

Sampling and Aliasing

This project will give you some hands-on experience with sampling 2-D signals and will demonstrate the effects of aliasing.

You should review your notes on sampling and aliasing before beginning this project. **NOTE: It is essential that you understand aliasing thoroughly and that you do each step accurately. Otherwise, you may do the entire project incorrectly and fail to learn anything from it.**

Exercises

1. Create a 512x512 image containing a vertical white line four pixels wide on a black background. Use `imrotate` with bicubic interpolation to rotate the image 15 degrees. Extract a 256x256 section out of the original image and the rotated image so that the line in the extracted image has ends on the top and bottom edges rather than cutting across a corner. Compare the two 256x256 images visually, and comment on the similarities or differences, if any. Use `true_size` so that each pixel in the display corresponds to a pixel in the image. (Make sure this is the case for every display in this exercise. See the help entry.) Examine the pixel values closely, and describe how the line at an angle is represented in a discrete array.
2. The Fourier magnitude of an image can be viewed by setting the colormap using `colormap(gray(256))`. You can view the the Fourier magnitude to see the stronger features using `imagesc(abs(fftshift(fft2(IMAGE))))` and then view the log-magnitude to see some of the finer details. To view a Fourier magnitude, use a dynamic-range compression mapping such as `log(abs(fftshift(fft2(IMAGE)))+0.01)`, which can then be viewed as an image using `imagesc`. The small constant is there in case any of the Fourier components are nearly zero and can be varied to change the “floor” of the mapping. Note that the origin is in the middle of the image in this display method. What you are viewing here is one period of the periodic frequency spectrum of the discrete image. To get a sense of what is happening with adjacent periods, you may want to view a 3x3 replication of the image using `repmat(IMAGE, [3 3])` substituted for `IMAGE` in the display command above. This display method may also be helpful in understanding some of the following exercises.

NOTE: When you view the log-magnitude, the darker components have very little significance to the issues in this project. Some lower-magnitude features occur as the result of using an FFT, which assumes that the image is periodic. Such features may appear as a strange texture background over the image. Those features will be considered artifacts for this project and not features of interest. You may want to try increasing the small constant to reduce the visibility of the background artifacts (try 1 or 10).

- (a) View the rotated and unrotated 256x256 images.
- (b) Explain using comparisons with simple Fourier transform pairs and properties why the strongest features in the Fourier magnitudes appear as they do in the two images. (Consider the 1-D Fourier transform of a constant, an impulse, and a pulse; the Fourier transform of separable signals; and the Fourier transform of a rotated image.)

- (c) Explain why the differences occur.
3. Consider the 256x256 images generated in Step 1 to represent continuous resolution.
 - (a) Create new 128x128 images that consist of every other sample of each of the 256x256 images. View the images, and explain what, if anything, has changed. Examine the pixel values, and describe what you find.
 - (b) View the Fourier magnitudes and explain in terms of aliasing. What happens to the strong features of the Fourier magnitude of the rotated line?
 4. Nonideal sampling can be simulated by averaging each 2x2 block in a “continuous” image to create a new image whose pixels are the block averages of the input image and whose size is one-half the original in each dimension.
 - (a) Write a **general** m-file function that will implement this operation on an input image of any size and return the downsampled output image. Please include your m-file in your report.
 - (b) Compare the rotated-line image resulting from this process to the rotated-line image resulting from the previous step, and explain the differences.
 - (c) Look at the Fourier log-magnitudes and explain the differences compared to the previous step.
 5. Consider sampling a sinusoid.
 - (a) Create a 256x256 image $f(m,n) = \sin(0.7\pi m - 0.2\pi n)$. Create a new image from every other sample of this image.
 - (b) View the two images and their Fourier magnitudes (clearer than log-magnitudes in this case). Explain the differences.
 - (c) What apparent frequencies are found in the downsampled image? (You should be able to calculate specific numerical frequencies in radians/sample. Use the original equation for $f(m,n)$ to determine this rather than the Fourier plot. The plot should correspond to what you find mathematically.)

Write a **one-page** memo that summarizes your findings. You will have to be selective in your discussion to meet the space requirements. That means you will need to leave some things out that you did in the exercise, and you will not be able to address every question in the report. Be sure to focus on the main ideas of the project. Include at least one image or image comparison and one code snippet and any items that I specifically ask you to include in the report. The memo should be written so that it makes sense without reference to the project instructions.

NOTE: All out-of-class work is to be done **independently** and should represent your work alone. Sharing of programming tips and discussing general concepts is ok. Collaborating on experiments or code-writing is not. **Any** such collaboration on these assignments will be considered an act of dishonesty and will be treated accordingly. Memos will be checked using Turnitin (TM) for excessive similarity to one another and to documents available online.

For further help:

- Matlab Primer
- Matlab Help Desk