years. Plots such as Figure 6 can also be used to test the sensitivity of the IRR value to assumptions. For example, at the project cost used in the original IRR calculation (c=2200, Figure 6), minimum annual savings of around 370 must be realized to attain the MARR of 10%, even at lifetimes in excess of 10 years. This compares with 492 used in the original calculation that leads to an IRR of 11%.

5. Summary and Conclusions

A method has been presented that integrates experimental design and metamodel analysis to study the nature of variation in project values as a function of variation in the components of the project value. The resulting information is richer and more compact than that obtained from simulation studies alone. This method is also more practical than enumerating all possible alterations in the factors and their result on project valuation. The most influential assumptions can be readily identified, and surface maps can be developed that provide visual identification of the conditions needed to meet the financial goals of the project. The resulting metamodel can also be used as a means of producing simulated project values for the purposes of assessing the financial risks associated with the project. Also, due to the compactness of metamodels, a wider variety of simulated inputs can be tried with greater ease than with a standard Monte Carlo approach.

Since a heavy burden is placed on the metamodel developed, care must be taken to select an appropriate experimental design to support the evaluation of a proper model form. Once a prospective model is obtained, it must be thoroughly validated to ensure its usefulness as an approximation to the project valuation relationship. While the form of the metamodel will vary depending upon the nature of the cash flows, the results of the study presented here suggest that a three-level central composite design in support of a full quadratic polynomial regression model is a good place to start. The usefulness of the metamodel can be assessed by comparing the predictions and residuals using a random set of conditions that are not part of the original experiment.

When depreciation is considered, discontinuities can be introduced when lifetimes not used in developing the predictive metamodel are considered. This is especially true for lifetimes that are less than the expected project life. This suggests that alternative designs and modeling techniques could be investigated in further work on this topic. For example, splines or neural networks might be considered as alternative metamodeling techniques. These methods, however, do not offer the strong statistical interpretation of significance and require much more skill on the part of the analyst to fit and validate an appropriate model compared to polynomial regression.

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Biographies

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Table 1. Cash Flows for Study 1 (all values in $1,000).

Year	Principle Balance	Debt Service	Payment on Princ.	Operating Savings	Other Revenue	Net Cash Flow	Present Value
0	7010	0	0	0	280	280	280
1	7010	(421)	0	350	140	70	66
2	7010	(421)	0	361	35	(25)	(23)
3	7010	(421)	0	371	0	(49)	(42)
4	7010	(421)	0	382	0	(38)	(31)
5	7010	(421)	0	394	0	(27)	(20)
6	7010	(421)	0	406	0	(15)	(11)
7	7010	(421)	0	418	0	(3)	(2)
8	7010	(421)	0	430	0	10	6
9	7010	(421)	0	443	0	23	14
10	7010	(421)	0	457	0	36	21
11	7010	(421)	0	470	0	50	28
12	7010	(421)	0	484	0	64	34
13	7010	(421)	0	499	0	78	39
14	7010	(421)	0	514	0	93	44
15	7010	(421)	0	529	0	109	49
16	7010	(421)	0	545	0	125	53
17	7010	(421)	0	562	0	141	57
18	7010	(421)	0	578	0	158	61
19	7010	(421)	0	596	0	175	64
20	7010	(421)	0	614	0	193	67
21	3505	(421)	(3505)	632	0	(3293)	(1079)
22	0	(210)	(3505)	651	0	(3064)	(952)

Table 2. Factors and Ranges for Experimental Design.

Factor	Lower Bound (L) (-)	Mid-Range (0)	Upper Bound (U) (+)
First Cost (*c*)	5860	7010	8160
Interest Rate (*i*)	4	6	8
Initial Operating Savings (*s*)	100	350	600
Inflation Rate (*r*)	1	3	5

Table 3. 2^4 Experimental Design and Results.

Pattern	First Cost (c)	Interest Rate (i)	Annual Savings (s)	Inflation (r)	Calculated NPV
- - - -	5860	4	100	1	(3135)
- - - +	5860	4	100	5	(2502)
- - + -	5860	4	600	1	3855
- - + +	5860	4	600	5	7653
- + - -	5860	8	100	1	(5855)
- + - +	5860	8	100	5	(5250)
- + + -	5860	8	600	1	927
- + + +	5860	8	600	5	4556
+ - - -	8160	4	100	1	(4946)
+ - - +	8160	4	100	5	(4309)
+ - + -	8160	4	600	1	2071
+ - + +	8160	4	600	5	5889
+ + - -	8160	8	100	1	(8605)
+ + - +	8160	8	100	5	(8007)
+ + + -	8160	8	600	1	(1869)
+ + + +	8160	8	600	5	1723

Table 4. Summary of Fit for Metamodel using 2^4 Composite Design.

R^2	0.999999
R^2 Adjusted	0.999995
RMS Error	10.70631
Mean of Response	(1112.75)
Observations, N	16

Table 5. Analysis of Final Metamodel for Example 1.

Source	DF	Sum of Squares	F Ratio	Prob>F
c	1	23589291	14990.23	<0.0001
i	1	51089201	32465.52	<0.0001
$c \times i$	1	985056	625.97	<0.0001
s	1	317931733	202035.30	<0.0001
$i \times s$	1	106602	67.74	<0.0001
r	1	21066541	13387.10	<0.0001
$i \times r$	1	13340	8.48	0.0114
$s \times r$	1	9554281	6071.43	<0.0001
$i \times s \times r$	1	6724	4.27	0.0577
$r \times r$	1	114832	72.97	<0.0001

R^2	0.999948
R^2 Adjusted	0.999911
RMS Error	39.66919
Mean of Response	(1158.32)
Observations, N	25

Table 6. Nelder Mead Minimization of Objective Function $Z = NPV^2$.

Factor, Response or Formula	Range	Initial Setting	Optimal Value
Factors			
c	5860 to 8160	7010	7064
i	4 to 8	6	5.7556
s	100 to 600	350	388.42
r	1 to 5	3	3.7398
Responses			
NPV			0.13174
Formulas			
Z	Minimize		0.017355

Table 7. Cash Flow Analysis for Industrial Equipment Example.

Year	Operating Cash (Savings)	Depreci-ation	Taxable Income	Tax at 40%	Gross ATCF*	Change in Fixed Assets	Net ATCF*
0	-	-	-	-	-	1249.00	(1249.00)
1	159	124.90	34.10	13.64	145.36	985.00	(839.64)
2	405	323.32	81.68	32.67	372.33	-	372.33
3	492	357.16	134.84	53.94	438.06	-	438.06
4	492	285.72	206.28	82.51	409.49	-	409.49
5	492	228.63	263.37	105.35	386.65	-	386.65
6	492	182.87	309.13	123.65	368.35	-	368.35
7	492	154.40	337.60	135.04	356.96	-	356.96
8	492	146.33	345.67	138.27	353.73	-	353.73
9	492	146.33	345.67	138.27	353.73	-	353.73
10	492	146.33	345.67	138.27	353.73	-	353.73
11	492	137.79	354.21	141.68	350.32	-	350.32

* ATCF = After Tax Cash Flows

Table 8. Factors and Ranges for Industrial Equipment Example.

Factor	Lower Bound (L)	Mid-Range	Upper Bound (U)
Total Cost (*c*)	2200	2420	2640
Savings (*s*)	246	369	492
Lifetime (*l*)	5	10	15

Table 9. Analysis of Variance on Significant Effects for IRR Data.

Source	DF	Sum of Squares	F Ratio	Prob>F
c	1	0.477687	13.71	0.0101
s	1	4.390294	126.02	<0.0001
l	1	39.934157	1146.32	<0.0001
$s \times c$	1	0.546012	15.67	0.0075
$s \times s$	1	0.303601	8.72	0.0255
$l \times c$	1	0.262813	7.54	0.0334
$l \times s$	1	3.795013	108.94	<0.0001
$l \times l$	1	76.713691	2202.09	<0.0001

R^2	0.999813
R^2 Adjusted	0.999564
RMS Error	0.186646
Mean of Response	1.726667
Observations, N	15

Term	Estimated Effect
s	3639.67
i	(1459.01)
c	(991.41)
r	936.90
$s \times r$	630.95
$c \times i$	(202.59)
$r \times r$	69.17
$i \times s$	(66.65)
$i \times r$	(23.58)
$i \times s \times r$	(16.74)

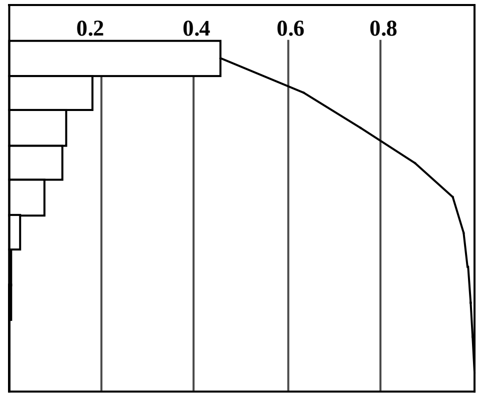

Figure 1. Pareto Plot of Estimated Effects on NPV for Example 1.

Figure 2a. NPV Response Surface for
i = 4% and *r* = 1%.

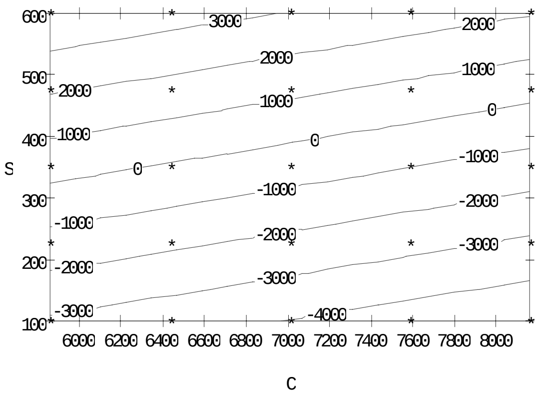

Figure 2b. NPV Response Surface for
i = 4% and *r* = 5%.

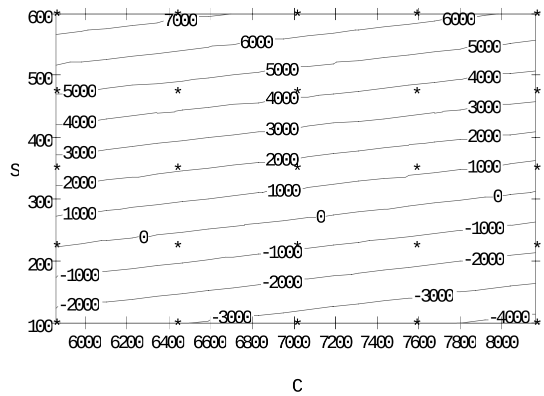

Figure 2c. NPV Response Surface for
i = 8% and *r* = 1%.

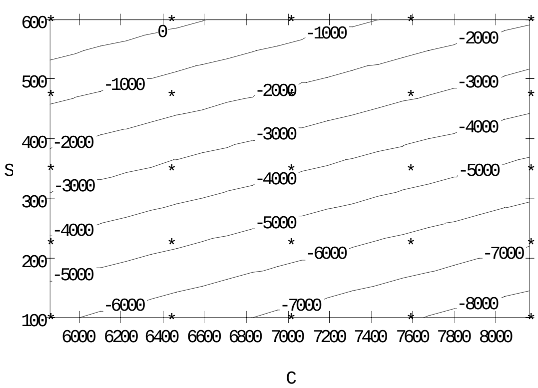

Figure 2d. NPV Response Surface for
i = 8% and *r* = 5%.

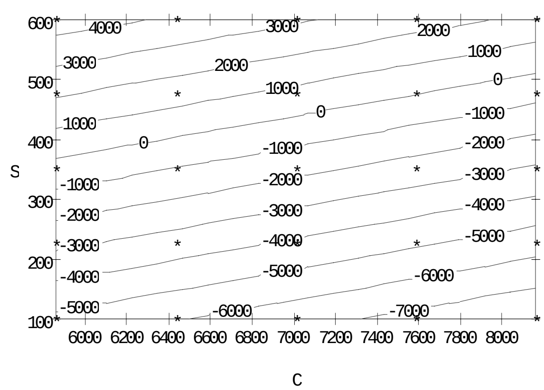

Figure 2. Contour Plots Using Metamodel for Example 1.

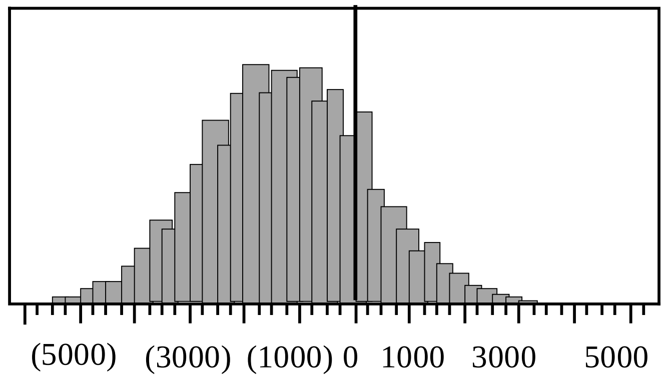

Figure 3. Histogram of 5000 Simulated NPV Results for Example 1.

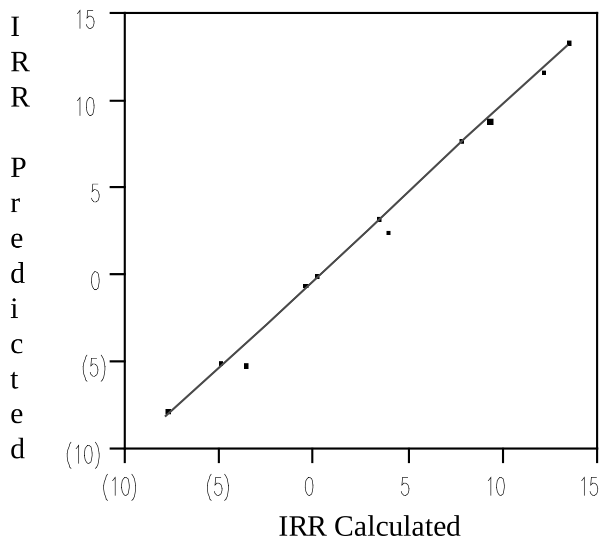

Figure 4. Predicted IRR versus Calculated IRR for Example 2.

Term	Estimated Effect
l	7.0822913
s	4.0620705
$l \times l$	(2.2614699)
$s \times s$	(1.1422132)
c	(1.0851240)
$l \times s$	(0.5029919)
$s \times c$	(0.1907900)
$l \times c$	0.1323663

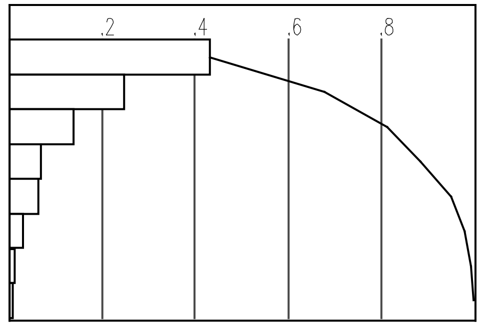

Figure 5. Pareto Plot of Effects, Industrial Equipment Project.

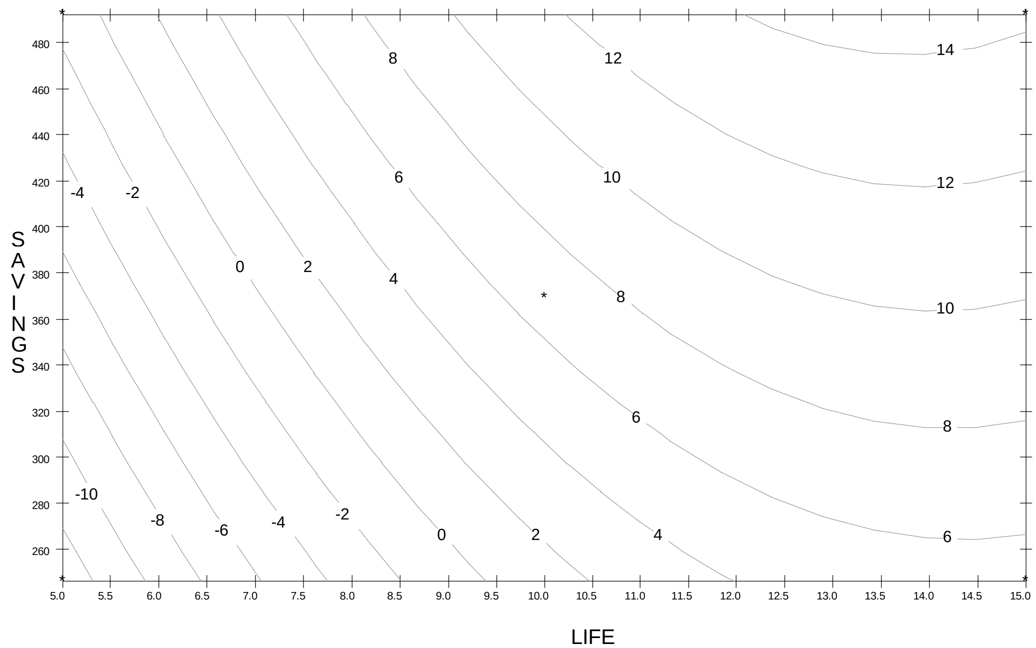

Figure 6. Contour Plot for Example 2 When c=2200.