

RF transistor mini project 1, updated 11/8 class. Save all graphs in a separate folder or paste them in a document.

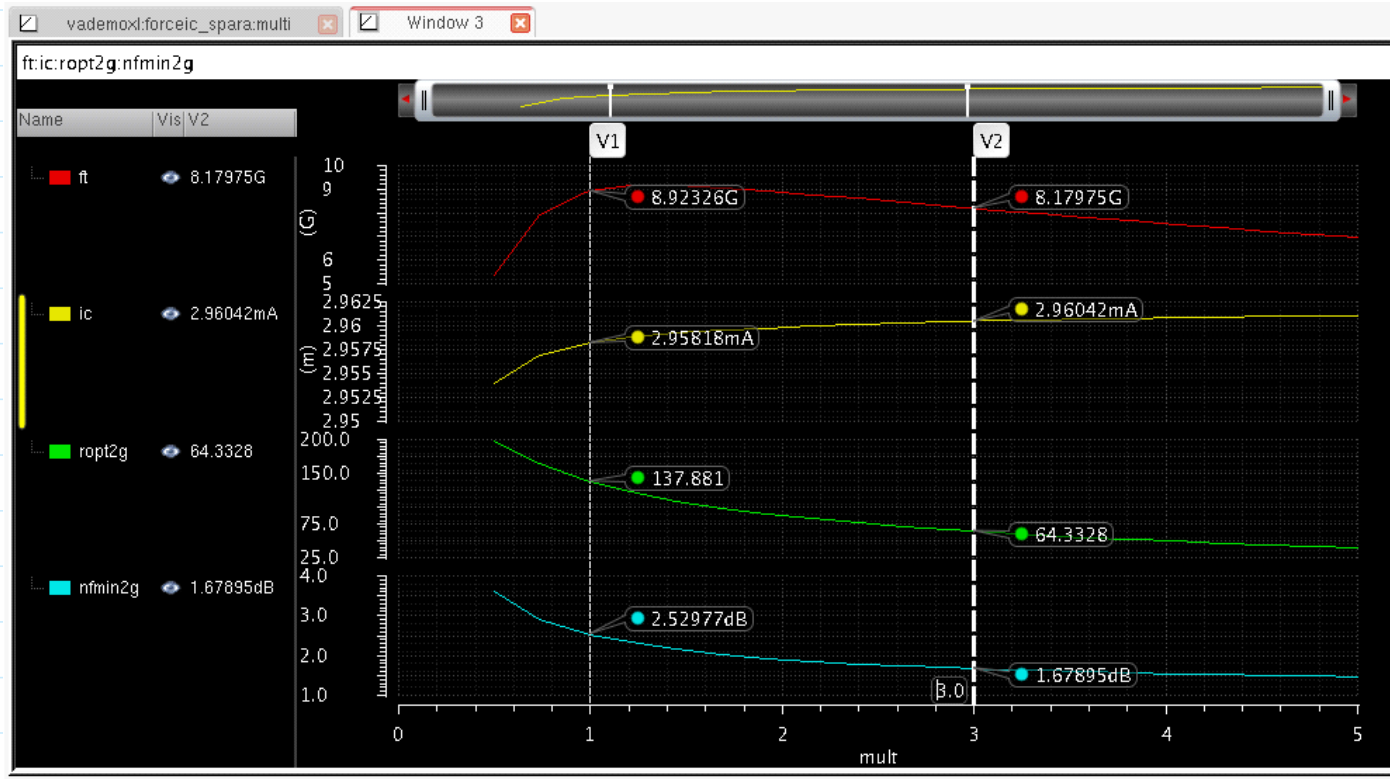
Tuesday, November 06, 2012 6:07 PM

1. Using the verilog-a Mextram transistor model, run s-parameter simulation to generate the following frequency dependence plots from 10MHz to 10GHz, for $V_{BE}=0.5, 0.85$ and $0.9V$. Set $V_{CE}=5.0V$. You can do this with calculator or ocean script.
 - a. All of the 4 y-parameters, real and imaginary part, versus frequency. Use a new subwindow for each parameter, i.e. you use 8 subwindows.
 - b. Repeat the above plot with log scale frequency
 - c. Real and imag S_{11}, S_{21}, S_{12} and S_{22} versus frequency.
 - d. Repeat the above plot with log scale frequency
 - e. S_{11} and S_{22} on Smith chart. S_{21} and S_{12} on polar plot. Move cursor and observe how the value of S_{11} and corresponding r and x values change with frequency.
 - f. Real and imag of z_{s11} - with z_{s11} defined as the impedance corresponding to a reflection coefficient of S_{11} .
 - g. $\text{Mag}(h_{21} \cdot \text{frequency})$ versus frequency
 - h. Real and imag of the Y_{21}/Y_{11} ratio.
 - i. Real and imag of h_{21} . Compare this with Y_{21}/Y_{11} ratio.
 - j. Plot out the mag of all s-parameters versus linear frequency on the same plot.
2. In "sp" analysis, set frequency to 3GHz. Sweep V_{BE} between 0.75 to 0.9 in 15 steps. $V_{CE}=5V$. Plot out $\text{mag}(h_{21}), \text{db}20(h_{21}), \text{ft.}$ and $\text{mag}(s_{21}), \text{db}20(s_{21})$ as a

function of V_{BE} .

3. Place two identical transistors in parallel. Re-run your frequency dependence simulation at the same $V_{BE}=0.85V$, $V_{CE}=5V$. Overlay the Y-parameter plots obtained using a single transistor and 2 transistors. Verify that all of your Y-parameters are exactly doubled.
4. Plot out f_t - V_{BE} for single transistor and double transistor.
5. (**newly added 11/8 class**) for the same transistor you have been using, create a ADE state that simulates f_t calculated from $\text{mag}(h_{21}) \cdot f$ at 2GHz versus mult for $I_C=3mA$, $V_{CE}=3V$. Your multi is a design variable, and should be swept from 0.1 to 5 in 10 steps. Change your "sp" analysis to sweep frequency from 1GHz to 3GHz using 3 steps to keep data file small. Use the technique we described today to hold your current constant as you change size. How much did your f_t change over this large size change?

Below is a sample result of mine, I also checked "noise" in "sp" analysis. Here I plot out minimum noise figure N_{fmin} at 2GHz, and f_t versus mult (size) for a fixed I_C . I also showed the actual I_C measured at the collector - it is indeed very close to our intended I_C , and is practically held constant as mult is swept.



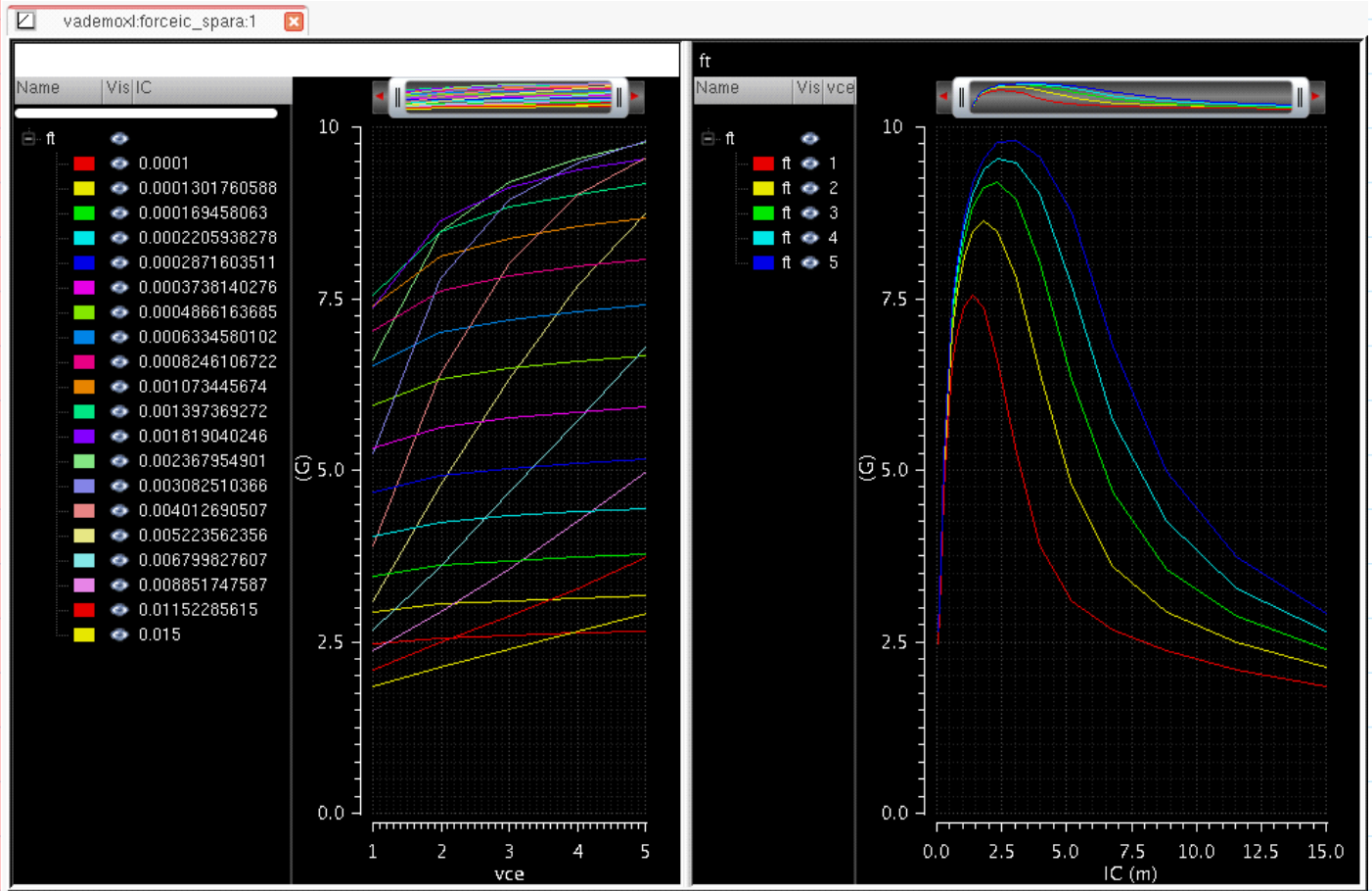
As you increase mult from 1 to 3, ft. decrease from 8.9GHz to 8.2GHz, however, Nfmin decreases from 2.5dB to 1.7dB, a much bigger change.

This type of plot is extremely useful in design of low-noise amplifiers. I also plotted out Ropt - the noise matching source resistance, just ignore that if you have not come across it before in circuit classes.

Required for tcad2 group students, optional for others - this will take a bit more time

1. Plot out $1/(2 \cdot \pi \cdot ft) - 1/IC$ for $V_{CE}=5V$. Determine the forward transit time τ_f .
2. Plot out $ft-IC$ for single transistor and double transistor.
3. (added 11/8) using the new circuit technique, sweep

IC and produce a smooth ft.-IC curve that covers the rise and fall of ft. nicely. Do this for multiple VCE=1, 3 and 5V. Your curves should look like this:



RF LNA design mini project 11/13/2012

Tuesday, November 13, 2012 5:34 PM

I have given you the LNA schematic. You can modify yours or use this one as is.

Use the same transistor you have been using.

Follow the case study we went over today to design a 2GHz LNA with 3mA current constraint. Set $V_{CE}=3V$.

I just found out there is no way to construct a complex waveform from two real waveforms in cadence. So you cannot write $y_{opt} = g_{opt} + j*b_{opt}$.

Cadence gives you g_{opt} and b_{opt} . You need to calculate $ropt$ yourself.

What I did is define an output "bopt" as:

```
getData("/Bopt" ?result "sp_noise"
```

Then "gopt" as:

```
getData("/Gopt" ?result "sp_noise"
```

From these two, calculate "ropt" which is real part of $z_{opt} = 1/y_{opt} = 1/(g_{opt} + jb_{opt})$

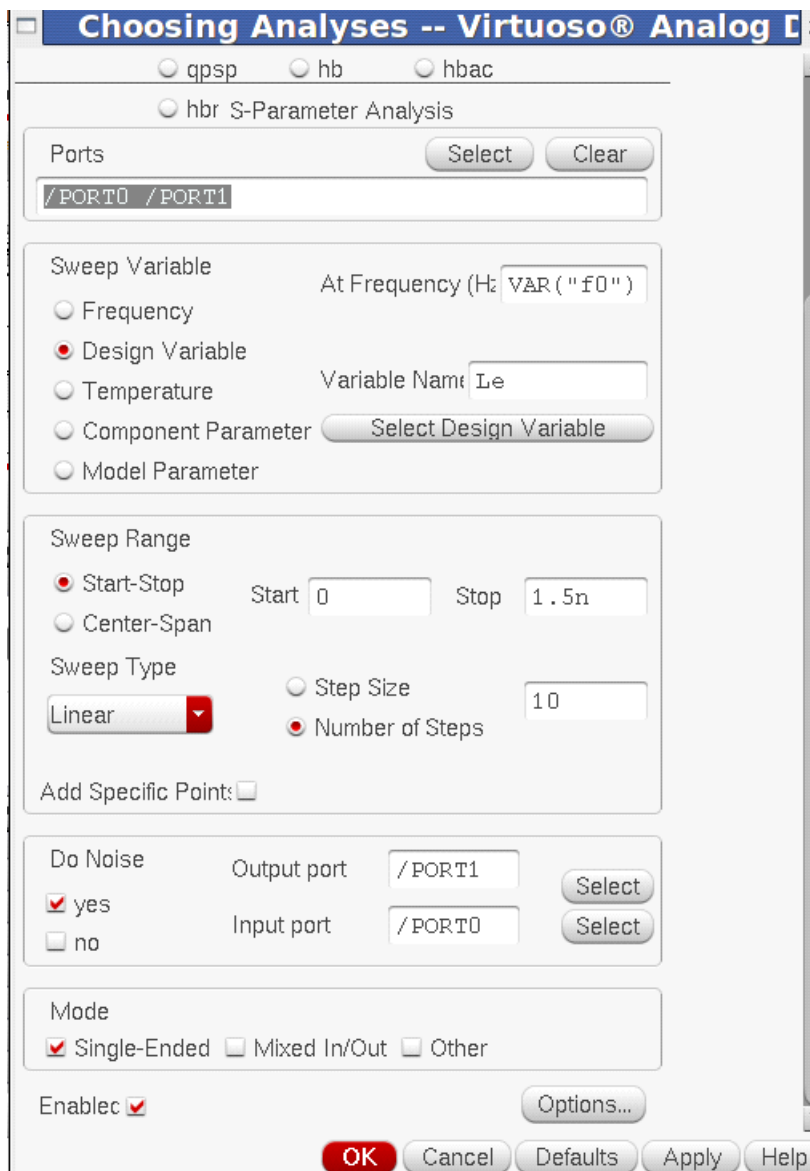
```
gopt / ((gopt**2) + (bopt**2))
```

Read today's notes. You will find the 3 steps leading to a good design.

1. Sweep "mult" to set $ropt=50\text{ohms}$
2. Sweep "Le" to set $zs11r = 1$ (normalized)
3. Sweep "Lb" to set $zs11i = 0$

Remember to check 'noise' for "sp" analysis.

Below is a screen shot of my "sp" analysis for sweeping "Le":



Go ahead and give this a try. You can in principle turn this into your final project.

Linear 2-port parameters

Saturday, October 27, 2012 9:28 PM

Y

Z

H

ABCD

S

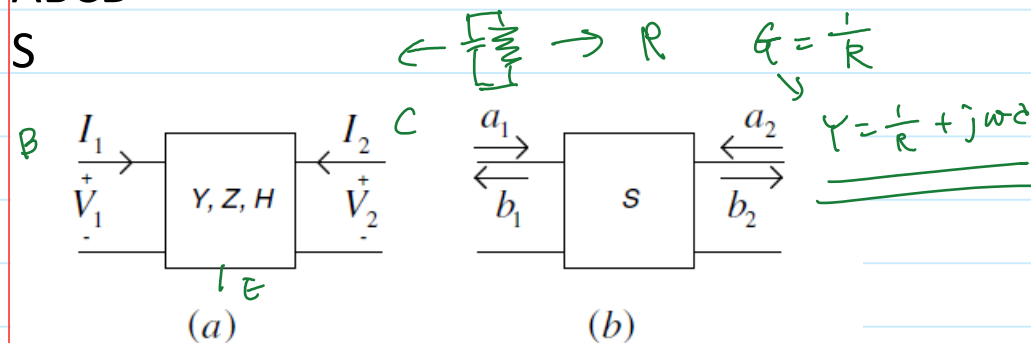


Figure 5.9 (a) Y -, Z - or H -parameters describe the relations among terminal currents and voltages of a linear network. (b) S -parameters describe the relations between the voltage waves, defined as independent linear combinations of terminal currents and voltages.

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$[Z]$

complex \checkmark

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$V_2 = 0, \quad I_1 = Y_{11} \cdot V_1$

The Y -parameters can be determined using short-circuit terminations at the input or the output.

$$\begin{pmatrix} V_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ V_2 \end{pmatrix}$$

H_{11} is essentially the input impedance with the output short circuited ($V_2 = 0$), and H_{21} is the current gain I_2/I_1 with the output short circuited. H_{11} is used to extract the base resistance, and H_{21} is used to extract f_T . Measurement of the H -parameters involves setting I_1 and V_2 to zero.

5.4.4 S-Parameters

At high frequencies, accurate open and short circuits are extremely difficult to achieve because of the inherent parasitic inductances and capacitances. Consequently, the device under test (DUT) often oscillates with open or short terminations. The interconnection between the DUT and test equipment is also comparable to the wave length, requiring the consideration of distributive effects. Because of these practical difficulties, *S*-parameters were developed and are almost exclusively used to characterize transistor RF and microwave performance.

S-parameters contain no more and no less information than the *Z*-, *Y*-, or *H*-parameters introduced above. The only difference is that the independent and dependent variables are no longer simple voltages and currents. Instead, linear combinations of the simple variables are used to produce four "voltage waves," which contain the same information since they are chosen to be linearly independent. These combinations are chosen such that they can be physically measured at high frequencies using transmission line techniques. One can understand this formulation as a simple transform of the *Y*-, *Z*- or *H*-parameters into a new form,

just like one can transform an impedance *Z* to a voltage reflection coefficient Γ

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} \quad (5.30)$$

where Z_0 is a characteristic impedance. Such a transform from *Z* to Γ is extremely useful in studying transmission lines, and the various definitions of two-port parameters provide a similar utility.

The newly defined voltage wave variables a_1 , b_1 , a_2 , and b_2 are shown in Figure 5.9(b), where *a* indicates incident, and *b* indicates reflection or scattering. The waves are related to port voltages and currents by

$$a_1 = \frac{V_1 + Z_0 I_1}{2\sqrt{Z_0}} \quad (5.31)$$

$$b_1 = \frac{V_1 - Z_0 I_1}{2\sqrt{Z_0}} \quad (5.32)$$

$$a_2 = \frac{V_2 + Z_0 I_2}{2\sqrt{Z_0}} \quad (5.33)$$

$$b_2 = \frac{V_2 - Z_0 I_2}{2\sqrt{Z_0}} \quad (5.34)$$

The voltage waves are defined using voltages and currents for a characteristic impedance Z_0 , similar to the definition of Γ in transmission lines. These voltage waves are not "voltages" per se, but voltages normalized to a $2\sqrt{Z_0}$ term such that when squared they have dimensions of power. The voltages a_1 and a_2 are called the incident waves, and b_1 and b_2 are called the scattered waves. The scattered waves are related to the incident waves by a set of linear equations, just as the port voltages are related to the port currents by the Z -parameters

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}. \quad (5.35)$$

The coefficients of these relationships are the S -parameters. One can mathematically prove that the resulting S -parameters are unique for a given linear network, just as they are for the Z -, Y -, and H -parameters.

The measurement of S -parameters involves setting a_1 and a_2 to zero, which is easily accomplished by terminating the ports with Z_0 . For instance, to set $a_2 = 0$, we terminate port 2 with Z_0 . As a result, $v_2 = -I_2 Z_0$, and thus $a_2 = 0$ according to the definition of a_2 . Using the definitions of a_1 and b_1 , S_{11} is then obtained as

$$S_{11} = \frac{b_1}{a_1} = \frac{V_1 - I_1 Z_0}{V_1 + I_1 Z_0} = \frac{Z_{in,0} - Z_0}{Z_{in,0} + Z_0}. \quad (5.36)$$

where $Z_{in,0} = V_1/I_1$ is the input impedance with $Z_l = Z_0$. We see then that S_{11} is therefore simply the reflection coefficient corresponding to the input impedance when the output is terminated with Z_0 . The required condition for S -parameter measurements is hence termination with the proper characteristic impedance, just as for short-circuit termination for Y -parameters, or open-circuit termination for Z -parameters. Similarly,

$$S_{21} = \frac{b_2}{a_1} = \frac{V_2 - I_2 Z_0}{V_1 + I_1 Z_0} = 2 \frac{V_2}{V_1 + I_1 Z_0} = 2 \frac{V_2}{V_s}. \quad (5.37)$$

where $V_s = V_1 + I_1 Z_0$ is equal to the source voltage if a source impedance Z_s is chosen to be Z_0 , as illustrated in Figure 5.10. We note that $Z_s = Z_0$ is indeed used in practical S -parameter measurements. We see that S_{21} is simply twice the ratio of V_{out} to V_s for a Z_0 source and a Z_0 load. This relationship provides a simple means of calculating S_{21} and S_{11} using the transistor equivalent circuit, and understanding the physical meanings of S_{21} and S_{11} in terms of impedance and voltage gain, which are familiar to analog designers. Another physical meaning of S_{21} is that $|S_{21}|^2$ gives the transducer gain for a Z_0 source and Z_0 load.

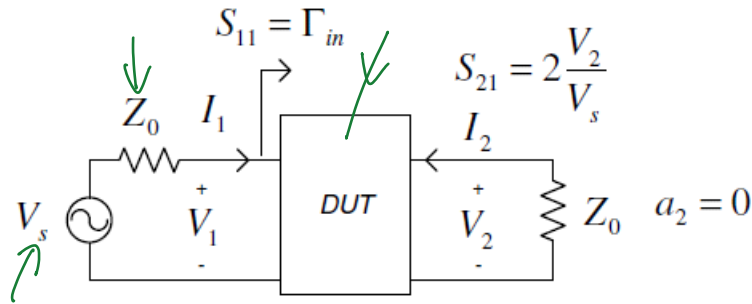


Figure 5.10 A simple method of calculating S_{11} and S_{21} . With a Z_0 drive and a Z_0 load, S_{11} is the input reflection coefficient looking into port 1, and S_{21} is twice the voltage gain V_2/V_s .

The measurements of S_{22} and S_{12} are similar. We terminate port 1 with a Z_0 load, and drive port 2 with a Z_0 source. S_{22} is essentially the output reflection coefficient looking back into the output port for a Z_0 source termination, S_{12} is the reverse gain, and $|S_{12}|^2$ is the reverse transducer gain for a Z_0 source and a Z_0 load.

Because of their intuitive relationship to the reflection coefficients, S_{11} and S_{22} are conveniently displayed on a Smith chart, while S_{21} and S_{12} are typically displayed on a polar plot. Figure 5.11(a) and (b) show an example of the S_{11} and S_{21} measured from 4 to 40 GHz for a SiGe HBT. Two collector currents of 1.26 mA and 25.0 mA are shown, with $V_{CB} = 1$ V. We see that the S_{11} for a bipolar transistor always moves clockwise as frequency increases on the Smith chart. The S_{11} data at higher I_C in general shows a smaller negative reactance, because of the higher EB diffusion capacitance. The S_{21} magnitude decreases with increasing frequency, as expected, because of decreasing forward transducer gain, while S_{21} is larger at higher I_C because of the higher f_T at that bias current. It follows from the above discussions that the S -parameters of a SiGe HBT will intimately depend on the transistor size, biasing condition, and operating frequency.

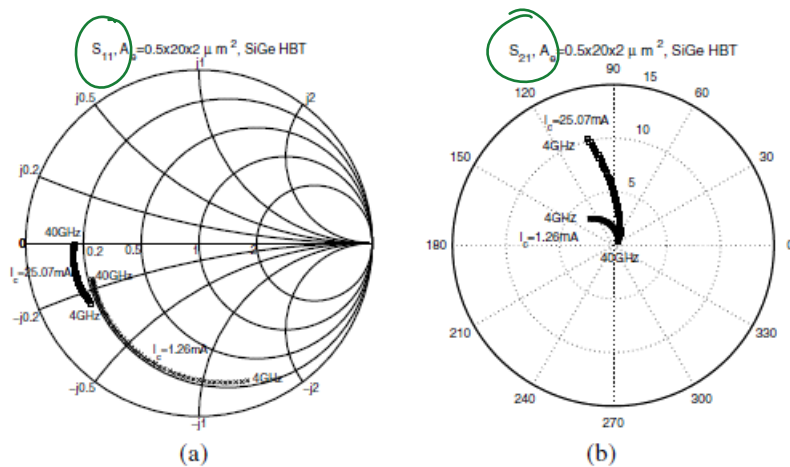


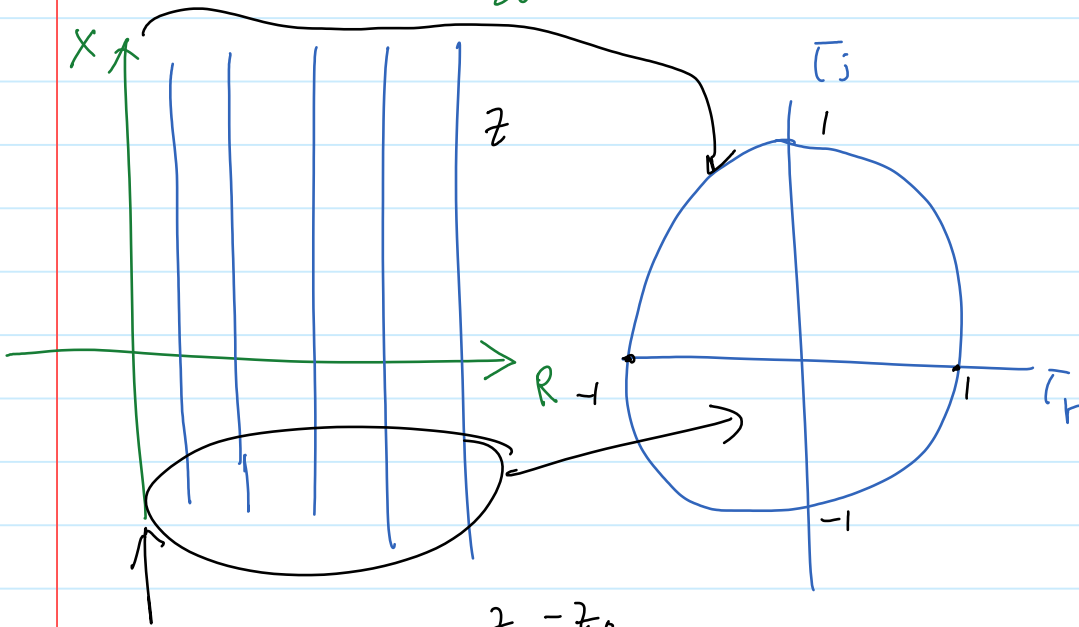
Figure 5.11 Example plots of (a) S_{11} and (b) S_{21} measured data for a SiGe HBT. Two traces are for $I_C = 1.26$ and 25 mA, with $V_{CB} = 1$ V. The frequency range is from 4 to 40 GHz, and $A_E = 0.5 \times 20 \times 2 \mu\text{m}^2$.

$$z \rightarrow \Gamma = \Gamma_r + j\Gamma_i$$

$$z \rightarrow \Gamma = \Gamma_r + j\Gamma_i$$

$$\frac{z}{z_0} = r + jx$$

$$\Gamma = \frac{z - z_0}{z + z_0} = \frac{\frac{z}{z_0} - 1}{\frac{z}{z_0} + 1}$$



$$\underline{\underline{r=0}} \quad \Gamma = \frac{z - z_0}{z + z_0}$$

$$z = r + jx$$

s-parameter simulation and measurement

Tuesday, October 30, 2012 9:38 AM

We need to combine dc bias with RF excitation using bias tees - essentially an inductor that passes DC and blocks ac (RF), and a capacitor that passes RF and blocks dc.

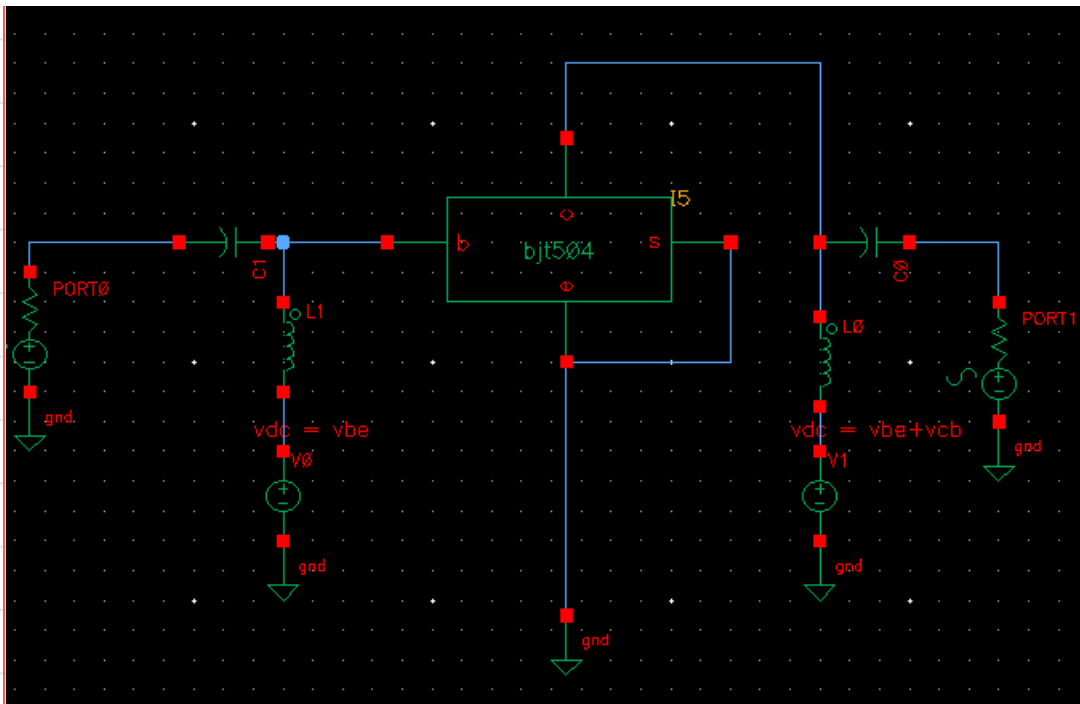
Copy the vademio folder from /scratch/8710/cadence

To your own \$HOME/cadence folder or another folder (if so, modify the cds.lib after copying)

Open terminal

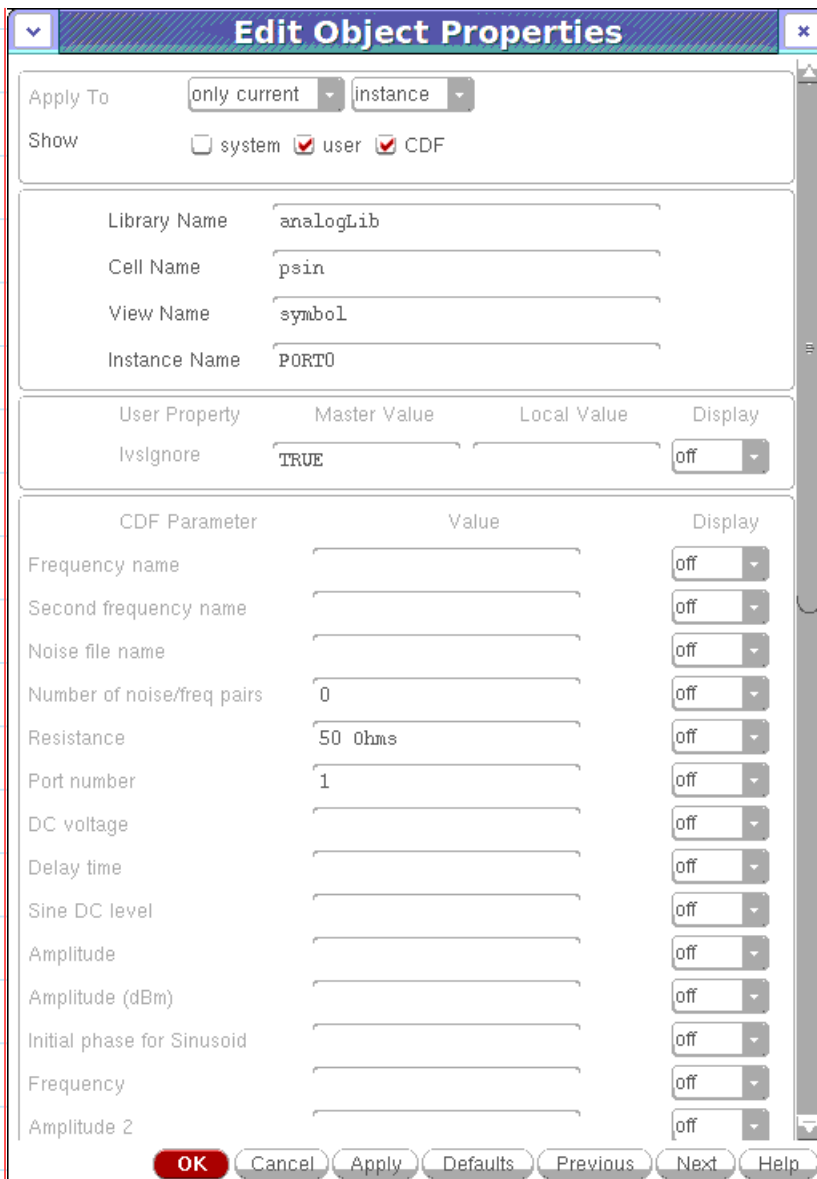
Then go to (cd) that folder of yours, type "source cadence6"

Then type "virtuoso" on command line.

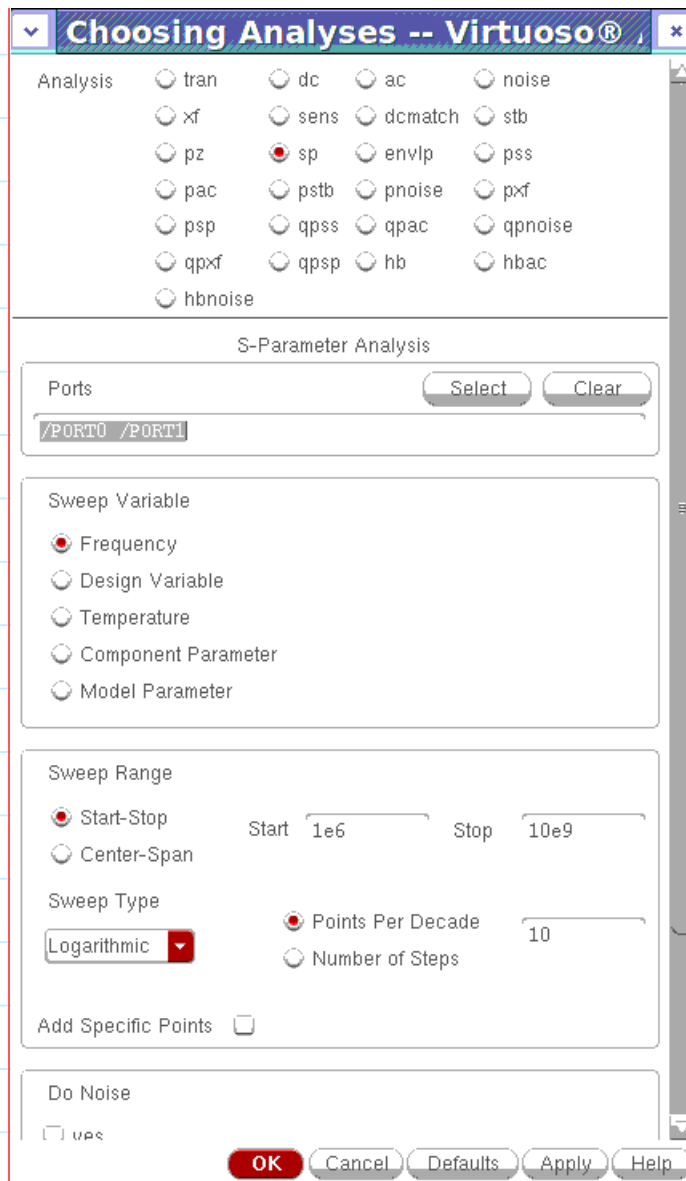


Pay attention to the "PORT0" and "PORT1" - they are from the analogLib, under sources.

Specify the port number as "1" and "2" for the input and output ports. An example is shown below:



Then you can set up "sp" analysis for s-parameters:



You can type in or select from schematic the ports, for most part, we specify two ports.

A frequency sweep is basic, of course, you can sweep other parameters like you did before.

Plotting and understanding transistor s-parameters

Thursday, November 01, 2012 9:46 AM

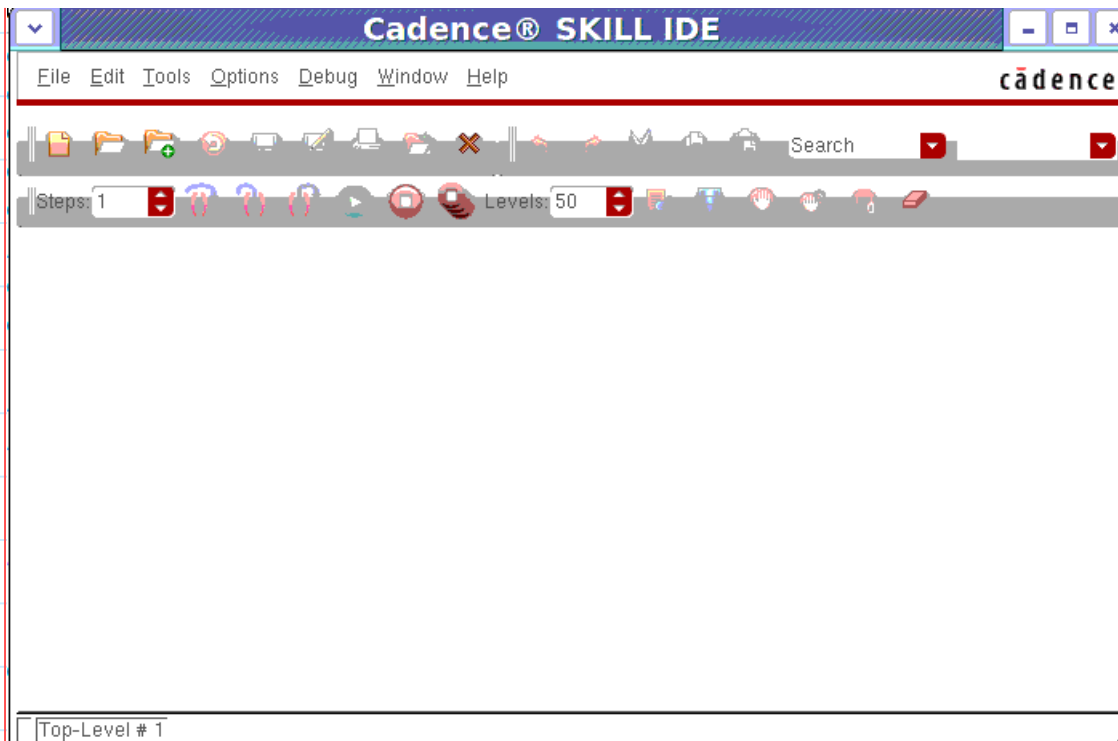
You can of course use the GUI to manually do plotting. Problem is that Cadence does not have a nice way of keeping all of your plot settings so you can reuse them when you run a new simulation.

Other tools like Agilent does better in this regard. However, verilog-a support is much better in cadence, also many design kits support cadence (there is ADS link too but that requires additional kit support you often do not have)

Cadence has scripting, so you can use ocean - but it is difficult for the average user, they are designers not programmers.

I experimented with it and came up with a piece of ocean program for teaching - you should find this useful for research too.

From the menu of the very first "virtuoso" window, "Tools-Skill IDE"



Then "File -> Open" to open the only plotbjtspara.ocn in the folder that you copied from me.

I wrote it so it will work with both cadence ic5 and ic6. Also I wrote it in a way that allows you to copy these expressions into your "Outputs" or "calculators" - so some expression options may seem unnecessary if you just use it for ocean.

You will see that I made conversion between reflection coefficient and impedance in two occasions.

1. S11 - I created "zs11" - which is the normalized impedance for a Gamma = S11. I plot out S11 on "Smith" chart , and then plot out real and imag of zs11.
 - a. Look at both plots together, use cursor to move

- along data on Smith chart, observe the "r+jx" number the program reports.
- b. Compare that with what is on the real(zs11) and imag(zs11) vs freq curves. You will find they are consistent
2. H11 - h11 is the input impedance for an output short circuit termination. So I convert H11 to a reflection coefficient "gh11" so that you can then plot it on Smith chart.
 - a. Move cursor along data on smith - watch the "r+jx" output, compare that with your own plotting of real and imag of h11, well you can normalize to z0 easily for exact apple-to-apple comparison
 3. Try to compare S11 and H11.

Current gain with output shorted and fT

Saturday, October 27, 2012 9:37 PM

5.3.1 Current Gain and Cutoff Frequency

The high-frequency current amplification capability of a SiGe HBT is typically measured by the small-signal current gain for a shorted output termination (i.e., h_{21}). Imagine driving the base terminal with a small-signal current $i_b = i_0 e^{j\omega t}$, and now short-circuit the output (collector), as shown in Figure 5.3. The node voltage v_b then equals

$$v_b = \frac{1}{g_{be} + j\omega(C_{be} + C_{bc})} i_b. \quad (5.4)$$

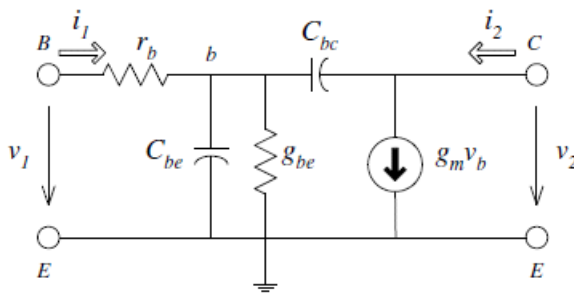


Figure 5.2 A simple high-frequency equivalent circuit model.

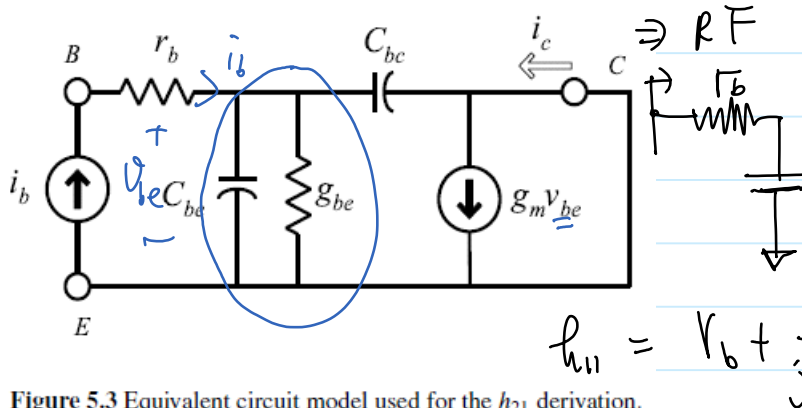


Figure 5.3 Equivalent circuit model used for the h_{21} derivation.

$$= r_b \sim j \frac{1}{\omega C}$$

The effective capacitive load for the input due to Miller capacitance C_{bc} is still C_{bc} because of the "zero" voltage gain resulting from the short-circuited output. Because the reverse-biased CB junction capacitance is far smaller than the forward-biased EB junction capacitance, we can neglect its contribution to the output current i_c

$$i_c \approx g_m v_b = \frac{g_m}{g_{be} + j\omega(C_{be} + C_{bc})} i_b. \quad (5.5)$$

Therefore, we have

$$h_{21} = \left. \frac{i_c}{i_b} \right|_{v_c=0} = \frac{g_m}{g_{be} + j\omega(C_{be} + C_{bc})} = \frac{\beta}{1 + j\omega(C_{be} + C_{bc})/g_{be}}. \quad (5.6)$$

Note that h_{21} is constant at low frequencies, and then decreases at higher frequencies. Obviously, the imaginary part increases with ω , and dominates at high fre-

cies. Obviously, the imaginary part increases with ω , and dominates at high fre-

quencies. Under these conditions the above equation becomes

$$h_{21} = \frac{g_m}{j\omega(C_{be} + C_{bc})}$$

$$h_{21} \cdot \omega = \omega_T = \frac{g_m}{C_{be} + C_{bc}} \quad (5.7)$$

which is equivalent to

$$h_{21} \times f = \frac{f_T}{j} \quad (5.8)$$

$$f_T = \frac{g_m}{2\pi(C_{be} + C_{bc})} \quad (5.9)$$

The $|h_{21} \times f|$ product is a constant over the frequency range where these assumptions hold. This constant is referred to as f_T , the transition frequency, or more commonly, the cutoff frequency. In practice, f_T is extracted by extrapolating the measured $|h_{21}|$ versus frequency data in a range where a slope of -20 dB/decade is observed. The frequency at which the extrapolated $|h_{21}|$ reduces to unity is defined to be f_T (i.e., the unity gain cutoff frequency). Practically speaking, the extrapolation is necessary here because we are usually not interested in operating transistors at the frequency of unity current gain, which can be different from the extrapolated f_T , depending on parasitics and other factors. Instead, we are interested in the gain available at much lower frequencies where the current gain is much higher than unity. In the frequency range where $|h_{21}|$ rolls off at -20 dB/decade, $|h_{21}|$ can be easily estimated as f_T/f .¹

State-of-the-art SiGe HBTs exhibit f_T values above 200 GHz [1], which is much higher than the operating frequencies of the bulk of existing wireless systems, which are typically below 10 GHz. In this case, caution must be exercised in estimating $|h_{21}|$ from f_T , because the operating frequency f may be below the frequency range over which $|h_{21} \times f| = f_T$. In this case, we then need to resort to (5.6) which applies to all frequencies below f_T and can be rewritten as follows using (5.9)

$$h_{21} = \frac{\beta}{1 + jf/f_\beta} \quad (5.10)$$

$$f_\beta = \frac{f_T}{\beta}$$

Here, $|h_{21}|$ is equal to β at low frequencies, reduces by 3 dB at $f = f_\beta = f_T/\beta$, and then drops off with increasing f at a theoretical slope of -20 dB/decade. Hence, for

¹We note that for the very high f_T SiGe HBTs being realized today (200+ GHz), instrumentation limitations place a practical upper bound on directly measuring f_T in any case, since the highest reliable measurement frequencies are in the 110-GHz range for commercially available test systems.

a SiGe HBT with $f_T = 100$ GHz and $\beta = 100$, the 3 dB frequency is $f_\beta = 1$ GHz. For a design frequency of 2 GHz, which is close to f_β , (5.11) needs to be used for $|h_{21}|$ estimation instead of f_T/f . Figure 5.4 shows an example of measured $|h_{21}|$ versus frequency from 2 to 110 GHz for a SiGe HBT. The extrapolated f_T is 117 GHz. A noticeable deviation from the 20-dB/decade straight line fit is observed below 7 GHz, necessitating the use of (5.11) for $|h_{21}|$ estimation. Note as well that a deviation from the 20-dB/decade slope is observed above 40 GHz.

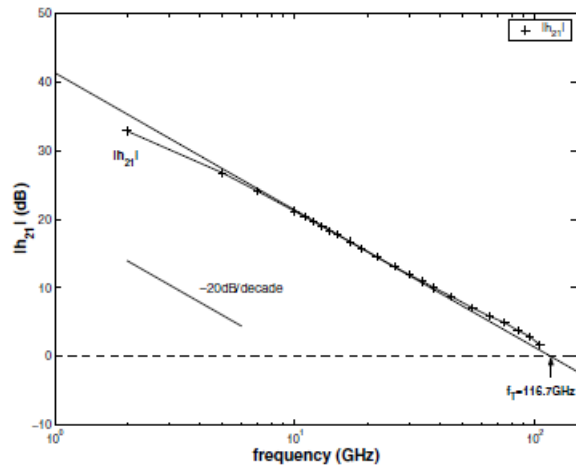


Figure 5.4 Measured $|h_{21}|$ versus frequency for a state-of-the-art SiGe HBT.

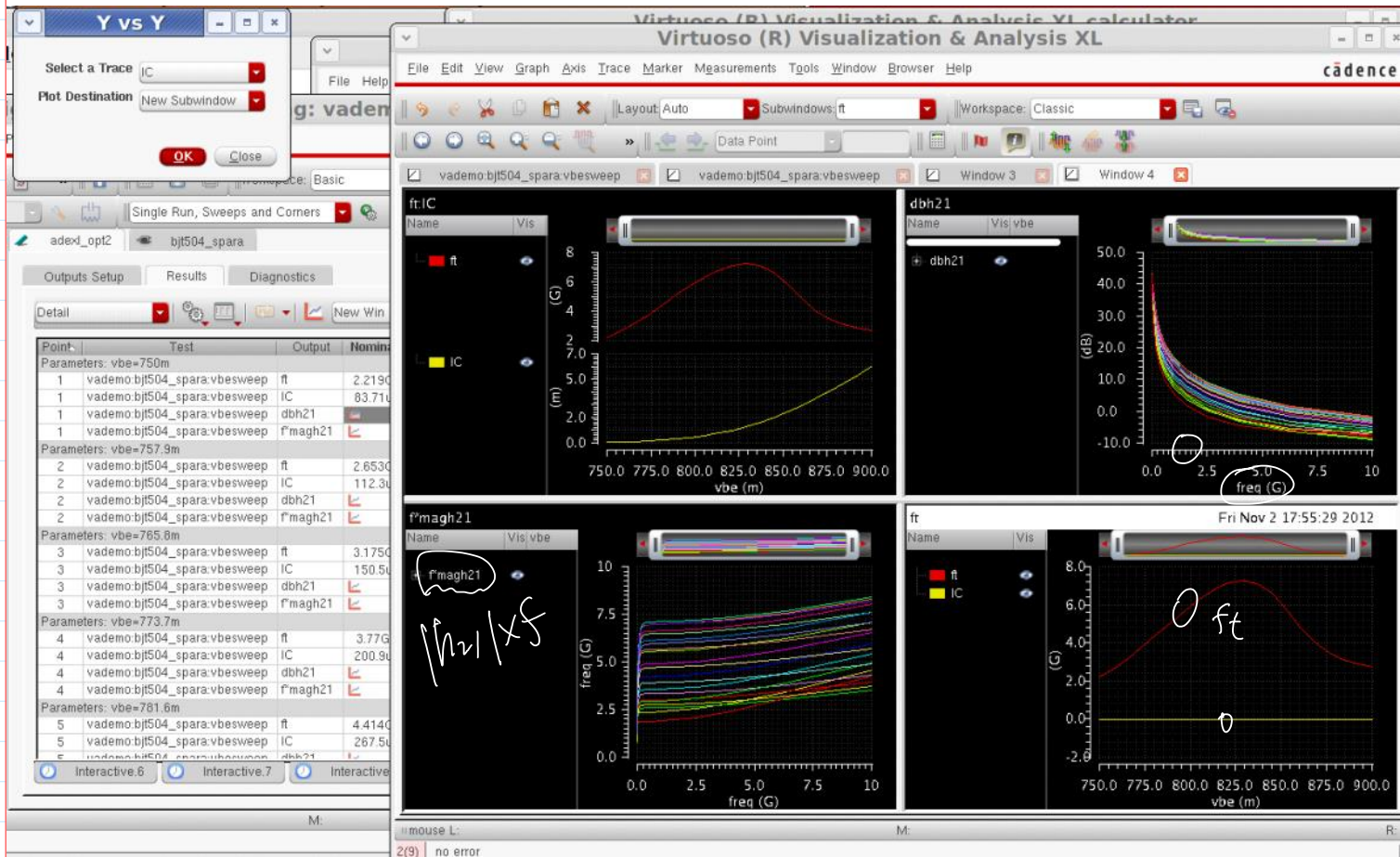
Ft extraction from h21 - frequency curve and how to plot ft-IC

Friday, November 02, 2012 5:56 PM

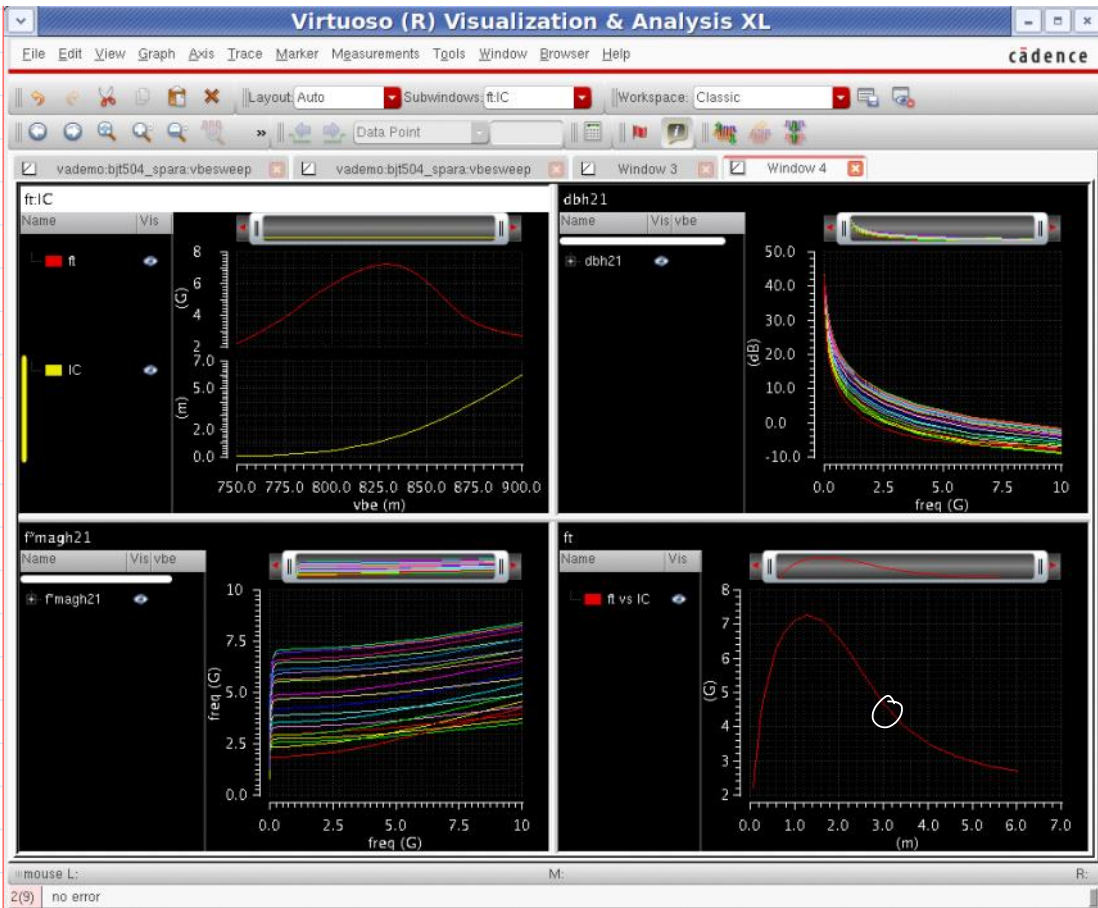
Look for a region where $\text{mag}h_{21}$ (mag of h_{21}) *
freq is frequency independent

Choose a freq point from this region, the $\text{mag}h_{21}$ *
freq product is f_t .

use "Y vs Y" from menu to make f_t -IC plot rather
than f_t -vbe plot:

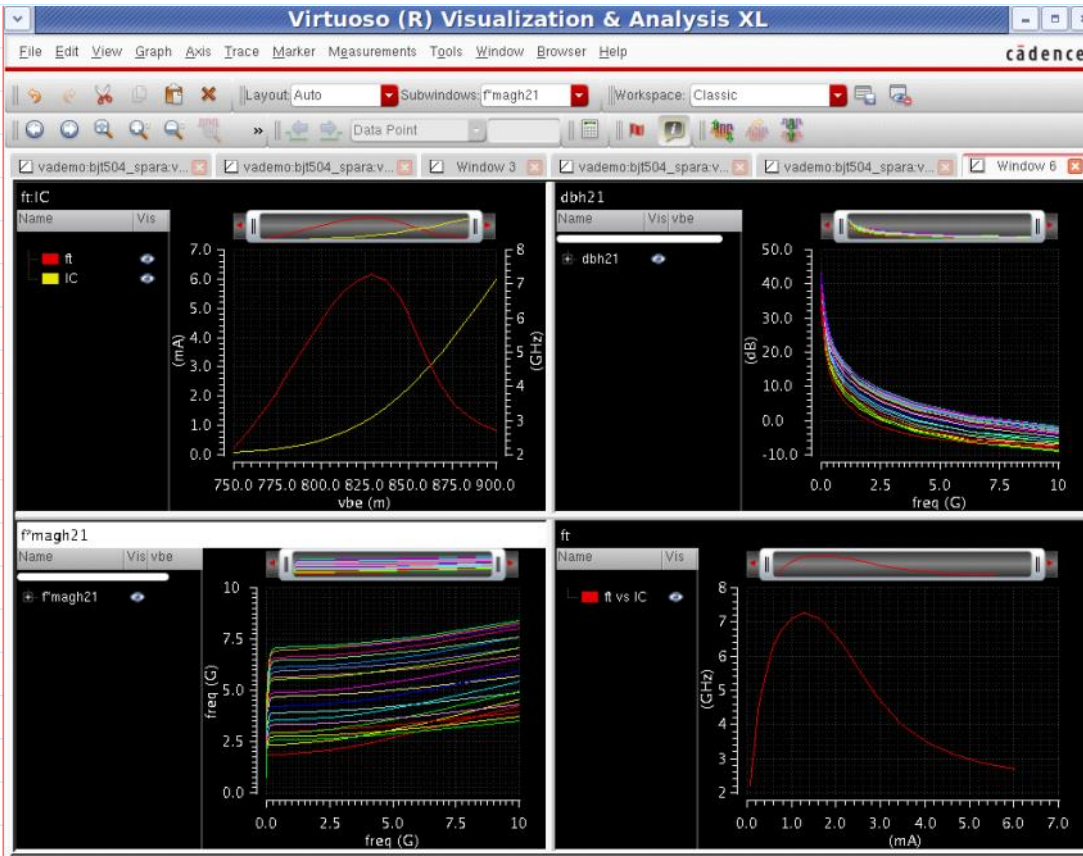


We then have f_t -IC plot:

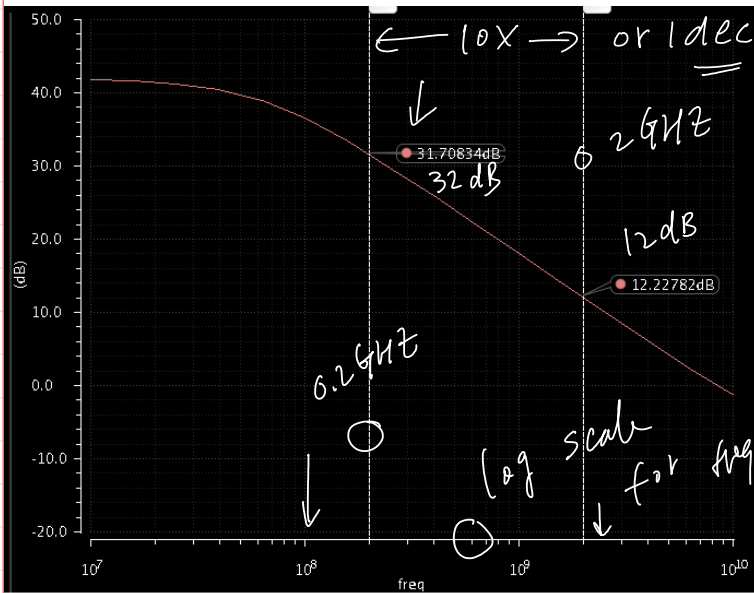


With a bit extra work in the "Outputs setup", we can add units and suffix to our like, e.g. using GHz for ft. and using mA for IC

This gives us a nice summary of our h21 and ft. data:



You can also inspect $db20(h21)$ vs log scale freq, and identify the 20dB/decade slope:



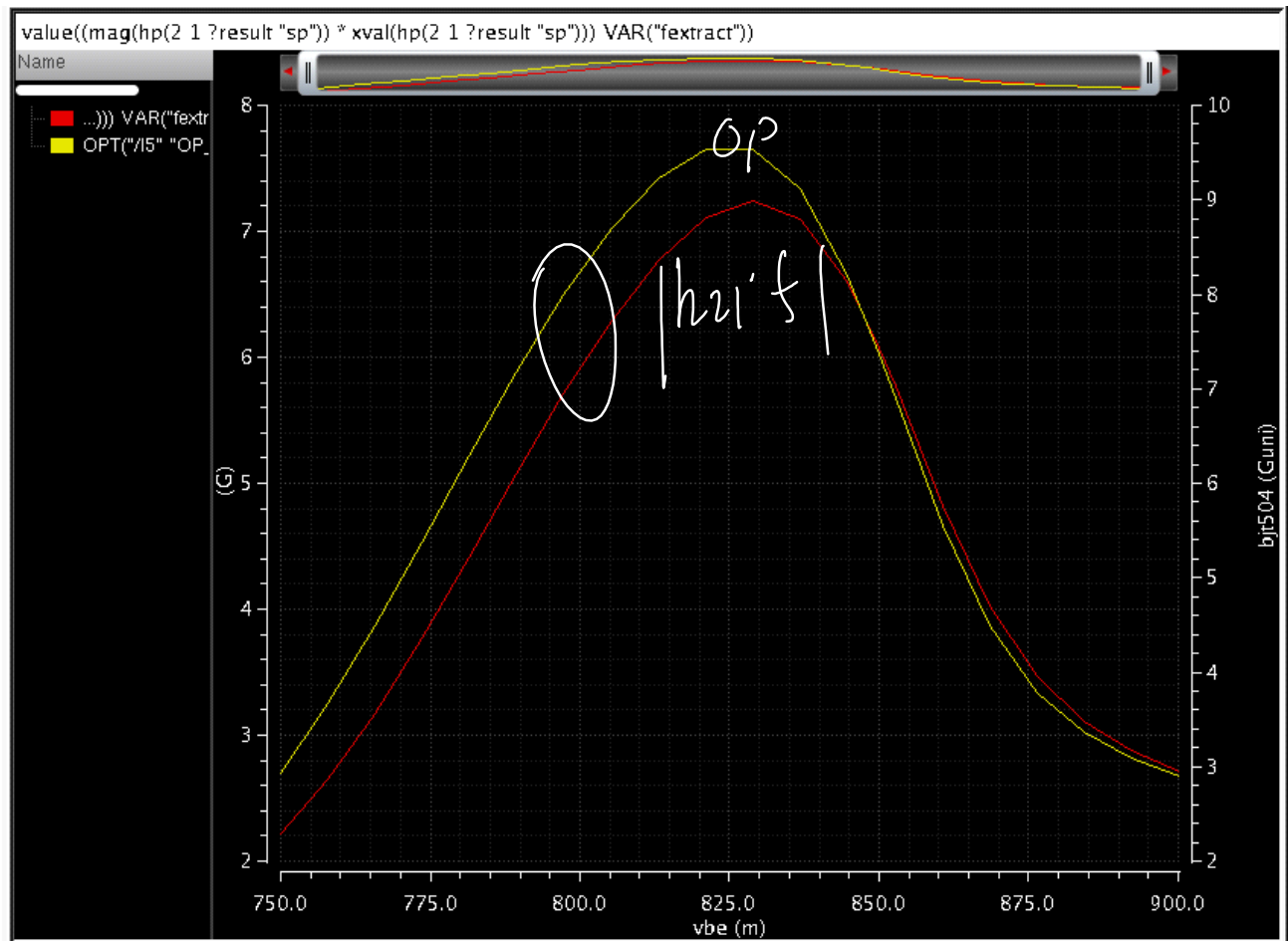
Ft from h21 and ft from OP point (approximate)

Saturday, November 03, 2012 3:20 PM

We can compare the ft. obtained from h21 (sp analysis) with the ft. obtained from dc operational point analysis (remember we did that first?)

They are not exactly the same but close.

Also the way we determine ft. should be changed "improved" to make a more fair comparison with the ft. from OP.



Ft and current density

Saturday, October 27, 2012 9:40 PM

5.3.2 Current Density Versus Speed

The fundamental nature of SiGe HBTs require the use of high operating current density in order to achieve high speed. The operating current density dependence of f_T is best illustrated by examining the inverse of f_T using (5.9)

$$\frac{1}{2\pi f_T} = \frac{C_{be} + C_{bc}}{g_m} \quad (5.12)$$

Since $C_{be} = g_m \tau_f + C_{te}$, $C_{bc} = C_{tc}$, and $g_m = qI_C/kT$, (5.12) can be rewritten as

$$\frac{1}{2\pi f_T} = \tau_f + \frac{kT}{qI_C} C_t \quad (5.13)$$

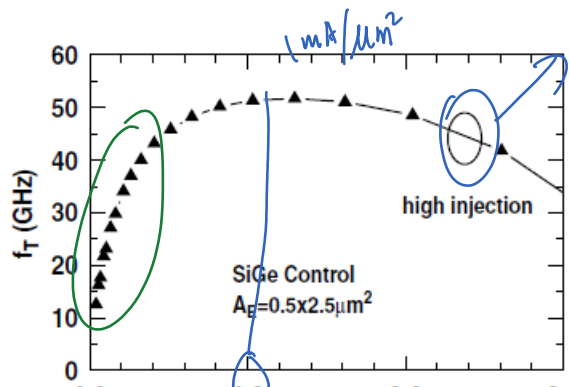
where $C_t = C_{te} + C_{tc}$. Since both C_{te} and C_{tc} are proportional to emitter area, (5.13) can be rewritten in terms of the biasing current density J_C as

$$\tau_{eff} \sim \frac{1}{J_C} \left(\frac{1}{2\pi f_T} \right) = \tau_f + \frac{kT}{qJ_C} C'_t \quad (5.14)$$

where $C'_t = C_t/A_E$ is the total EB and CB depletion capacitances per unit emitter area, and $J_C = I_C/A_E$ is the collector operating current density. Thus, the cutoff frequency f_T is fundamentally determined by the biasing current density J_C , independent of the transistor emitter length. For very low J_C , the second term is very large, and f_T is very low regardless of the forward transit time τ_f . With increasing J_C , the second term decreases, and eventually becomes smaller than τ_f . At high J_C , however, base push-out (Kirk effect, refer to Chapter 6) occurs, and τ_f itself increases with J_C , leading to f_T roll-off. A typical f_T versus J_C characteristic is shown in Figure 5.5 for a first generation SiGe HBT.

The values of τ_f and C'_t can be easily extracted from a plot of $1/2\pi f_T$ versus $1/J_C$, as shown in Figure 5.6. Near the peak f_T , the $1/2\pi f_T$ versus $1/J_C$ curve is nearly linear, indicating that C'_t is close to constant for this biasing range at high f_T . Thus, C'_t can be obtained from the slope, while τ_f can be determined from the y-axis intercept at infinite current ($1/J_C = 0$).

To improve f_T in a SiGe HBT, the transit time τ_f must be decreased by using a combination of vertical profile scaling as well as Ge grading across the base. At the same time, the operating current density J_C must be increased in proportion in order to make the second term in (5.14) negligible compared to the first term (τ_f). That is, the high f_T potential of small τ_f transistors can only be realized by using sufficiently high operating current density. This is a fundamental criterion for high-speed SiGe HBT design. The higher the peak f_T , the higher the required operating J_C . For instance, the minimum required operating current density has increased from 1.0 mA/ μm^2 for a first generation SiGe HBT with 50-GHz peak f_T to 8–10



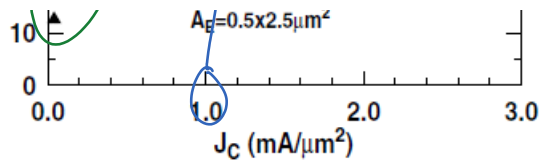


Figure 5.5 A typical $f_T - J_C$ behavior for a SiGe HBT.

$\text{mA}/\mu\text{m}^2$ for >200-GHz peak f_T third generation SiGe HBTs [1]. Higher current density operation naturally leads to more severe self-heating effects, which must be appropriately dealt with in compact modeling and circuit design [2]. Electromigration and other reliability constraints associated with very high J_C operation have also produced an increasing need for copper metalization schemes.

In order to maintain proper transistor action under high J_C conditions, the collector doping must be increased in order to delay the onset of high injection effects. This requisite doping increase obviously reduces the breakdown voltage. At a fundamental level, trade-offs between breakdown voltage and speed are thus inevitable for all bipolar transistors (Si, SiGe, or III-V). Since the collector doping in SiGe HBT is typically realized by self-aligned collector implantation (as opposed to during epi growth in III-V), devices with multiple breakdown voltages (and hence multiple f_T) can be trivially obtained in the same fabrication sequence, giving circuit designers added flexibility.

Another closely related manifestation of (5.14) is that the minimum required J_C to realize the full potential of a small τ_f transistor depends on C'_i . Both C'_{te} and C'_{ic} thus must be minimized in the device and are usually addressed via a combination of structural design, ground-rule shrink, and doping profile tailoring via selective collector implantation. This reduction of C'_{ic} is also important for increasing the power gain (i.e., maximum oscillation frequency - f_{max}).

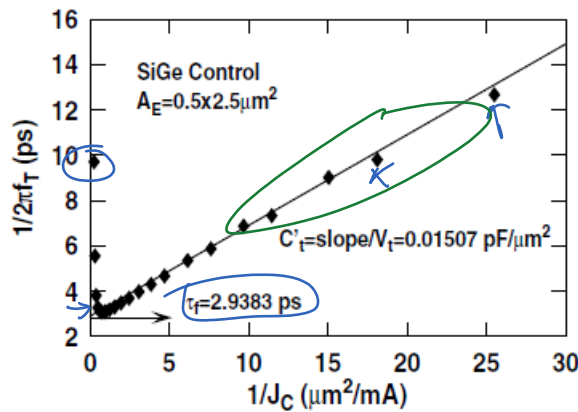
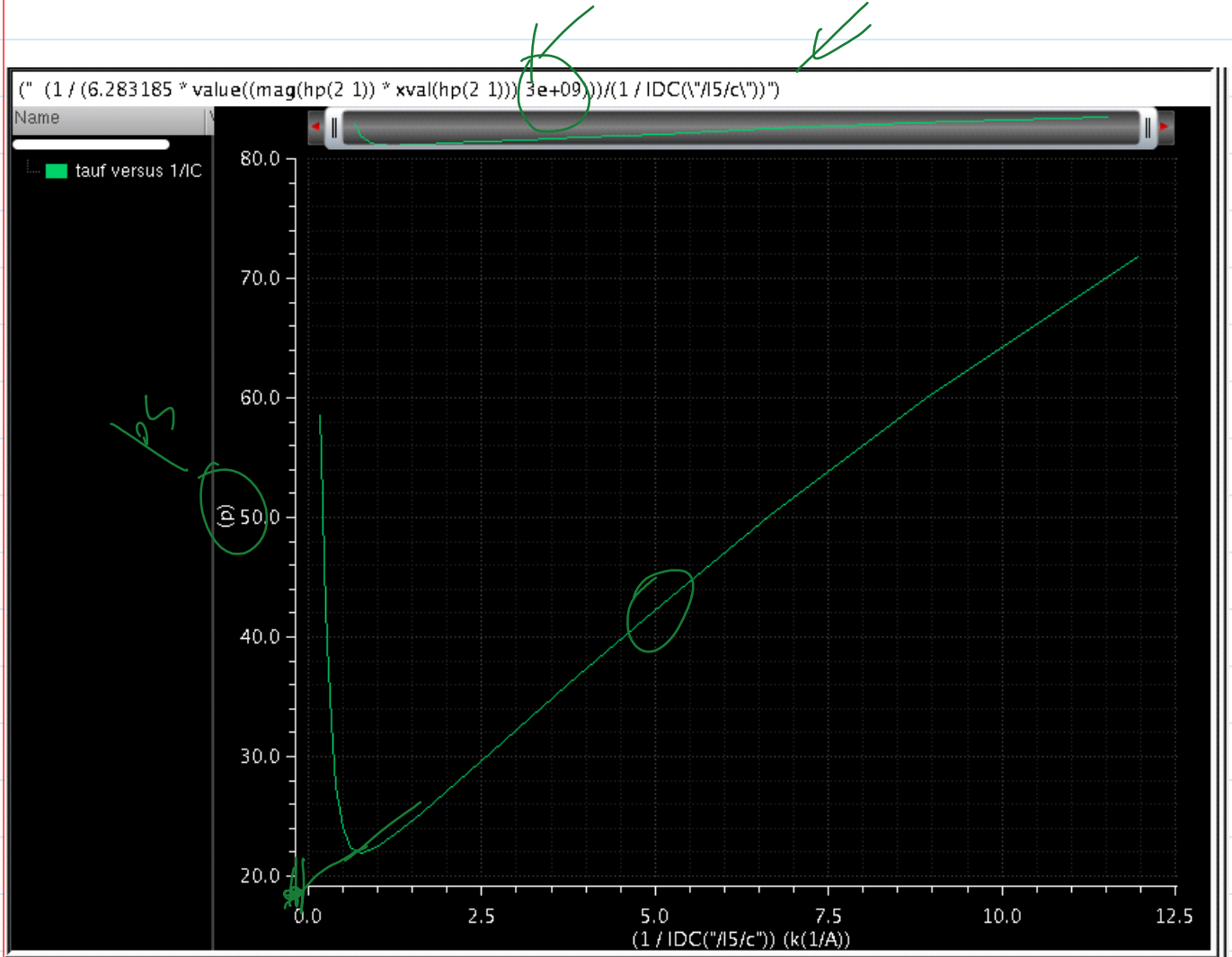


Figure 5.6 Illustration of C'_i and τ_f extraction in a SiGe HBT.

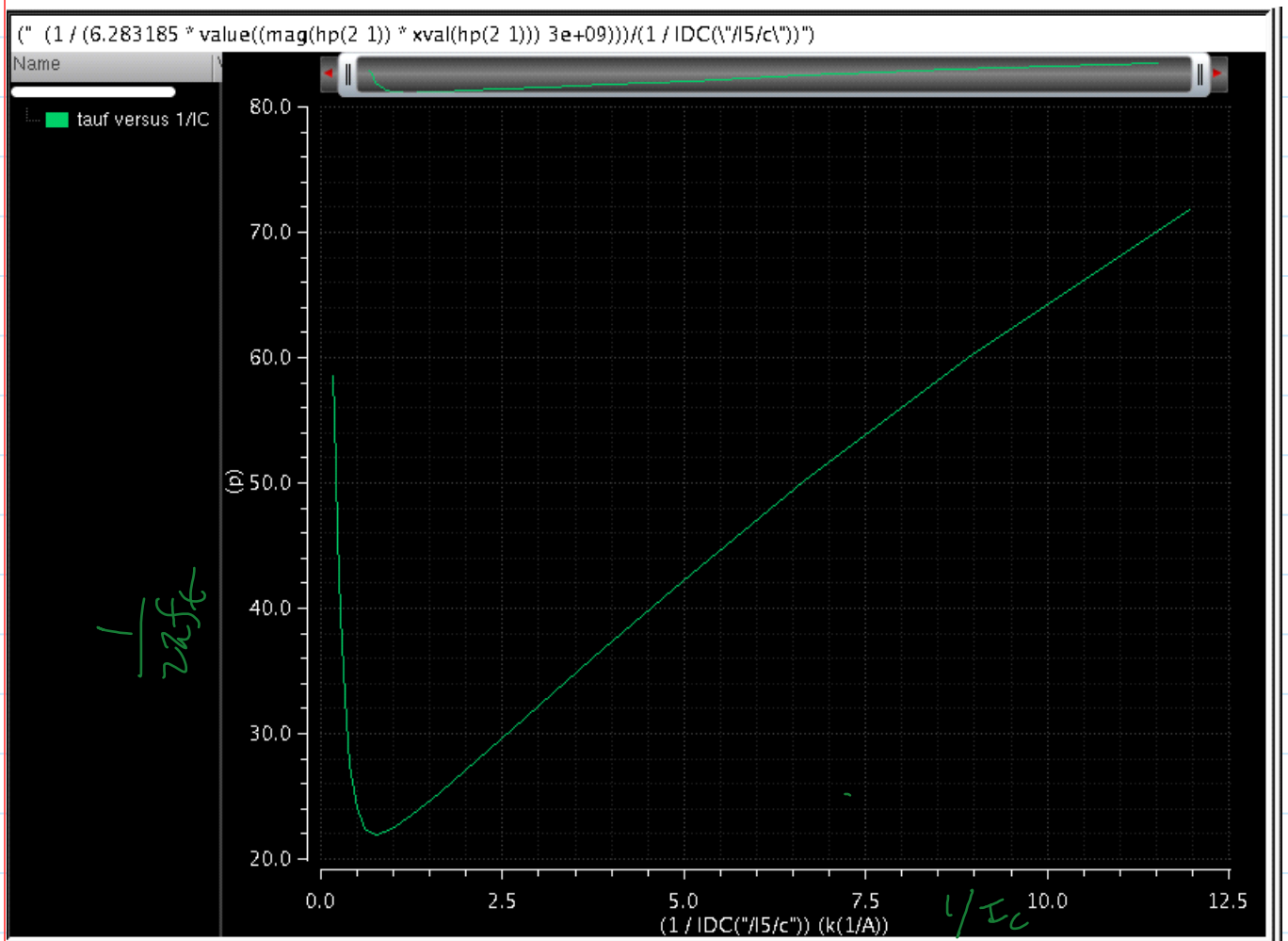
$1/(2\pi \cdot ft) - 1/IC$

Saturday, November 03, 2012 3:23 PM



Tauf versus ϕ_t/IC

Saturday, November 03, 2012 3:25 PM



Base resistance extraction using h_{11} semi circles

Saturday, October 27, 2012 9:41 PM

5.3.3 Base Resistance

Observe that the base resistance r_b does not directly enter the h_{21} expressions, simply because r_b is in series with the ideal transistor (without r_b). In practice, however, r_b limits transistor power gain and noise performance, because it consumes input power and produces thermal noise directly at the base terminal, the worst possible place for the location of a noise source! As a result, minimization of the various components of the base resistance is a major challenge in SiGe HBT structural design, fabrication, and process integration. The base resistance is a key parameter for both process control and circuit design, and deserves careful attention. Unlike many bipolar parameters, base resistance is particularly challenging (and time consuming) to extract in a robust manner.

A popular technique to extract r_b is to use the input impedance with a shorted output, which by definition is equal to h_{11} . An inspection of Figure 5.3 shows

$$h_{11} = Z_{in}|_{v_c=0} = r_b + \frac{1}{g_{be} + j\omega C_i} \quad (5.15)$$

$C_i = C_{be} + C_{bc}$

The real and imaginary parts of h_{11} are

$$\begin{aligned} x &= \Re(h_{11}) = r_b + \frac{g_{be}}{g_{be}^2 + (\omega C_i)^2} \\ y &= \Im(h_{11}) = -\frac{\omega C_i}{g_{be}^2 + (\omega C_i)^2} \end{aligned} \quad (5.16)$$

Using (5.16), one can easily prove that the (x, y) ordered pairs at different frequencies form a semicircle on the complex impedance plane

$$\begin{aligned} (x - x_0)^2 + y^2 &= r^2 \\ x_0 &= r_b + 1/2g_{be} \quad r = 1/2g_{be} \end{aligned} \quad (5.17)$$

The (x, y) impedance point moves clockwise with increasing frequency. The base resistance is then determined to be the high frequency intercept between the fitted impedance semicircle and the real axis, which appears on the left. This is the so-called "circle impedance" base resistance extraction method. In the above analysis, the emitter resistance r_e is neglected for simplicity, but it can be shown that the extracted r_b is actually the sum of the transistor r_b and r_e . Figure 5.7 shows an example of such an r_b extraction for a typical first generation SiGe HBT with an effective emitter area of $0.5 \times 40 \mu\text{m}^2$. The h_{11} data was measured from 0.5 to 15 GHz in order to make a meaningful fit to a semicircle. Choosing a proper measurement frequency range is important in reliable r_b extraction, as can be seen from Figure 5.7. In this case, had we used a frequency range of 15–50 GHz, the data would have formed only a tiny portion of the semicircle, making fitting and r_b extraction much more difficult. Deviation from circular behavior is often observed at frequencies close to f_T , and those data should be discarded in the r_b extraction. Given the I_C dependence of f_T , the frequency range over which r_b extraction is made can be varied with I_C in order to obtain an accurate I_C dependence of r_b , which is needed in compact modeling.

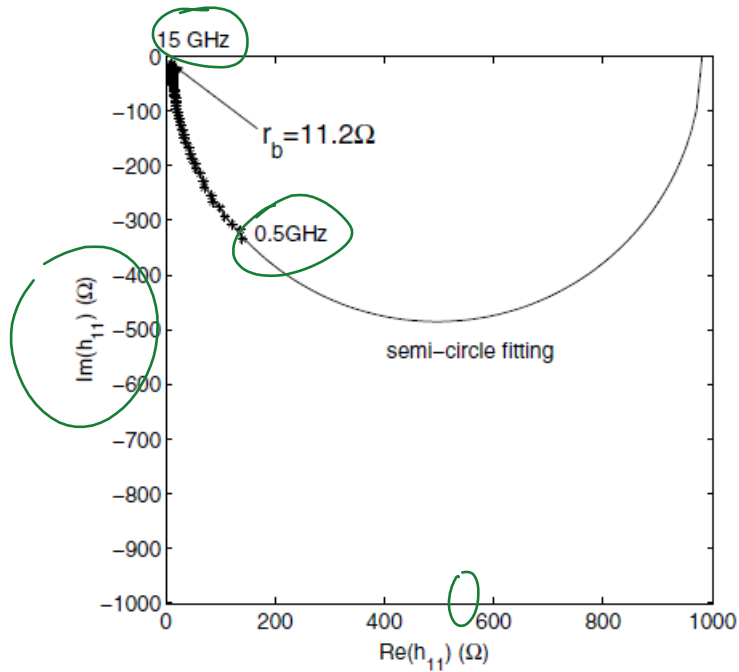
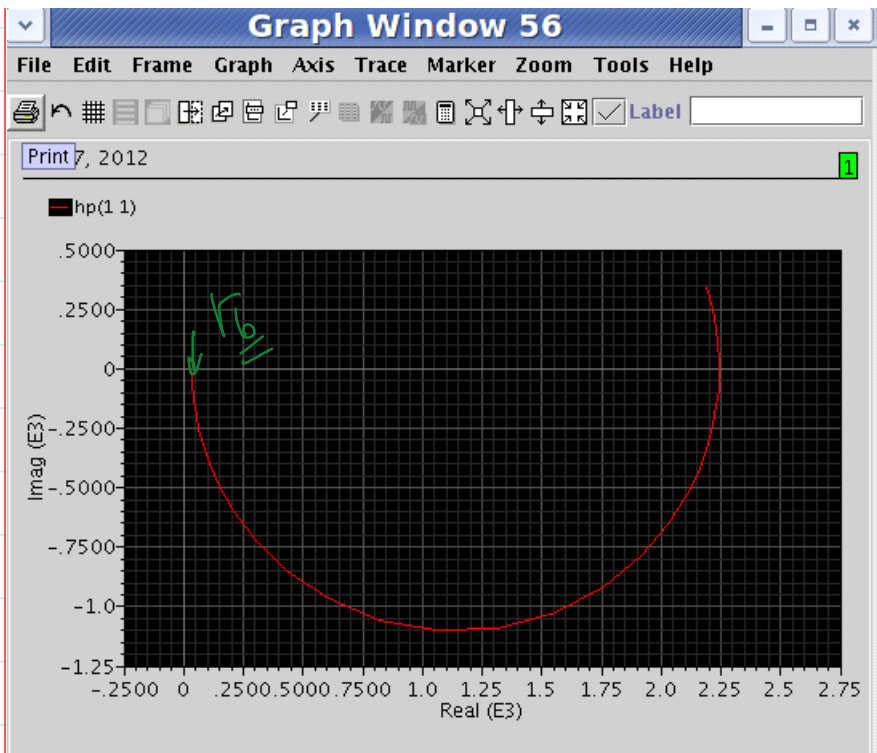


Figure 5.7 Extraction of r_b using the circle impedance method. The measured h_{11} data forms a semicircle. The frequency increases clockwise.

Example:

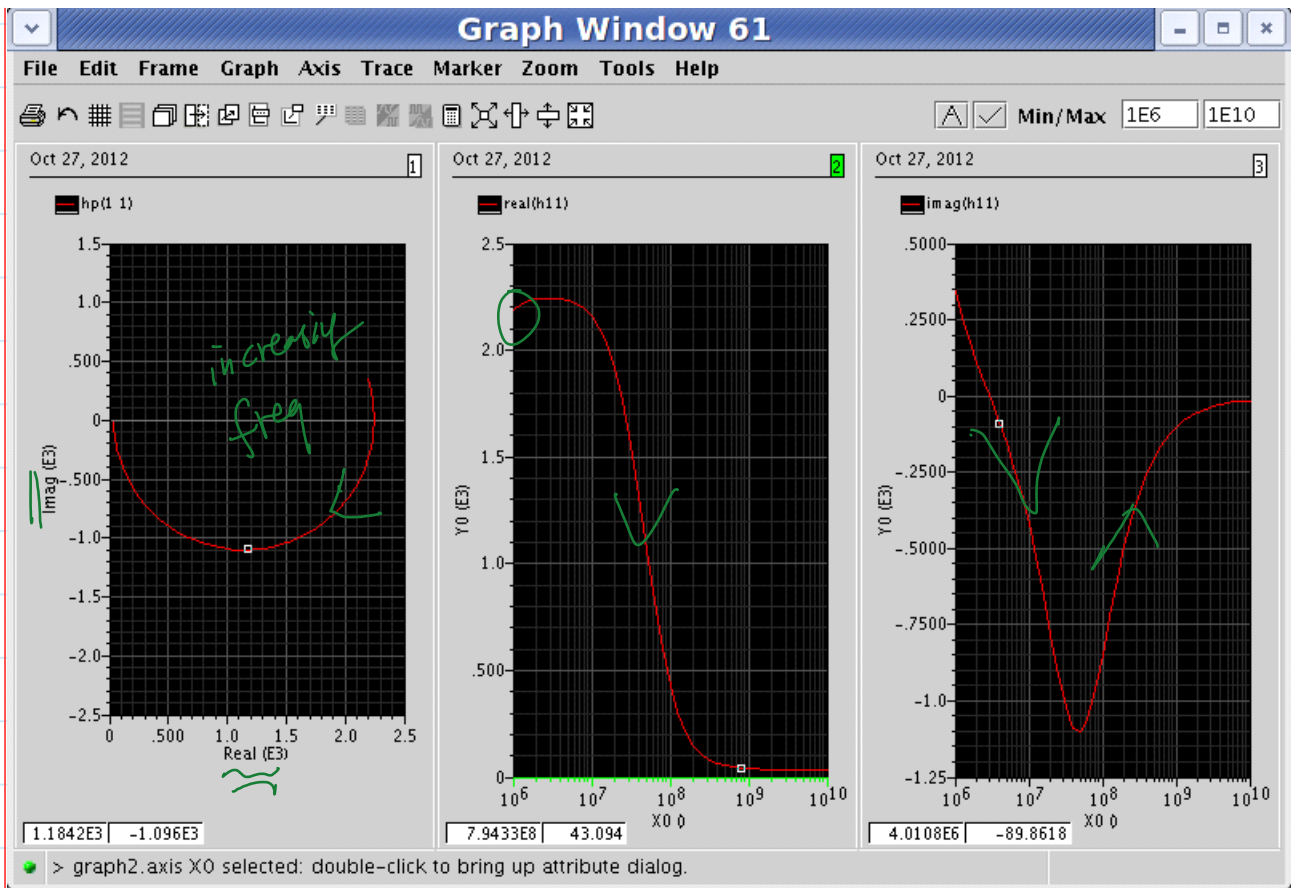
Simulate the h_{11} behavior for currents around peak ft. of the bjt504 device you have been using.

Plot out Imag-Real h_{11} as shown below.



Of course with all the parasitics, the simulated h11 does not precisely follow our first order derivation, but it shows the overall behavior nicely. It is indeed close to a semi circle.

In this simulation, frequency is from 1MHz to 10GHz.



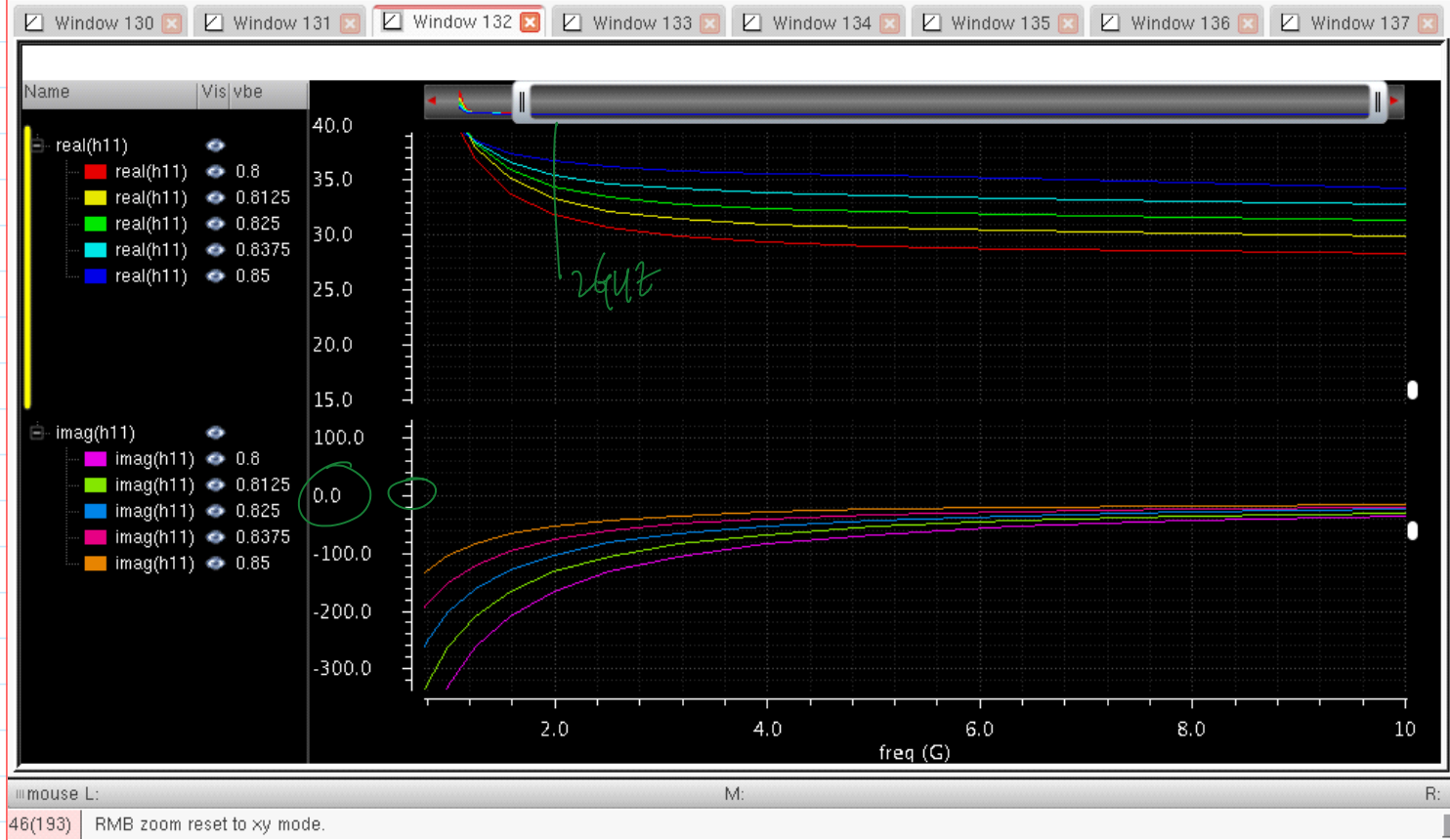
In measurement, it is typical to go from 2GHz to 10GHz (or 26GHz, 40GHz).

Can you explain why at several GHz, real(h11) is nearly a constant, while imag(h11) is very small?

h11

Tuesday, October 30, 2012 8:54 AM

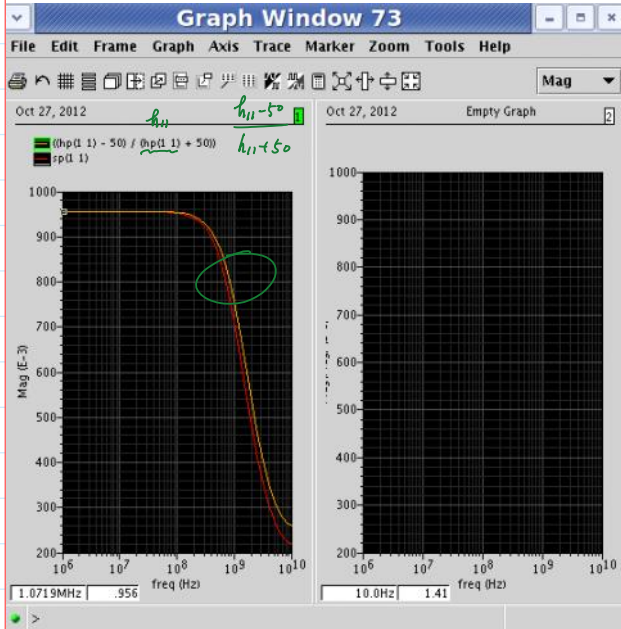
The real part of zs11 first. Now we add more VBE's.
Zoom in plots to show higher freq behavior of real
& imag h11



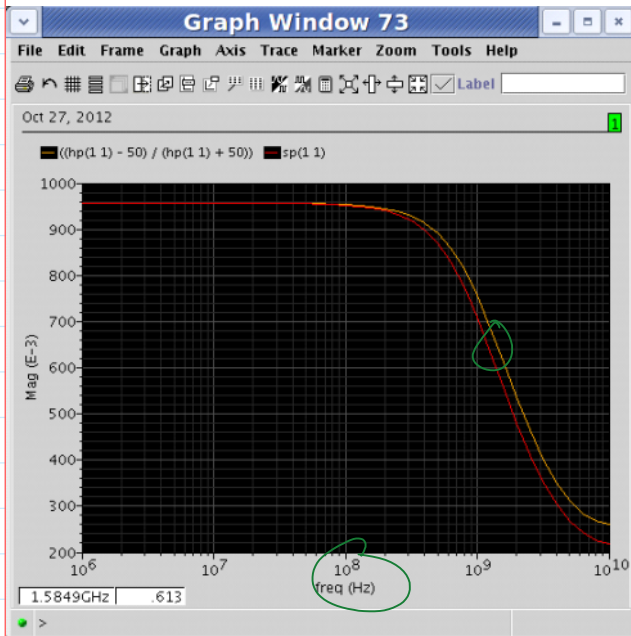
H11 and s11 comparison

Saturday, October 27, 2012 10:28 PM

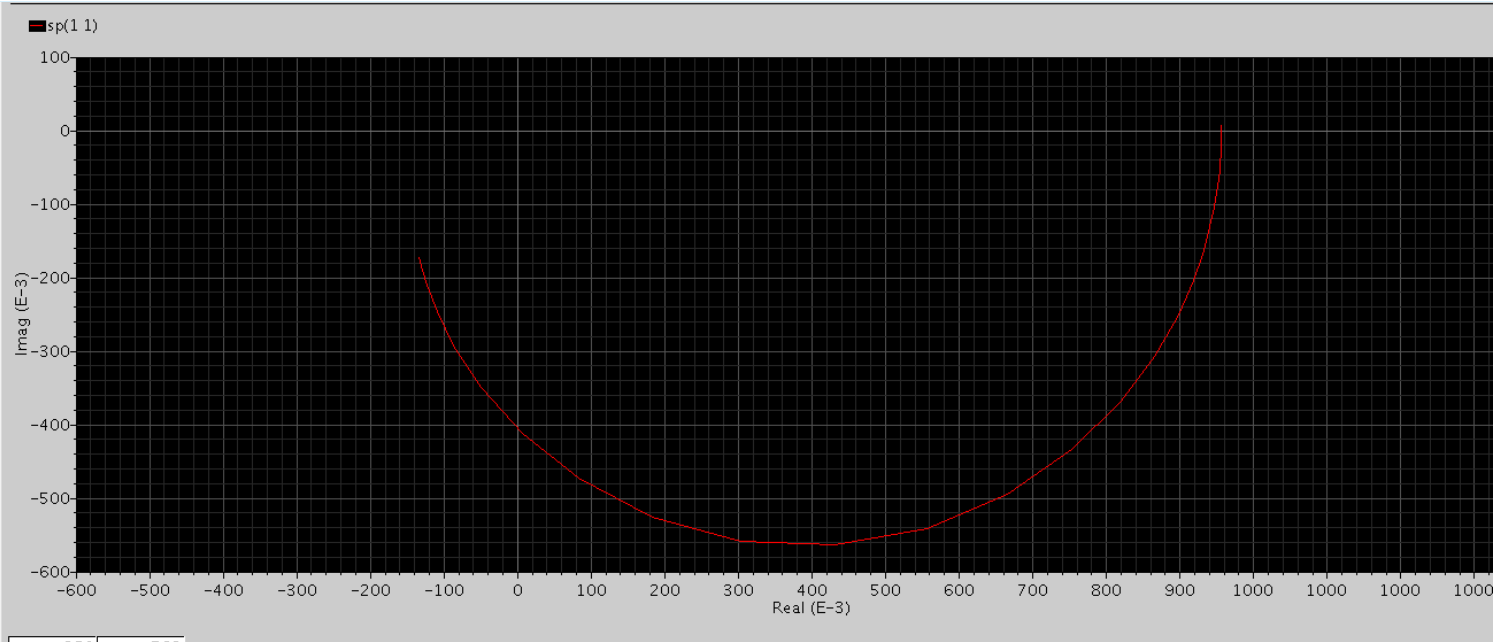
Let us compare s11 and h11 by converting h11 to a reflection coefficient



Some papers use s11 instead of using h11 to extract rb - this can cause some errors.



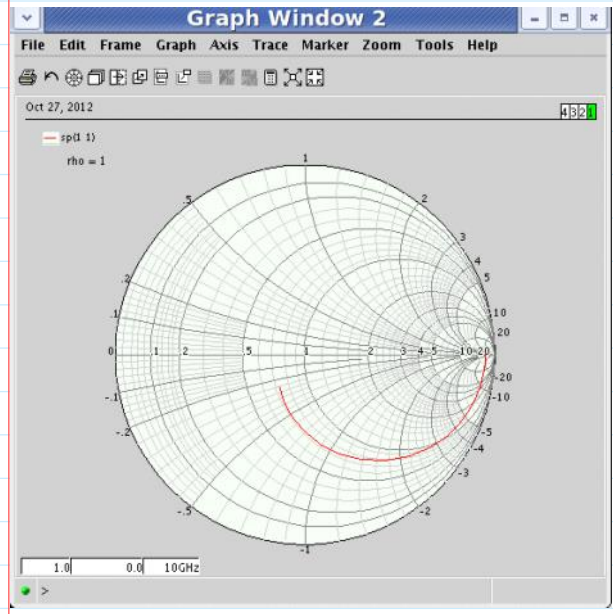
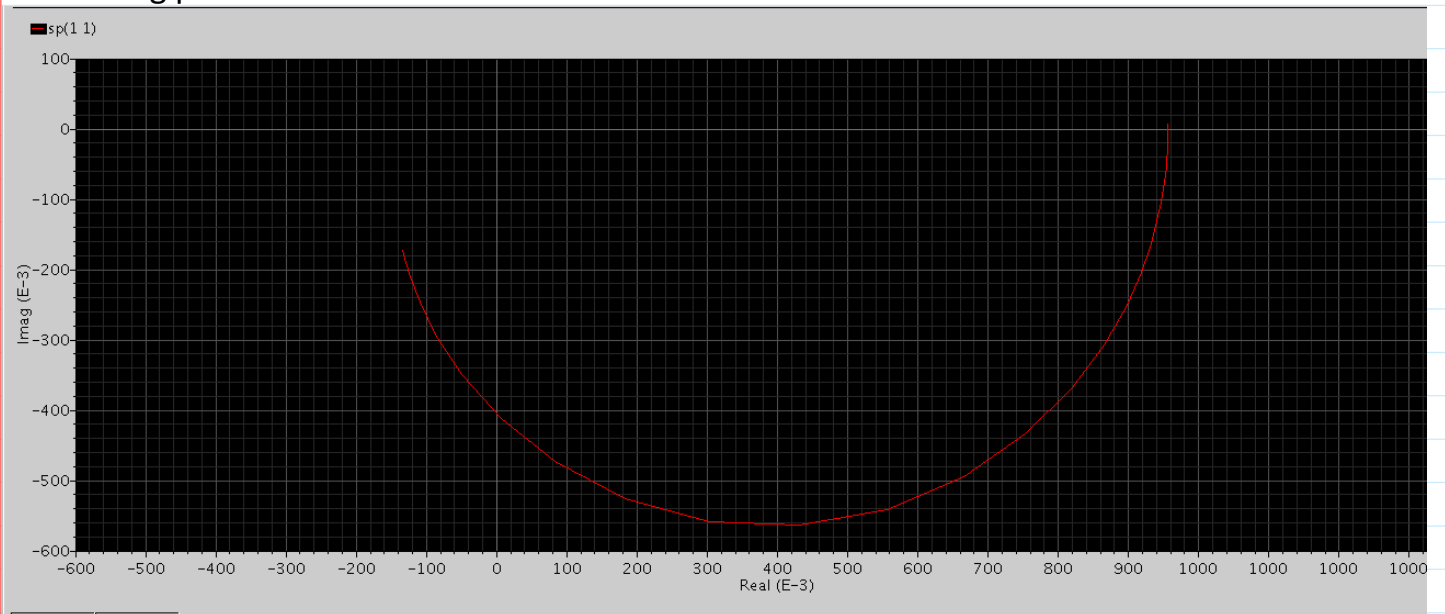
S11, real-imag plot:



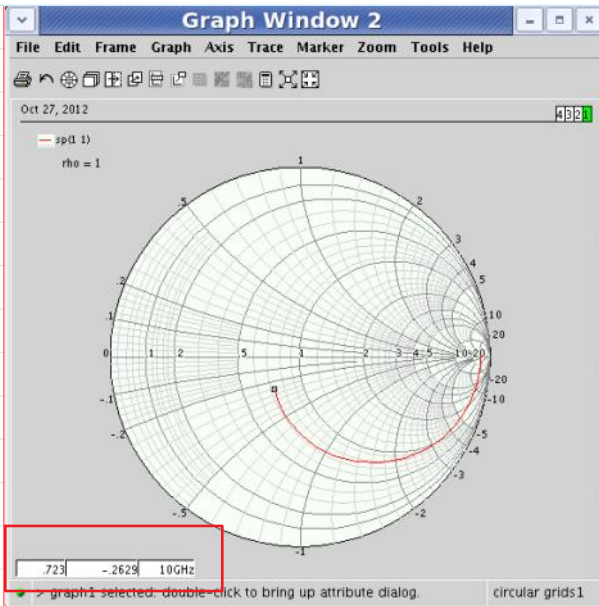
S11 behavior - this also helps you understand smith chart

Saturday, October 27, 2012 11:08 PM

Real - Imag plot:

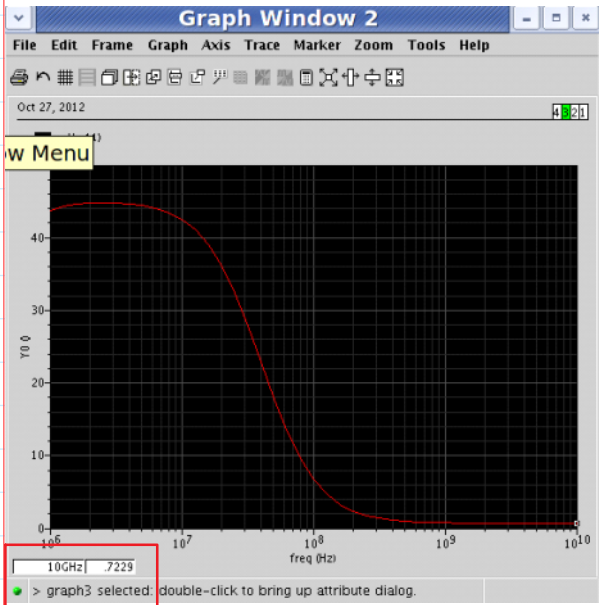


At 10GHz:

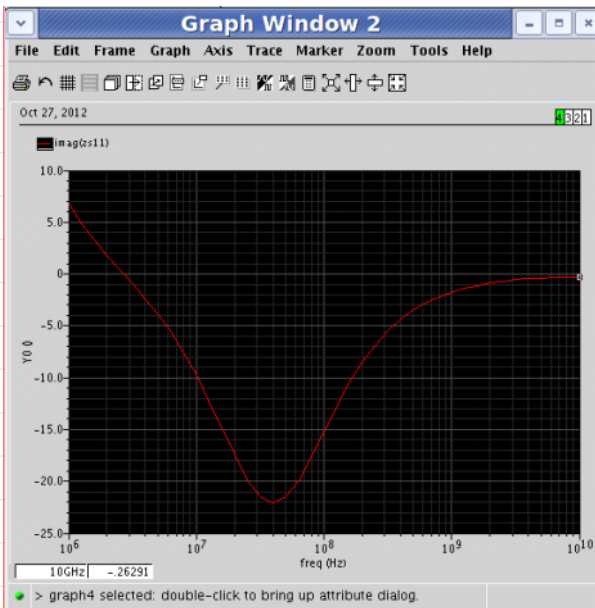


Note: $r=0.723$, $x=-0.2629$

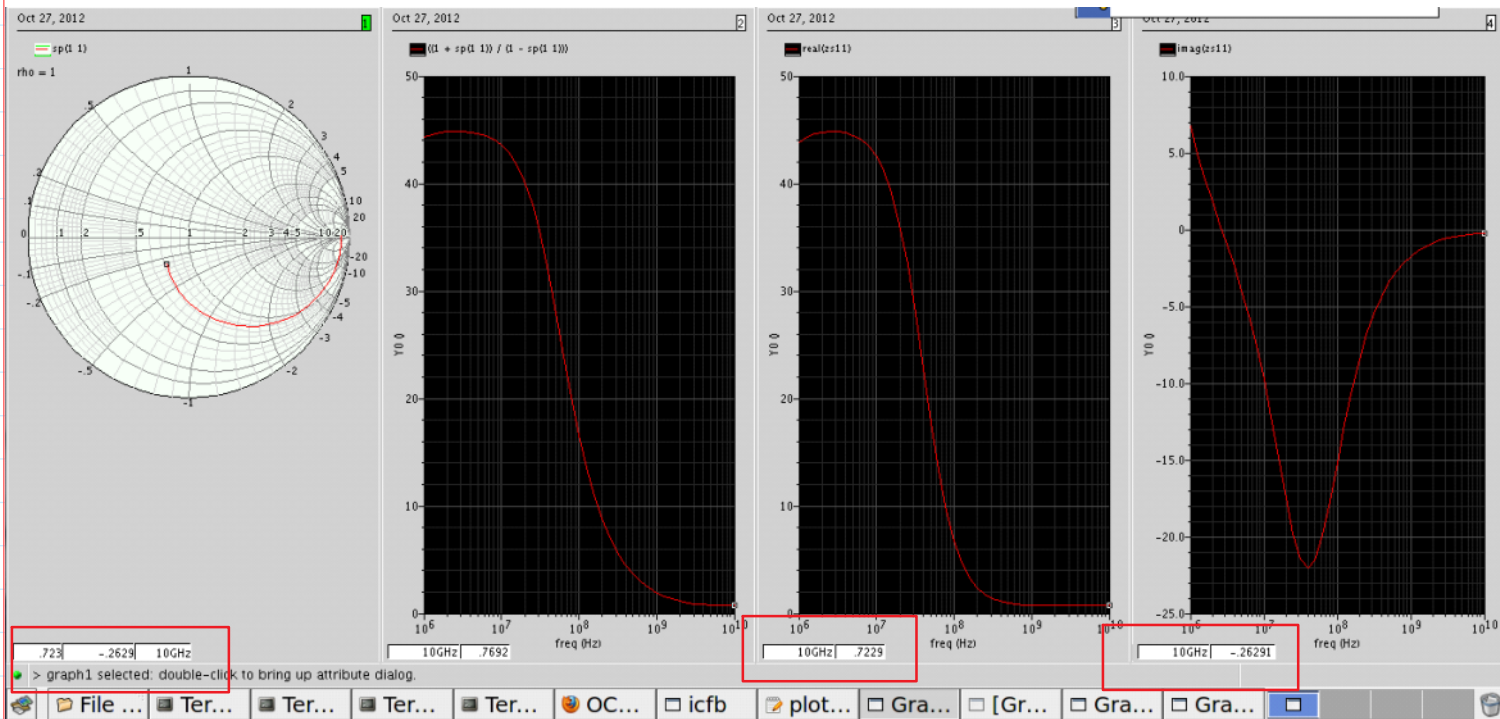
We can calculate z_{s11} by converting s_{11} to normalized impedance, $z_{s11} = (1+s_{11})/(1-s_{11})$, then looking at real and imaginary part. Let us look at the real part of z_{s11} first. I have set cursor to 10GHz, see lower left corner:



Now the imaginary part, again, note the lower left corner display of $\text{imag}(z_{s11})$:

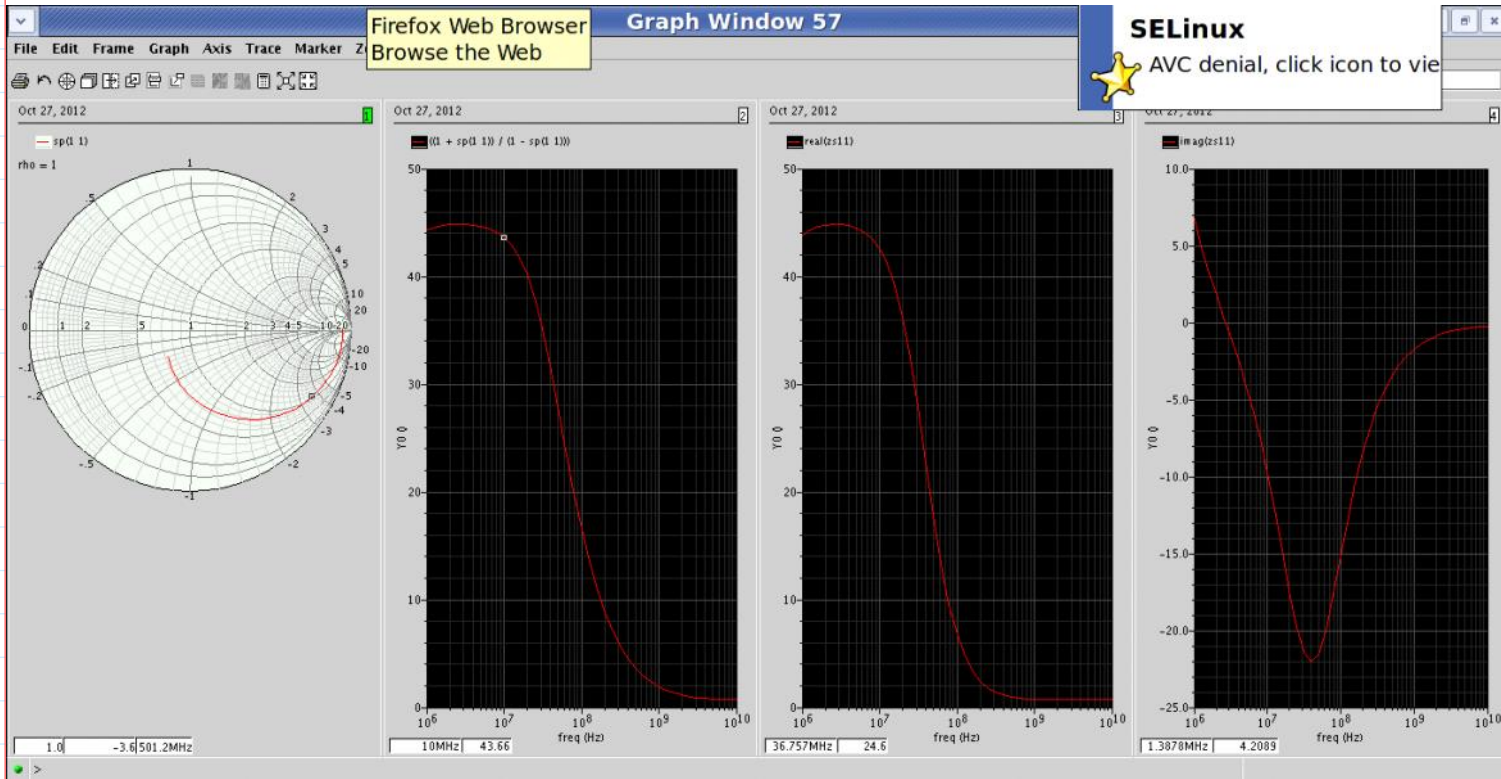


This is indeed what is displayed as "x" value when we view smith chart of s11.



Frequency response of s11 and h11

Saturday, October 27, 2012 9:22 PM



Move cursor and demonstrate the meaning of s_{11} on smith chart using $z_{s11} = (1+s_{11})/(1-s_{11})$

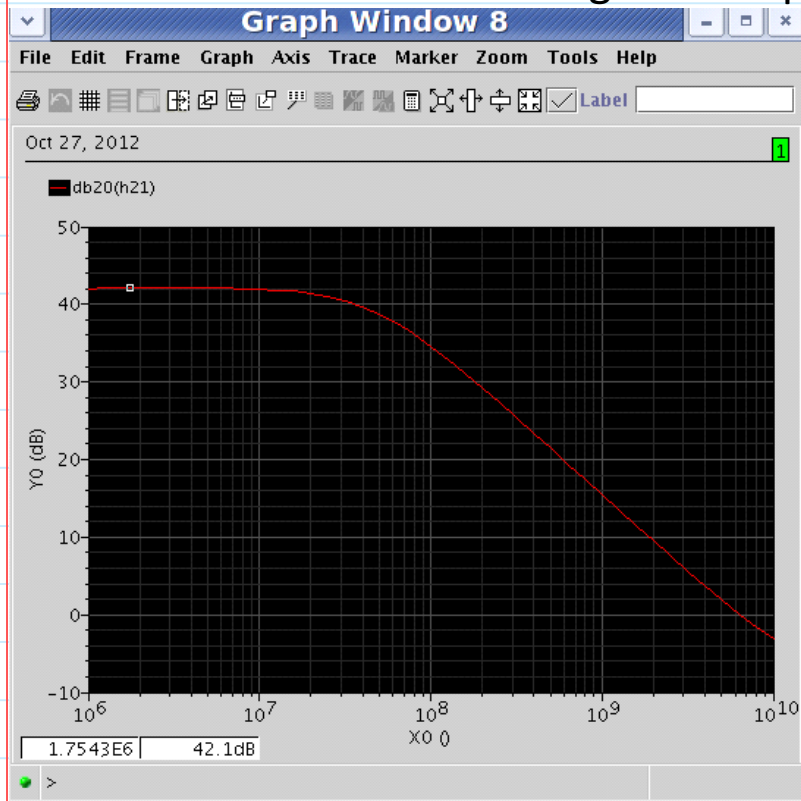
Explain why above 2GHz, real part of z_{s11} - essentially the "r" value of s_{11} on smith chart is nearly constant in transistors.

h21

Saturday, October 27, 2012 11:37 PM

Use db20() function.

Also often it is best to use "log" for freq axis.

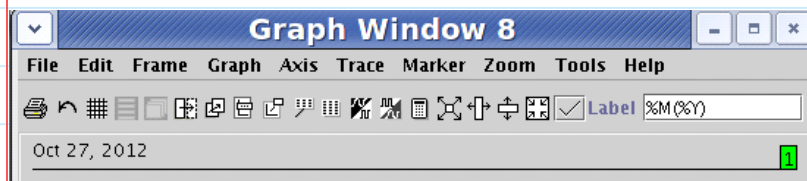


Low-frequency, h21 is flat.

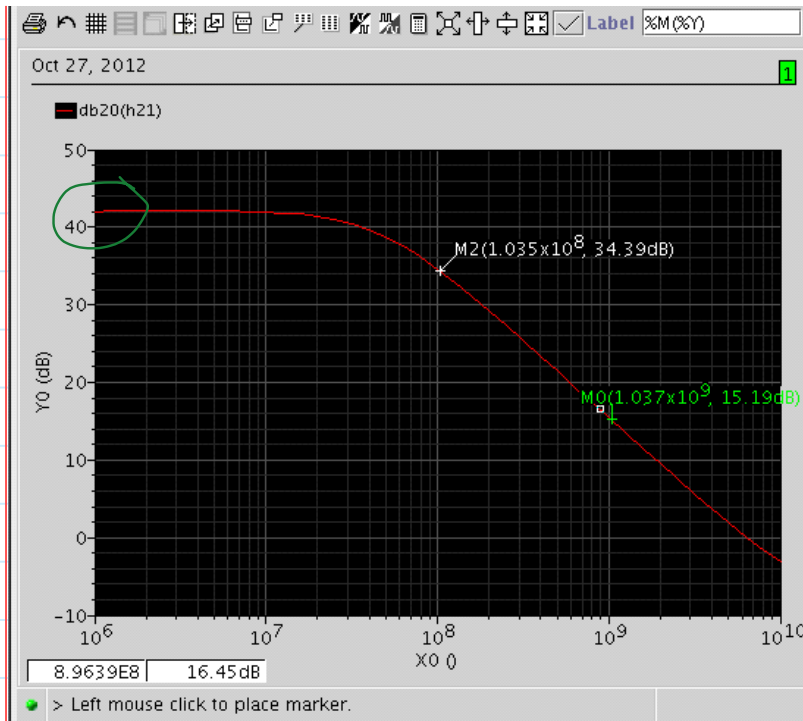
High-frequency, it drops.

The slope is -20dB per decade increase of frequency.

In this region, $h21 * f = \text{constant}$, or ft.

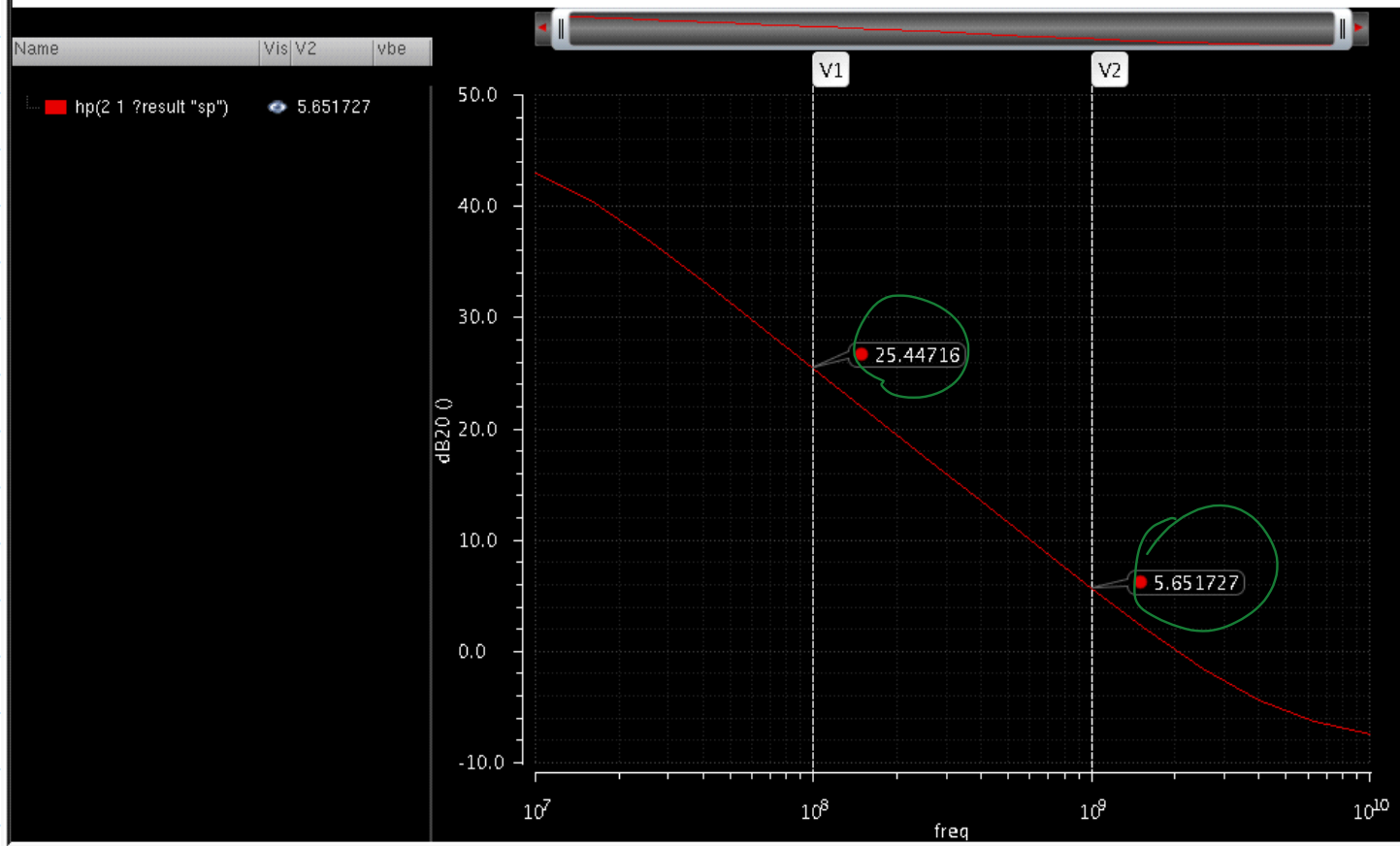


0 1



$h_{21} / \omega f \rightarrow \beta$
 ≈ 100 or more

In IC615, see the marker I made at 100MHz and 1GHz, and observe how the y-value differs by about 20dB for 1 decade increase in frequency



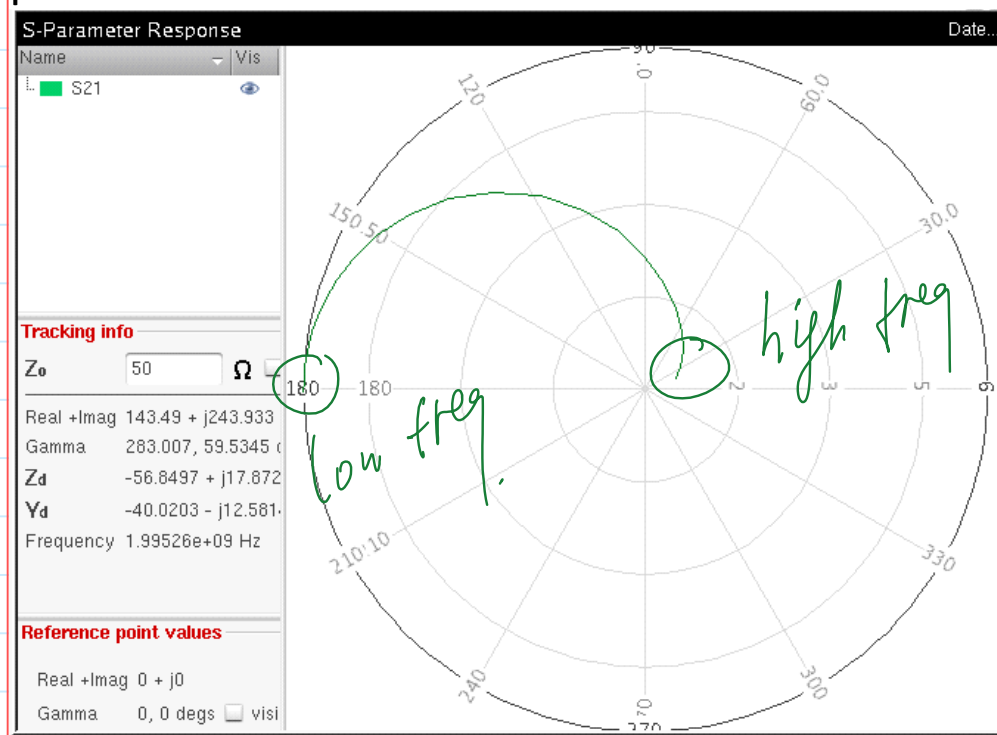
S21, polar, mag, and db20

Sunday, October 28, 2012 5:13 PM

Recall that s_{21} itself is a voltage gain, so db_{20} is power gain - more accurately, power delivered to a z_0 load divided by power available from a z_0 source.

Polar showing magnitude and phase:

It is typical to plot the original s_{21} number (without taking db) on polar plot that shows magnitude and phase:

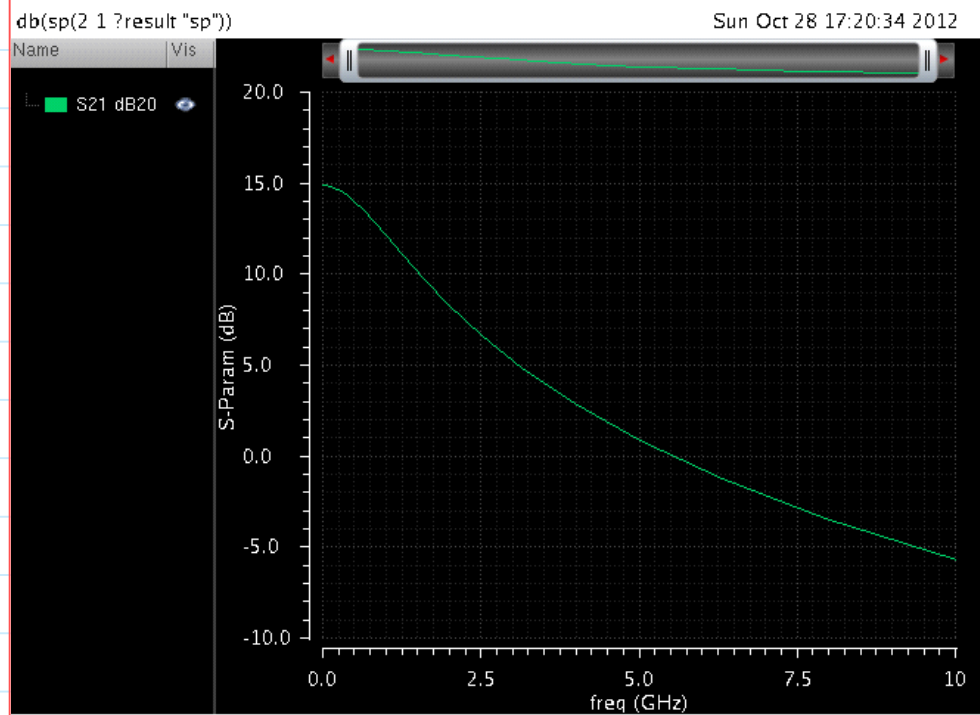


Note that at very low frequency, the phase is 180 degrees, as the amplifier is inverting (180 degree phase diff)

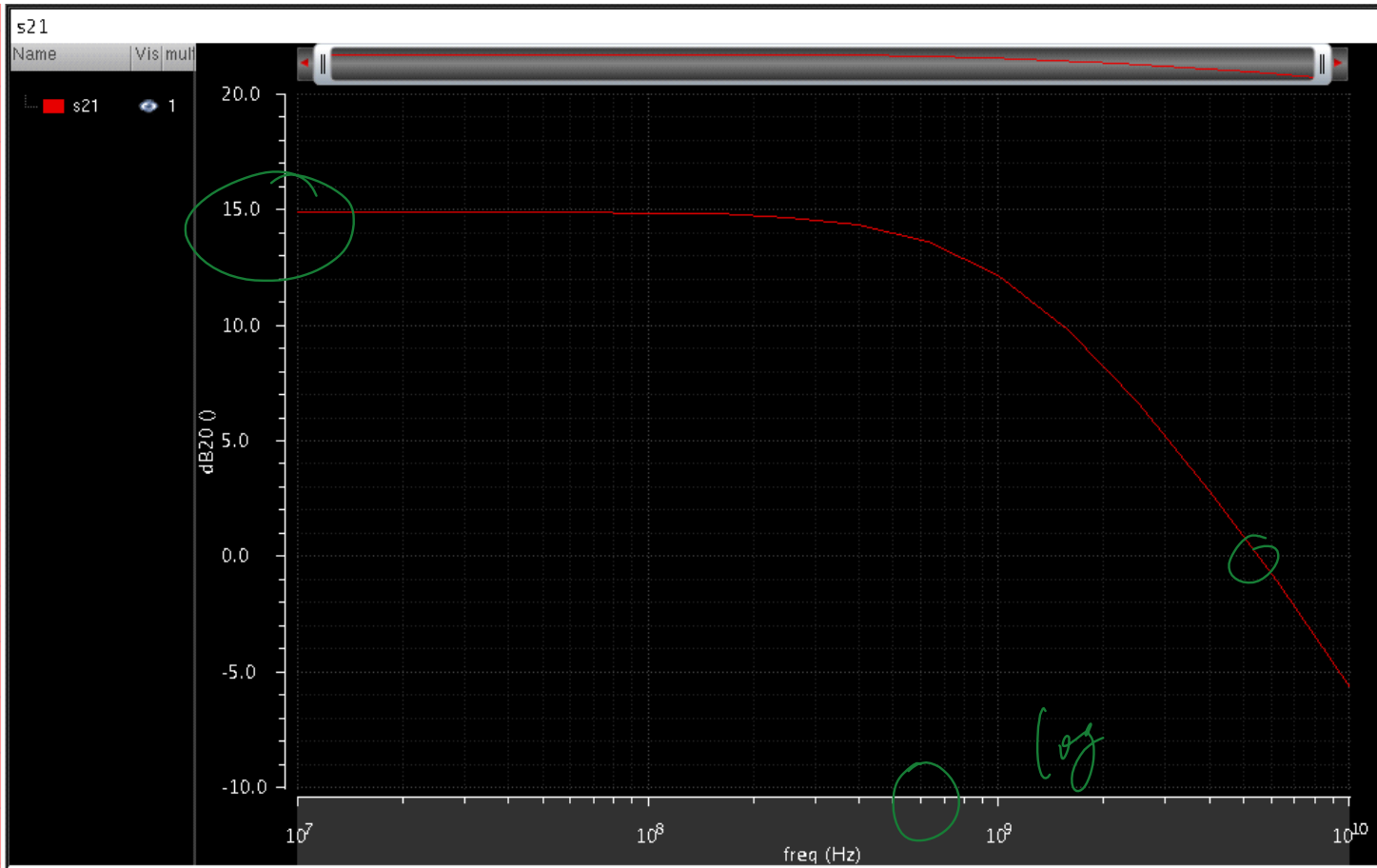
Db20

Use `db20(s21)` to show its db value.

To just show the magnitude, we can take `db20`, and use log scale for frequency axis:



Db20 with log freq axis



Like h_{21} , there is 20db/decade slope at high frequency.

Comparing h21 and s21

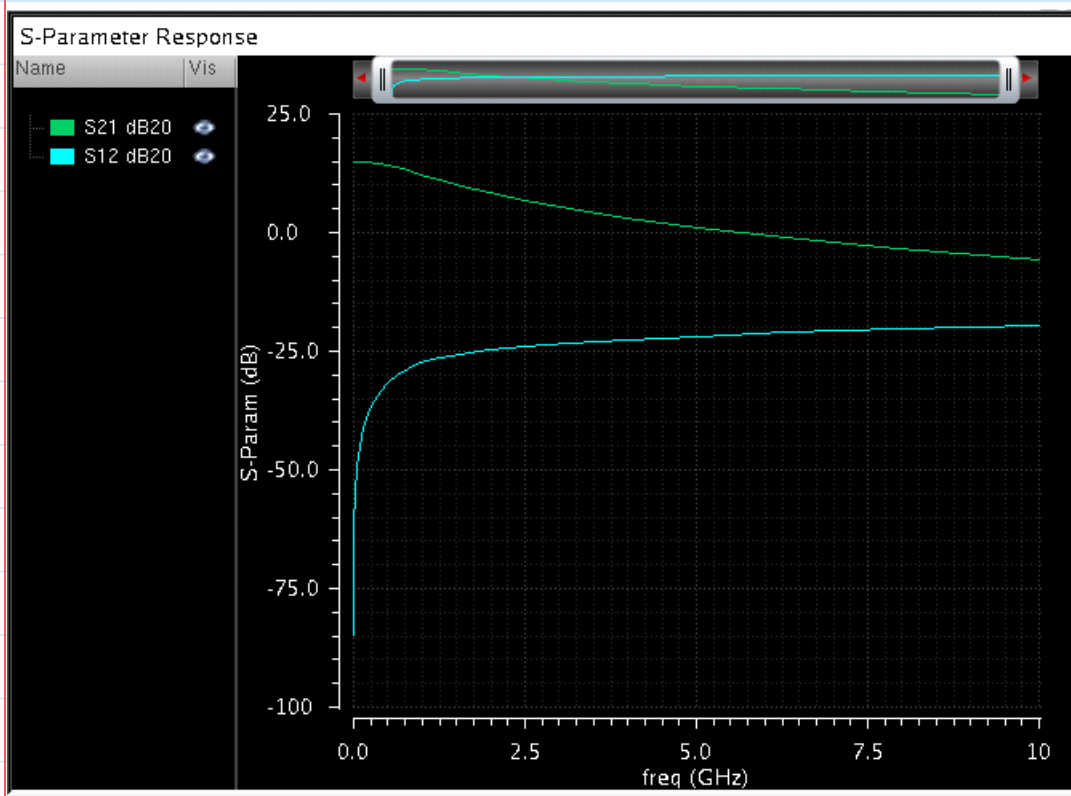
Tuesday, November 06, 2012 10:24 AM

s12

Sunday, October 28, 2012 5:23 PM

S12 is reverse transmission coefficient - for transistors in forward mode, this should be small.

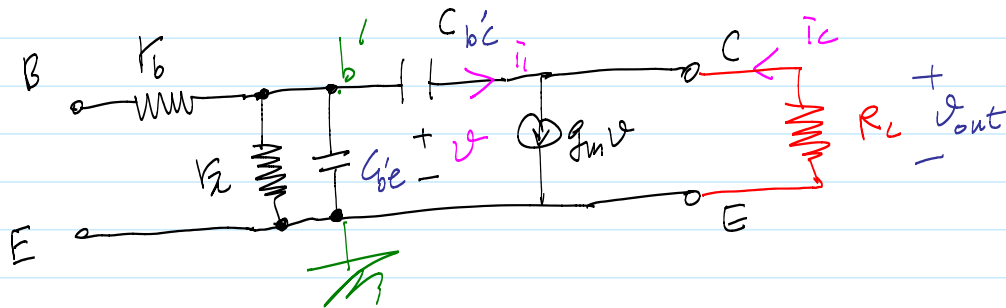
To show this we plot out s21 and s12 in db20 together:



Transistor s-parameter analytical calculation and intuitive understanding

Sunday, November 04, 2012 2:31 PM

Consider bipolar transistor (MOS is simpler)



$$i_c + i_1 = g_m v \Rightarrow i_c = g_m v - i_1$$

$$\text{Typically } i_1 \ll g_m v, \quad i_c \approx g_m v$$

$$v_{out} = -i_c R_c = -g_m R_c v$$

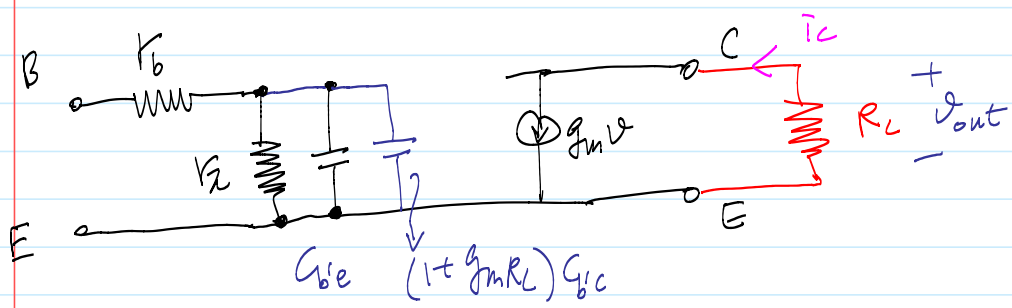
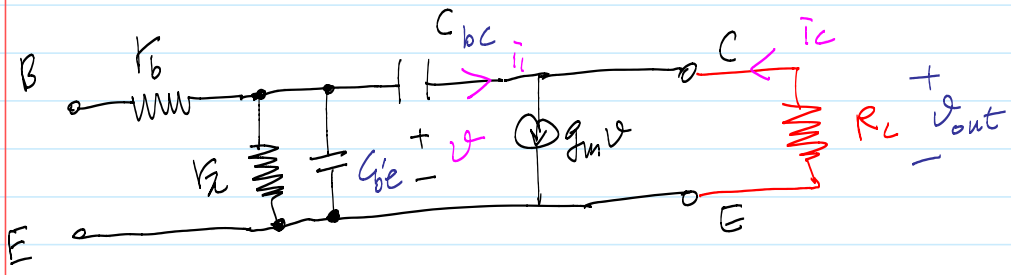
i_1 : Current thru $C_{b'c}$ is

$$\begin{aligned} i_1 &= j\omega C_{b'c} v_{b'c} = j\omega C_{b'c} (v_{b'} - v_c) = j\omega C_{b'c} (v - v_{out}) \\ &= j\omega C_{b'c} (1 + g_m R_c) v \end{aligned}$$

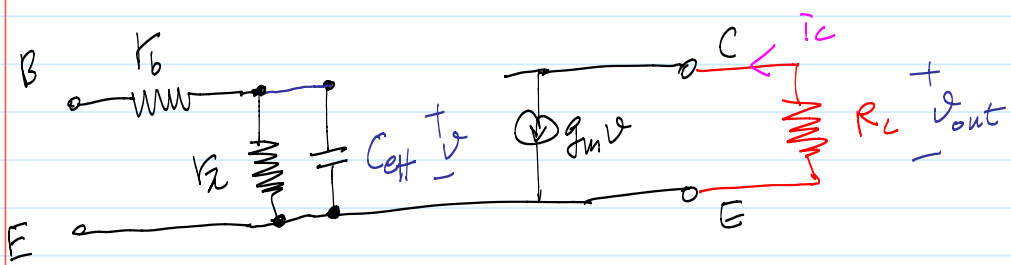
The input admittance due to $C_{b'c}$ is thus

$$\frac{i_1}{v_1} = j\omega C_{b'c} (1 + g_m R_c)$$

This is equivalent to replacing $C_{b'c}$ with $(1 + g_m R_c) C_{b'c}$ that is parallel to $C_{b'e}$.



That is :



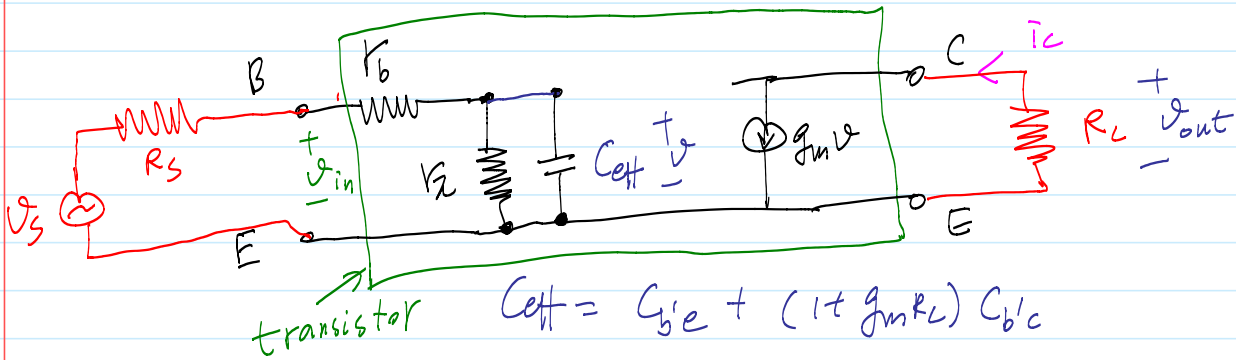
$$C_{eff} = C_{be} + (1 + g_m R_L) C_{bc}$$

You might have heard "Miller effect" before. This is it.

C_{bc} is a "Miller capacitance".

The key here is that we assumed $i_i \ll g_m v$. This works for not too small a current (so g_m is large).

if g_m is too small, transistor is not amplifying.



let us add a R_s source, find $\frac{V_{out}}{V_s}$

$$V = V_s \frac{r_{\pi} \parallel C_{eff}}{r_{\pi} \parallel C_{eff} + R_s + r_b} \quad \left. \vphantom{\frac{V}{V_s}} \right\} \Rightarrow$$

$$V_{out} = -g_m R_L V$$

$$\frac{V_{out}}{V_s} = -g_m R_L \frac{r_{\pi} \parallel C_{eff}}{r_{\pi} \parallel C_{eff} + R_s + r_b}$$

$$= -g_m R_L \frac{1}{1 + (R_s + r_b) \frac{1}{r_{\pi} \parallel C_{eff}}}$$

$$= -g_m R_L \frac{1}{1 + (R_s + r_b) \left(\frac{1}{r_{\pi}} + j\omega C_{eff} \right)}$$

$$= -g_m R_L \frac{r_{\pi}}{r_{\pi} + (R_s + r_b) (1 + j\omega C_{eff} r_{\pi})}$$

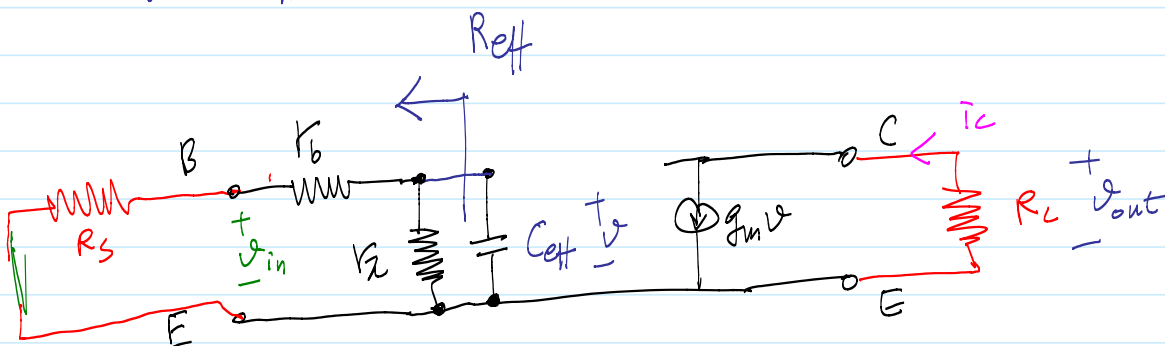
$$= -g_m R_L \frac{r_o}{r_o + (R_s + r_b) + j\omega C_{eff} r_o (R_s + r_b)}$$

$$= -g_m R_L \frac{r_o}{(r_o + R_s + r_b) \left(1 + j\omega C_{eff} \frac{r_o (R_s + r_b)}{r_o + (R_s + r_b)} \right)}$$

let us define $R_{eff} \triangleq \frac{(r_b + R_s) r_o}{r_o + (R_s + r_b)}$

$$\frac{V_{out}}{V_s} = -g_m R_L \frac{r_o}{r_o + R_s + r_b} \frac{1}{1 + j\omega C_{eff} R_{eff}}$$

note this R_{eff} is simply the effective Thévenin resistance seen by C_{eff} !



$$\frac{V_{out}}{V_s} = -g_m R_L \frac{R_2}{R_2 + R_S + R_b}$$

$$\frac{1}{1 + j\omega C_{eff} R_{eff}}$$

this is low frequency voltage gain

this is frequency response roll-off with $f \uparrow$

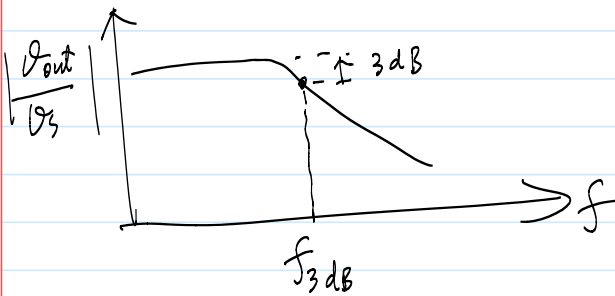
When $\omega C_{eff} R_{eff} = 1$

$$\frac{V_{out}}{V_s} = \frac{V_{out}}{V_s} \Big|_{f=0} \cdot \frac{1}{1 + j1}$$

$$\left| \frac{V_{out}}{V_s} \right| = \frac{1}{\sqrt{2}} \left| \frac{V_{out}}{V_s} \right|_{f=0}$$

voltage gain drops by $\frac{1}{\sqrt{2}}$ x

or 3 dB



you might say -3 dB change
or 3 dB drop

So $\omega_{3dB} C_{eff} R_{eff} = 1$

$$f_{3dB} = \frac{1}{2\pi C_{eff} R_{eff}}$$

Recall: $R_{eff} = (R_b + R_S) \parallel R_2$

Recall: $R_{eff} \triangleq \frac{(r_b + R_s) r_\pi}{r_\pi + (R_s + r_b)}$ \Rightarrow

$$C_{eff} = C_{b'e} + (1 + g_m R_L) C_{b'c}$$

$$f_{3dB} = \frac{r_\pi + r_b + R_s}{2\pi \left[(1 + g_m R_L) C_{b'c} + C_{b'e} \right] (r_b + R_s) r_\pi}$$

$$\frac{V_{out}}{V_s} = -g_m R_L \frac{r_\pi}{r_\pi + R_s + r_b} \frac{1}{1 + j\omega C_{eff} R_{eff}}$$

To find s-parameters,

$$V_s = 2V, \quad R_L = 50\Omega, \quad R_s = 50\Omega$$

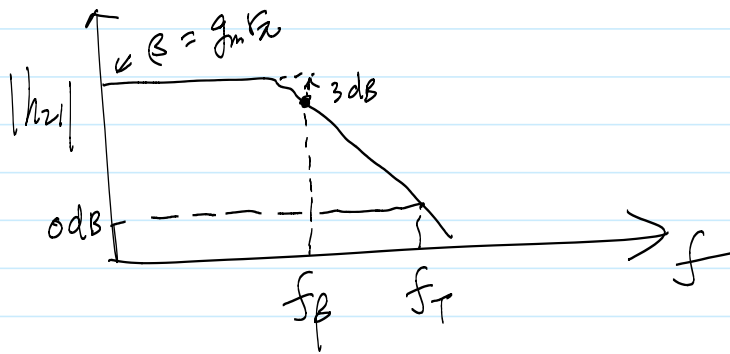
$$S_{21} = -2 g_m \cdot 50 \frac{r_\pi}{r_\pi + 50 + r_b} \frac{1}{1 + j\omega C_{eff} \frac{(r_b + 50) r_\pi}{r_b + 50 + r_\pi}}$$

Example of S_{21} for RF Transistors

Let us examine some typical values, get a good feel

of S_{21} .

We are all familiar with h_{21} which we define f_T with.



$$h_{21} = \beta \frac{1}{1 + j \frac{f}{f_T}}$$

$$f_T = \frac{g_m}{2R(G'_{be} + G'_{bc})}$$

Numerical example of a medium current RF transistor

Sunday, November 04, 2012 2:43 PM

Medium Current RF Transistor, $I_c = 10 \text{ mA}$

say $f_T = 5 \text{ GHz}$, $\beta = 100$, $C_{b'c} = 1 \text{ pF}$, $r_b = 10 \Omega$.

$$g_m = \frac{I_c}{V_t} = \frac{10 \text{ mA}}{25 \text{ mV}} \quad \frac{1}{g_m} = 2.5 \Omega \quad (\text{also called } r_e)$$

or $r_e = 2.5 \Omega$

$$r_x = \frac{I_B}{V_t} \quad \text{or}$$

$$= \beta \cdot \frac{1}{g_m} = 250 \Omega$$

$$\text{From } f_t = \frac{g_m}{2\pi(C_{b'e} + C_{b'c})}, \quad C_{b'e} + C_{b'c} = \frac{g_m}{2\pi f_t}$$
$$= \frac{\frac{1}{2.5}}{2\pi \times 5 \times 10^9} \text{ F} = 12.6 \text{ pF}$$

As $C_{b'c} = 1 \text{ pF}$,

$$C_{b'e} = 12.6 - 1 = 11.6 \text{ pF}$$

For s-para,

$$C_{\text{eff}} = (1 + g_m \cdot \overset{50 \Omega}{z_o}) C_{b'c} + C_{b'e}$$

$$= \left(1 + \frac{50}{2.5}\right) \cdot 1 \text{ pF} + 11.6 \text{ pF}$$

$$= 32.6 \text{ pF} \quad \uparrow$$

pay attention to this term,

your $C_{eff} \Rightarrow C_{b'e} + G_{b'c}$ (what is indicated by f_t)

even if $G_{b'c}$ is only 1 pF .

$$S_{21} = -2 g_m \cdot 50 \frac{r_x}{r_x + 50 + r_b} \frac{1}{1 + j\omega C_{eff} \frac{(r_b + 50)r_x}{r_b + 50 + r_x}}$$

at low frequency

$$S_{21} \Big|_{f \rightarrow 0} = -2 \frac{1}{2.5} 50 \frac{250}{250 + 50 + 10} = -32.25$$

at high frequency it will drop by 20 dB/decade

like in $|h_{21}|$, as

$$S_{21} = S_{21} \Big|_{f \rightarrow 0} \cdot \frac{1}{1 + j\omega \cdot C_{eff} R_{eff}}$$

you can define S_{21} cut off if you want

3 dB frequency of S_{21}

$$f_{3dB} = \frac{\cancel{250\Omega} + \cancel{10\Omega} + \cancel{R_b} + R_s \quad 50\Omega}{2\pi \left[\underbrace{(1 + g_m R_L) C_{bc} + C_{be}}_{32.6 \times 10^{-12} \text{ F}} \right] \left(\underbrace{R_b}_{10\Omega} + \underbrace{R_s}_{50\Omega} \right) \pi \quad \leftarrow 250\Omega$$

$$= 10 \text{ MHz}$$

Numerical Example of Lower Current RF Transistor

Sunday, November 04, 2012 2:45 PM

Lower current RF Transistor - 1mA

$$f_t = 5 \text{ GHz}, \quad \beta = 100, \quad I_c = 1 \text{ mA}, \quad C_{b'c} = 0.2 \text{ pF}$$

$$\text{Now } \frac{1}{g_m} = \frac{V_t}{I_c} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \Omega$$

$r_b \approx 100 \Omega$
For typical 1mA device

$$r_x = 2500 \Omega$$

Following similar procedure,

$$C_{b'e} + C_{b'c} = \frac{g_m}{2\pi f_t} = 1.26 \text{ pF}$$

$$C_{b'e} = 1.26 \text{ pF} - 0.2 \text{ pF} = 1.06 \text{ pF}$$

$$C_{\text{eff}} = (1 + g_m \cdot 50) C_{b'e} + C_{b'e}$$

$$= (1 + 2) \cdot 0.2 \text{ pF} + 1.06 \text{ pF} = 1.66 \text{ pF}$$

$$f_{3\text{dB}} = \frac{r_x + r_b + R_s}{2\pi C_{\text{eff}} (r_b + R_s) r_x}$$

$$= \frac{2500 + 100 + 50}{2\pi \times 1.66 \times 10^{-12} \times (100 + 50) \cdot 2500}$$

$$= 678 \text{ MHz}$$

$$S_{21} \Big|_{f \rightarrow 0} = -2 \cdot g_m \cdot 50 \frac{r_{\pi}}{r_{\pi} + R_b + 50}$$

$$= -2 \cdot \frac{50}{25} \frac{2500}{2500 + 100 + 50} = -3.77$$

BFG25A is such a transistor.

A significant amount of information can be obtained

from f_T , β , G_{bc} (cb capacitance), R_b , and I_c .

$$\text{Low frequency } S_{21} = -2 g_m R_c \frac{r_{\pi}}{r_{\pi} + R_b + 50} \approx -2 g_m R_c \approx -2 \frac{z_o}{r_e}$$

$$f_{3dB} = \frac{r_{\pi} + R_b + 50}{2\pi C_{eff} (R_b + 50) r_{\pi}}, \quad C_{eff} = \left(1 + \frac{z_o}{r_e}\right) G_{bc} + G_{be}$$

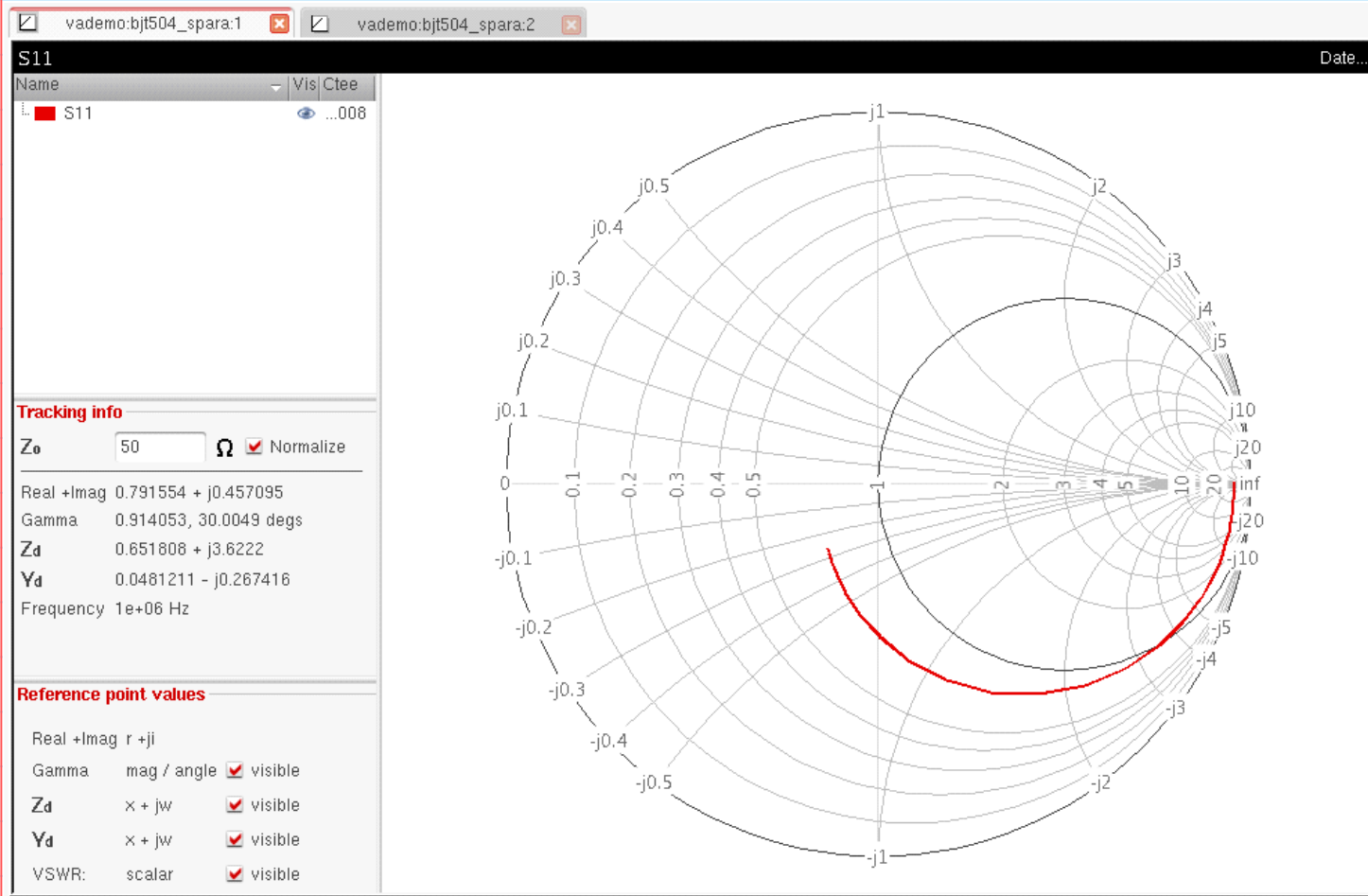
Bias dependence

Sunday, October 28, 2012 7:29 PM

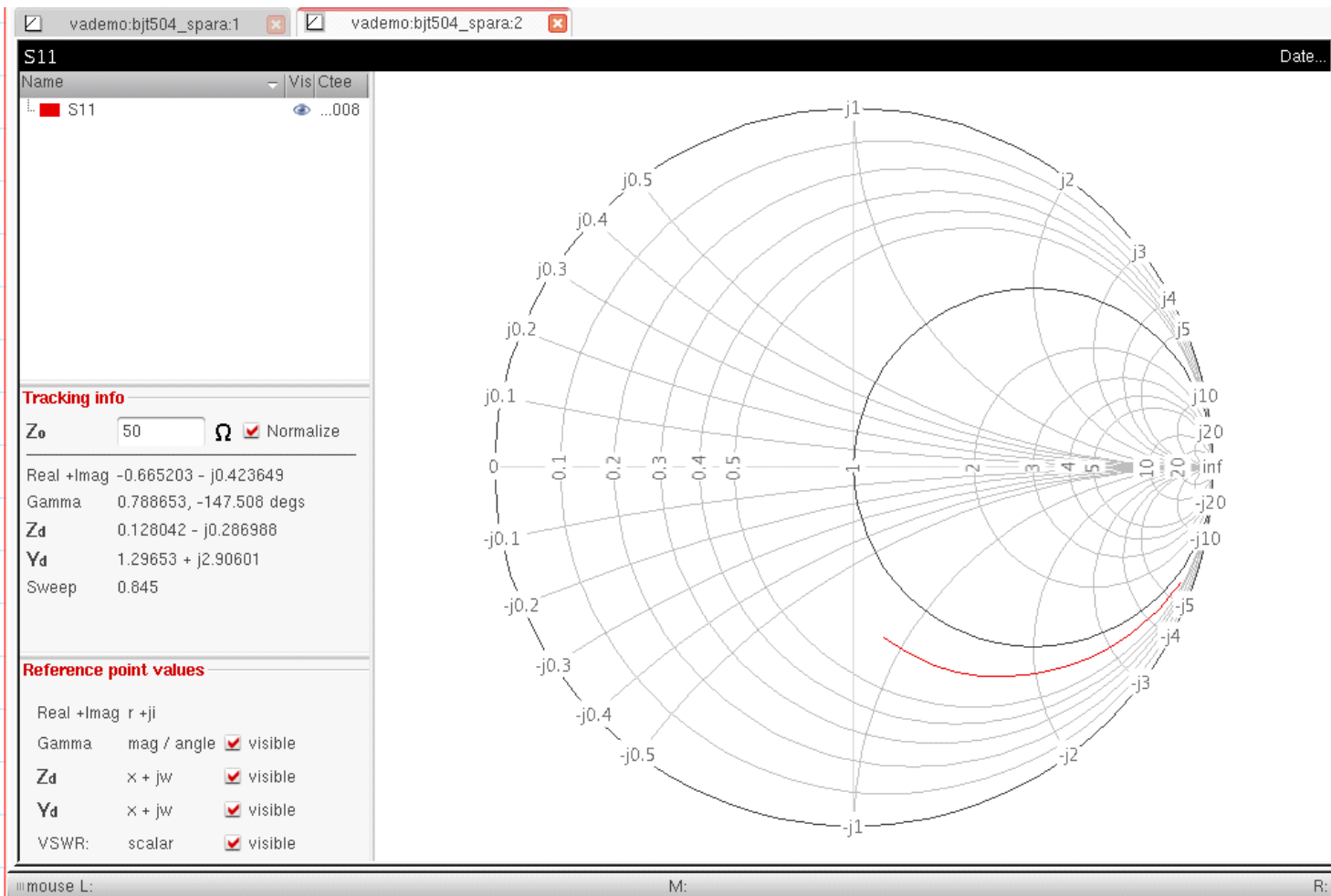
s11

Sunday, October 28, 2012 7:29 PM

Recall frequency dependence of s11 on smith:



Now let us sweep bias (vbe) for a frequency of 2GHz - a useful frequency for cellular and wifi

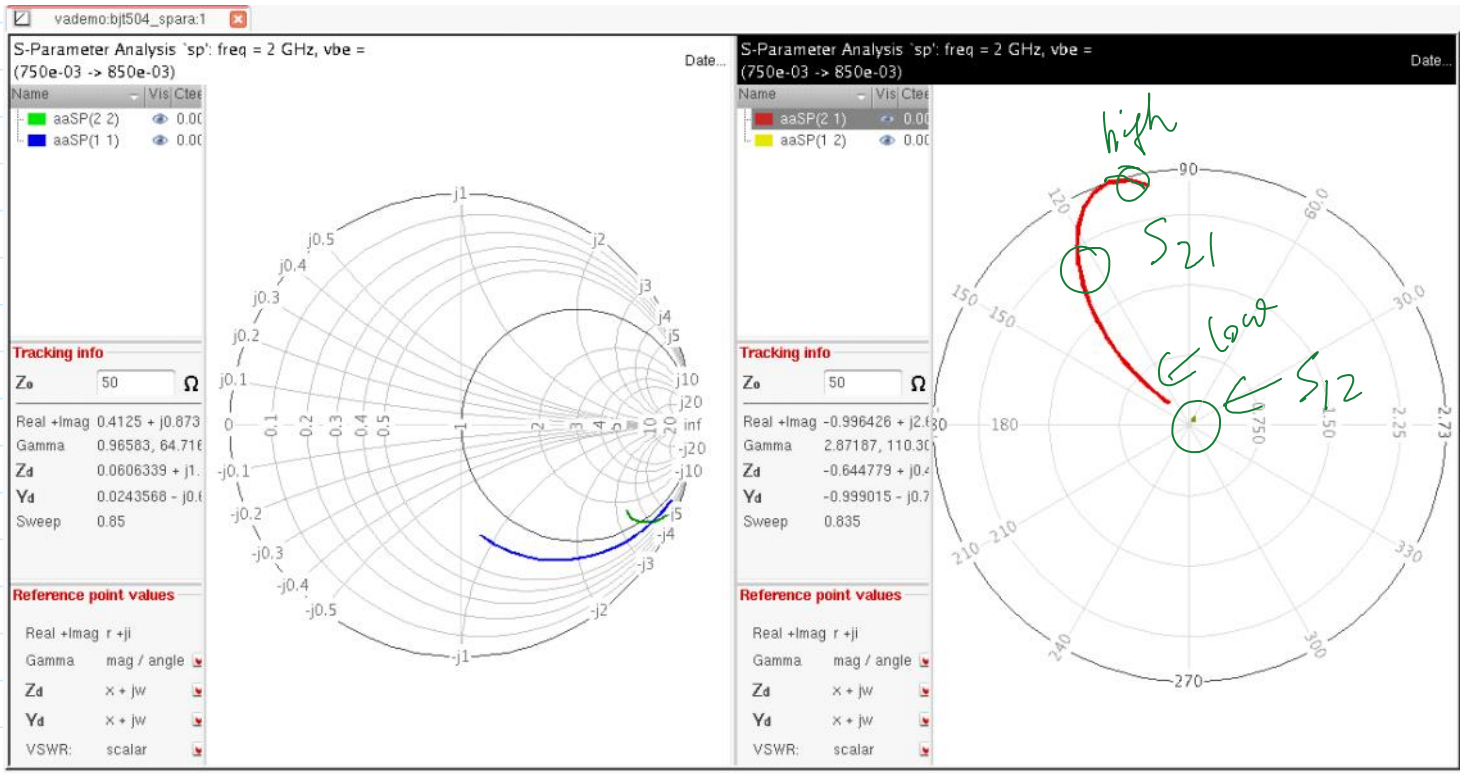


Based on your understanding of s11 and transistor equivalent circuit, can you figure out which end of the curve is higher vbe?

S11 s22 s21 and s12

Sunday, October 28, 2012 8:03 PM

Vbe dependence of all s-parameters, see if you can tell what is what, and which end is low vbe which end is high vbe based on your understanding:

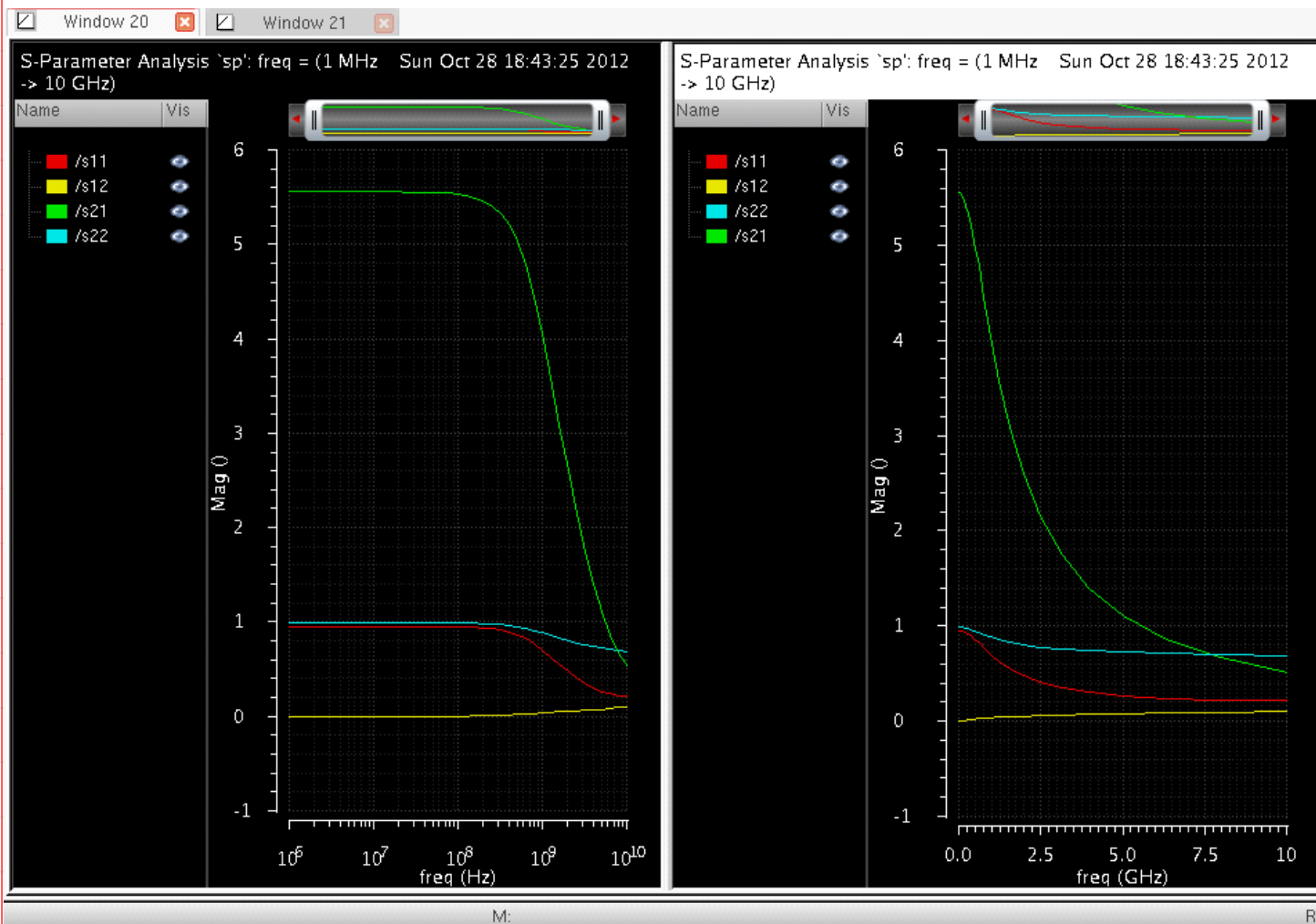


Magnitude of all s-parameters vs frequency

Sunday, October 28, 2012 9:19 PM

At useful biases (not too far from peak ft.), if transistor is not too small, the magnitude of all s-parameters vary with freq like shown below.

Here I'll first show the values of all s-parameters without taking db20, but I'll use both log scale and linear scale for frequency,



Your mag S21 can be easily larger than "1" at low frequency, and in general decreases with increasing

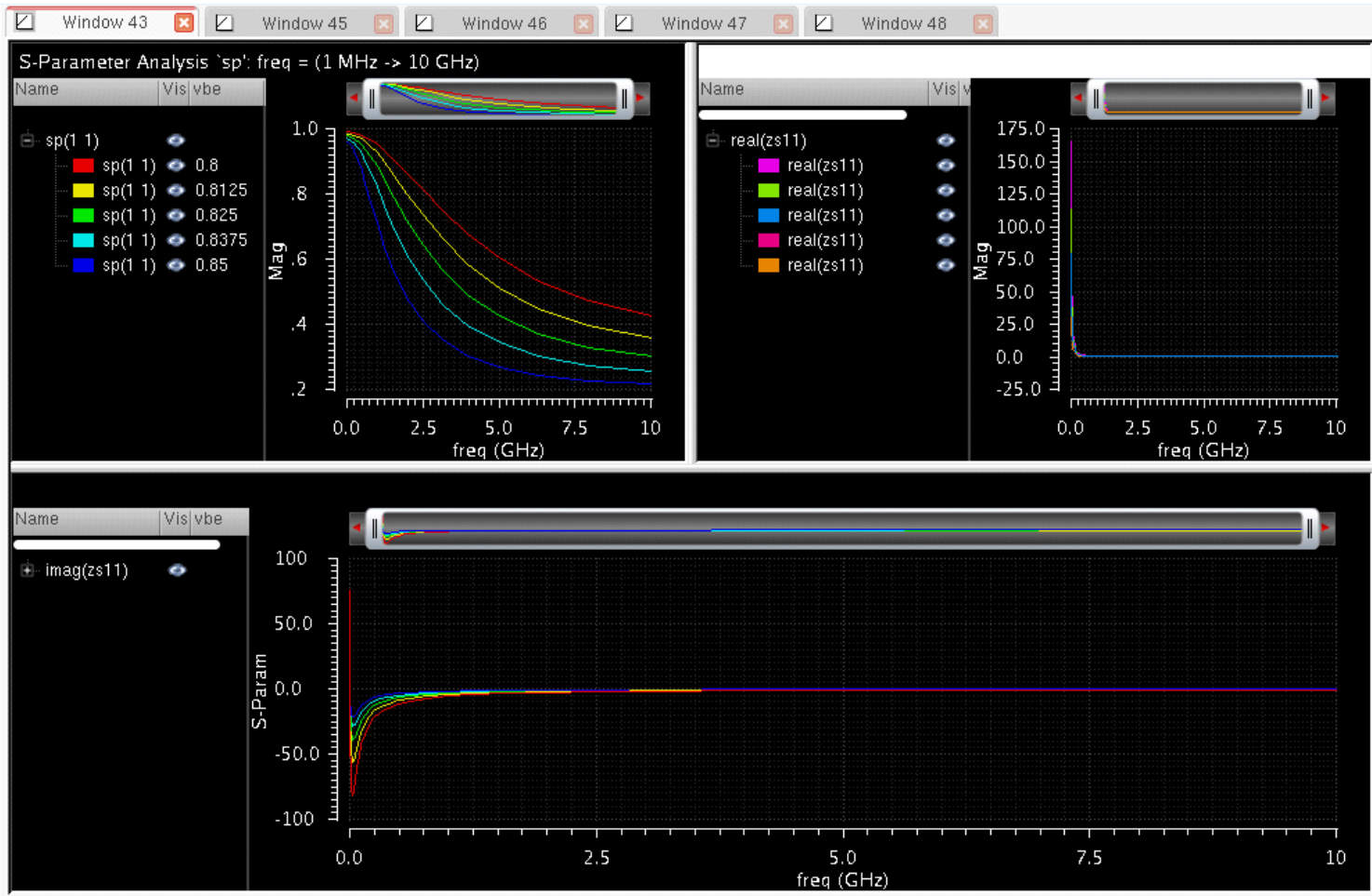
frequency

Your mag of s11 and s22 should in general be less than "1" - on smith chart, it is within the r = 0 circle.

Your mag s12 should in general be very small, much smaller than s21.

s11

Sunday, October 28, 2012 10:37 PM

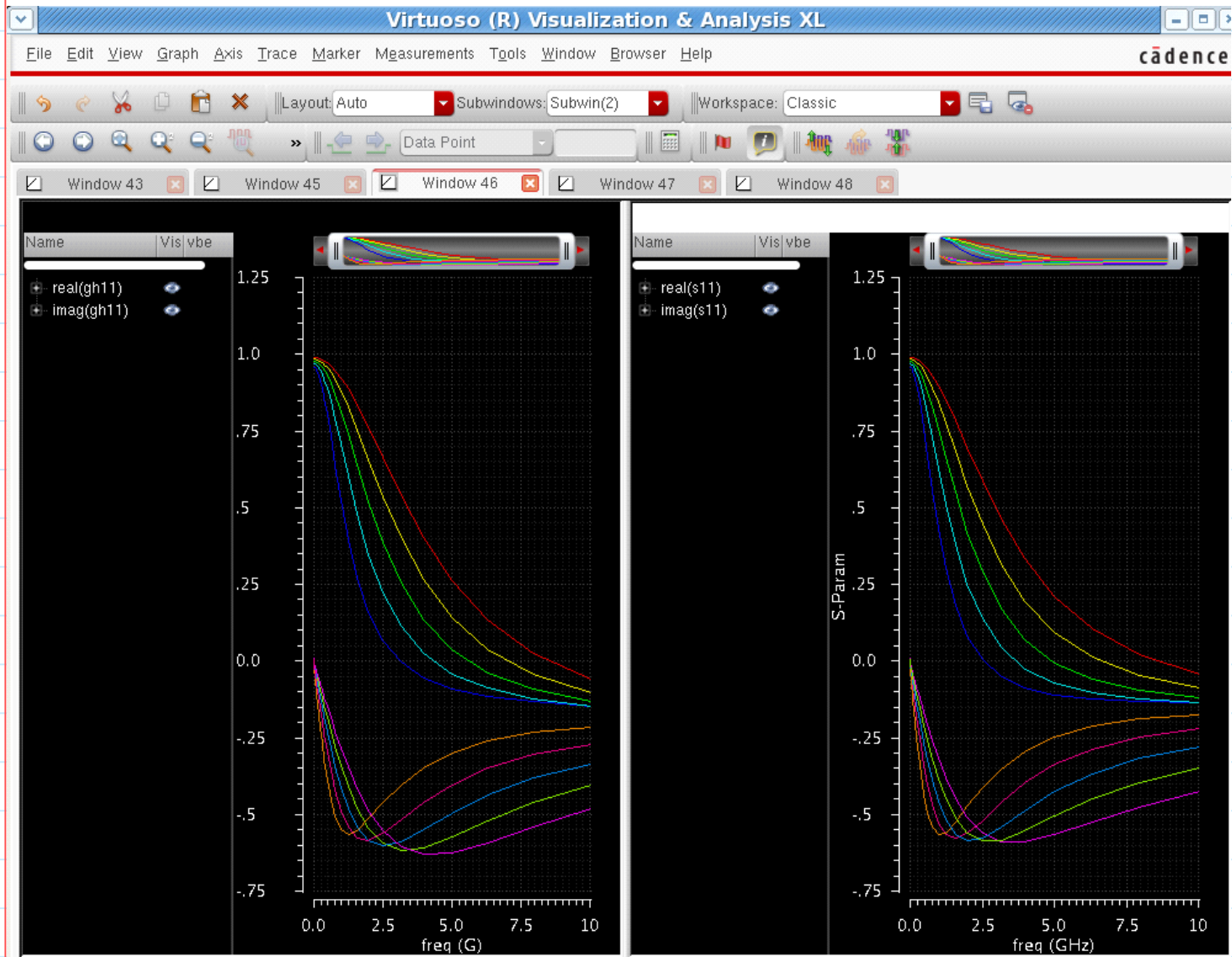


Frequency dependence with bias sweep

Sunday, October 28, 2012 10:37 PM

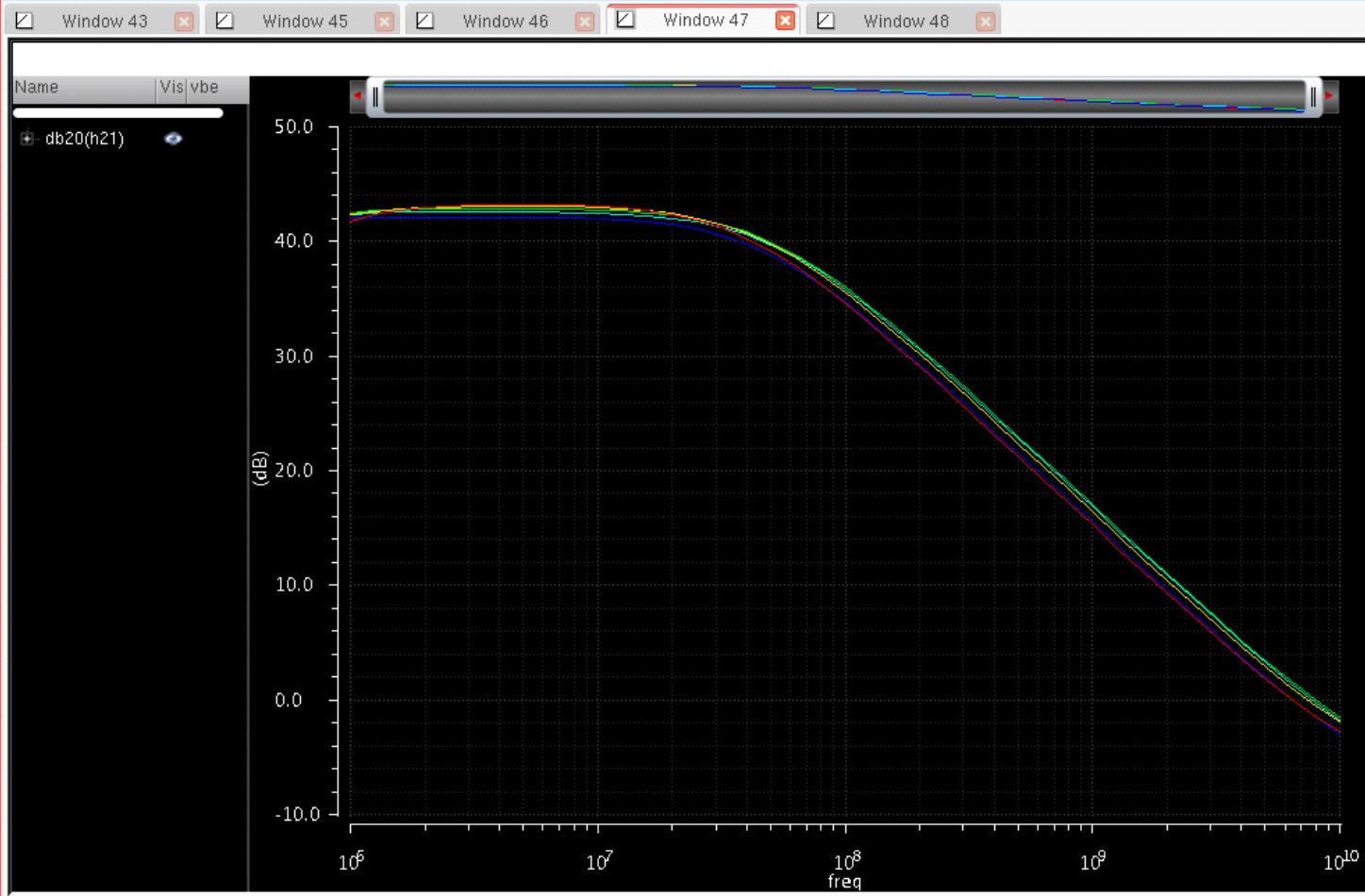
Gamma(h11) and s11

Sunday, October 28, 2012 10:37 PM



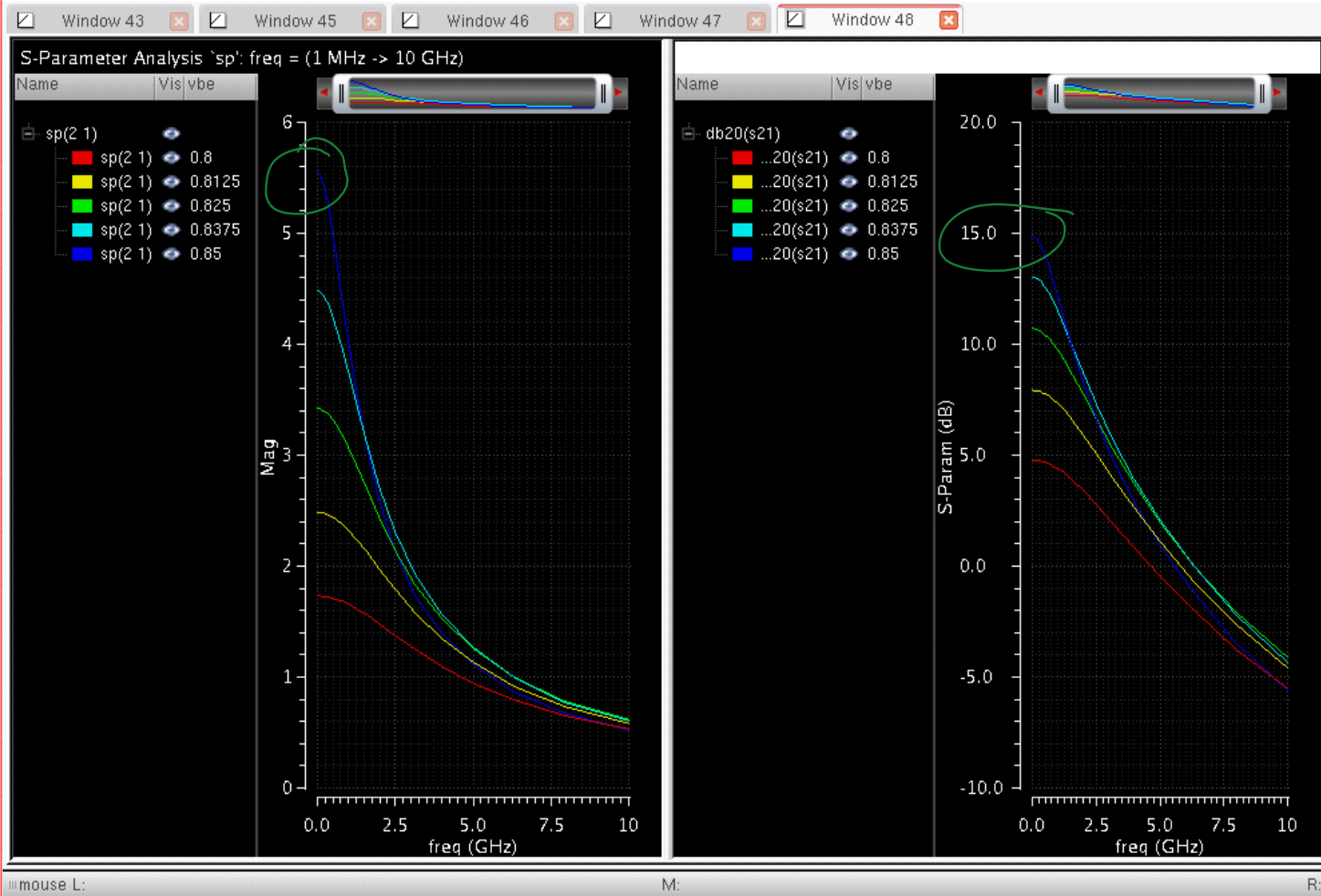
db20(h21)

Sunday, October 28, 2012 10:39 PM



S21 and db20(s21)

Sunday, October 28, 2012 10:39 PM



Transistor sizing - how to choose my transistor size

Tuesday, October 30, 2012 9:37 AM

For the same V_{BE} , current density is the same.

If you put two identical transistors in parallel, for the same voltage, all currents are doubled.

So all conductance/admittance will be doubled ($2x$) - e.g. all of your y -parameters.

All resistances/impedances will be halved ($1/2x$) - e.g. h_{11} , all the your z -parameters.

If you heard of noise matching admittance Y_{opt} and noise matching impedance Z_{opt} , they also obey the same rule.

All current gains remain the same, e.g. h_{21} which is a function of current density, or V_{BE} .

For bipolar transistors, changing size typically means changing the emitter length and/or the number of emitter fingers.

Of course, it is possible to use parallel connection of multiple unit transistors.

Your s -parameters, however, do not have a simple

scaling rule, simply because it involves not only voltages and currents, but also a reference impedance Z_0 .

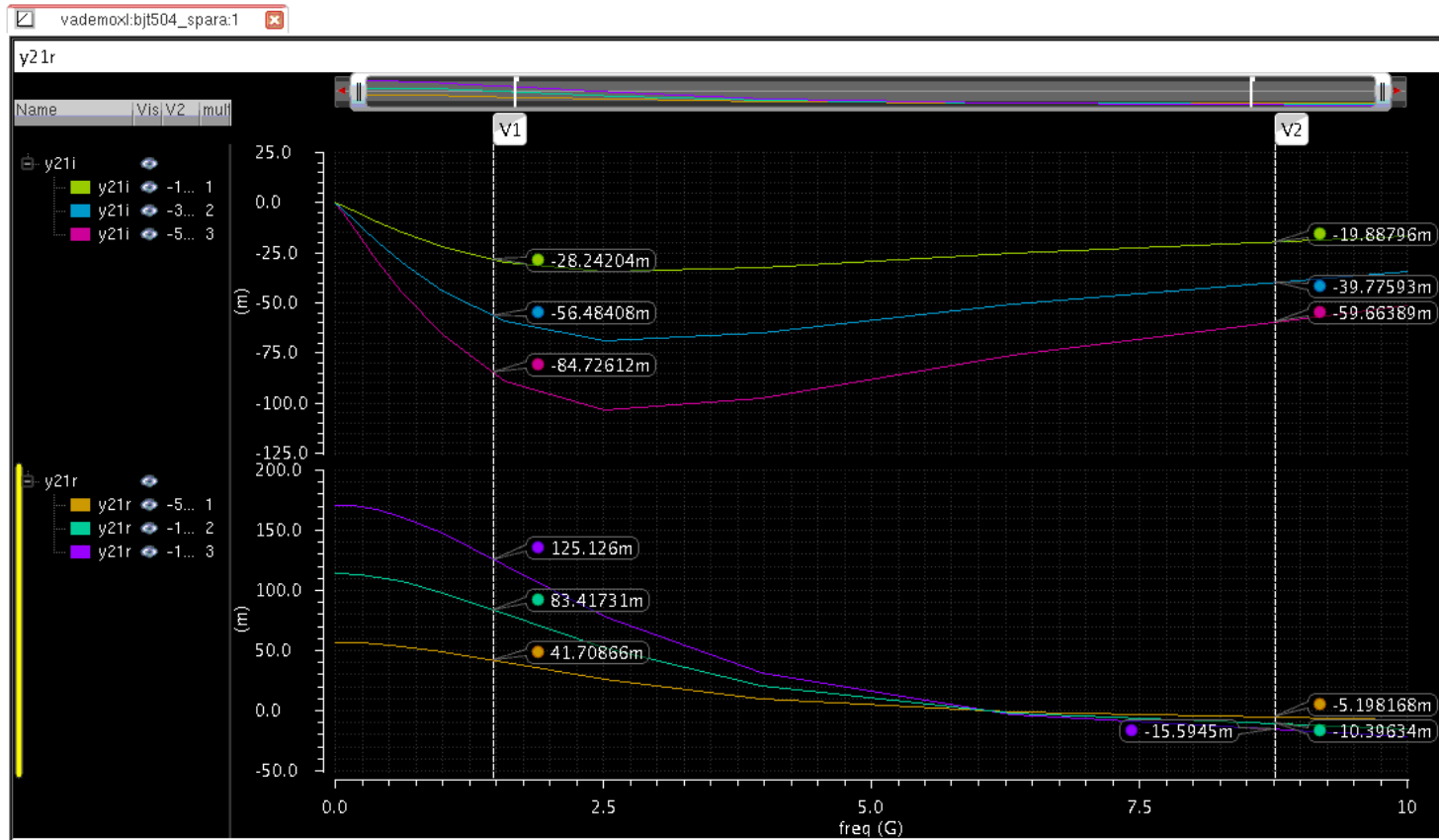
So if you have a device that is too small or too large, its s_{11} can be too close to "OPEN" circuit or "SHORT" circuit in comparison to Z_0 (50ohm).

You always want to measure devices with "reasonable" size, meaning, not too far away from 50 ohms.

Real(y21) imag(y21) scaling example

Sunday, November 04, 2012 10:23 PM

Same VBE, we set the "mult" parameter of our verilog-a model. Mult=2 is equivalent to 2 devices in parallel, mult=3 means 3 in parallel.

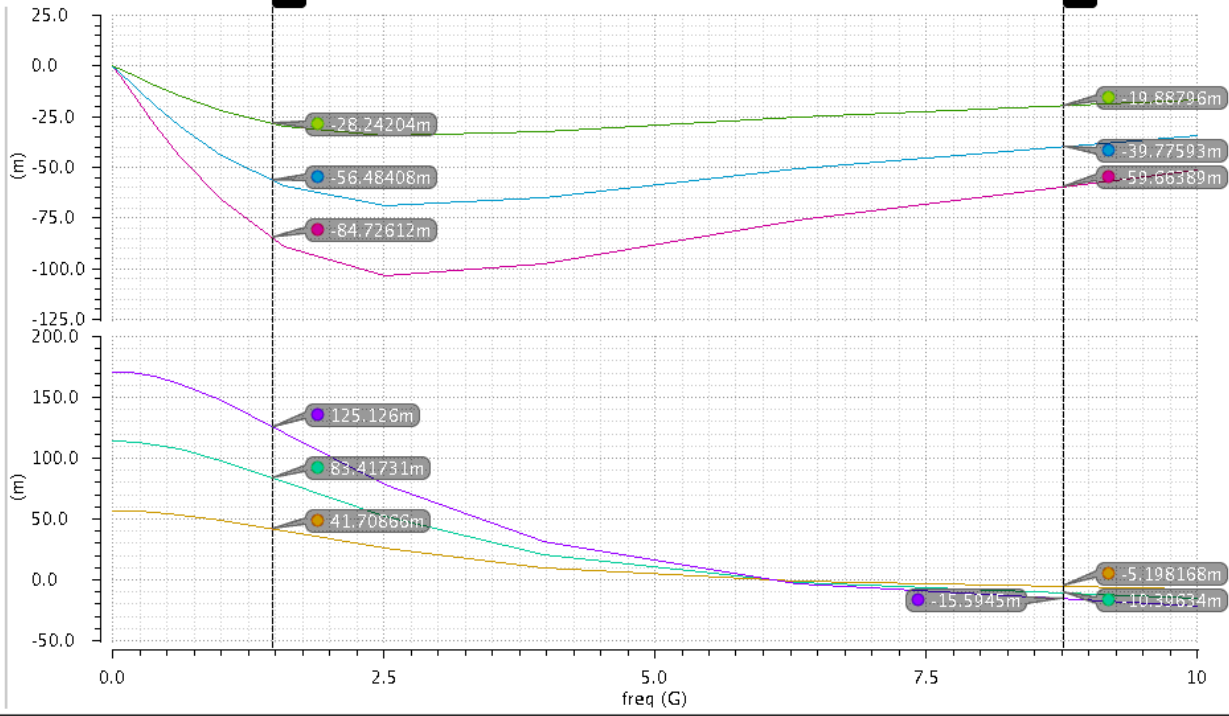


y21r

Name | Vis | V1 | mul

y21i

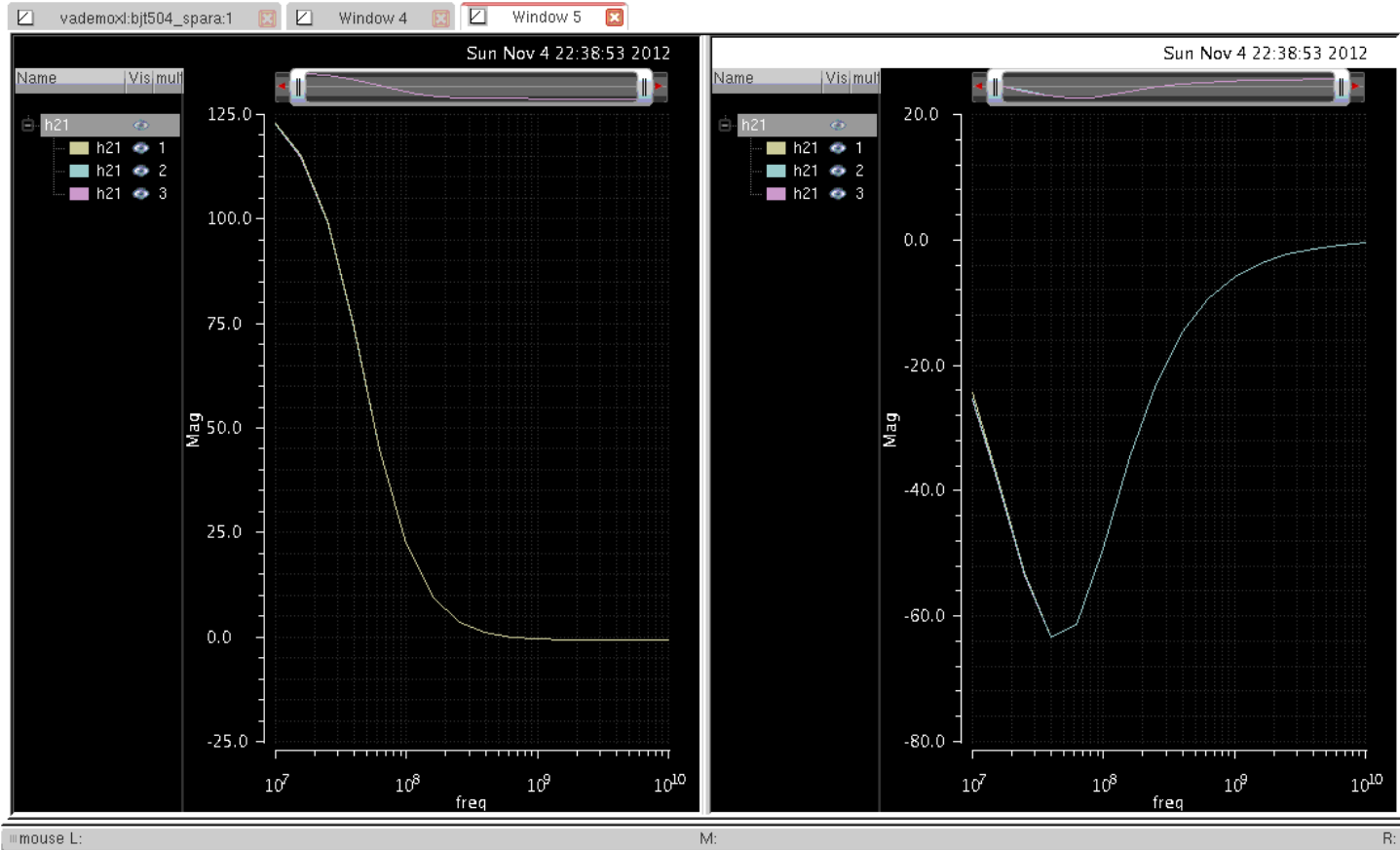
y21r



h21

Sunday, November 04, 2012 10:37 PM

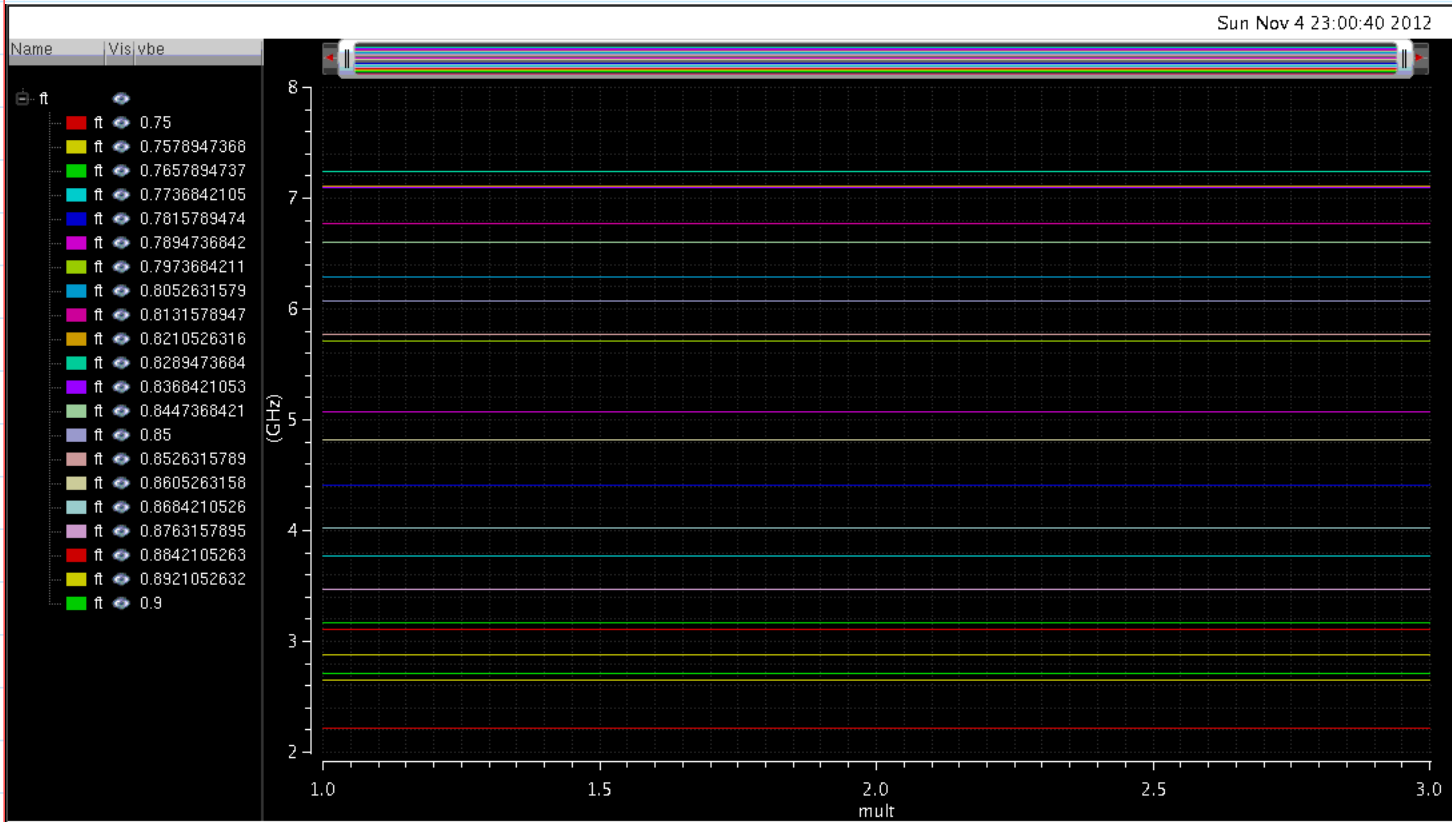
Same VBE. Mult=1,2,3, note that all three devices have the same h21.



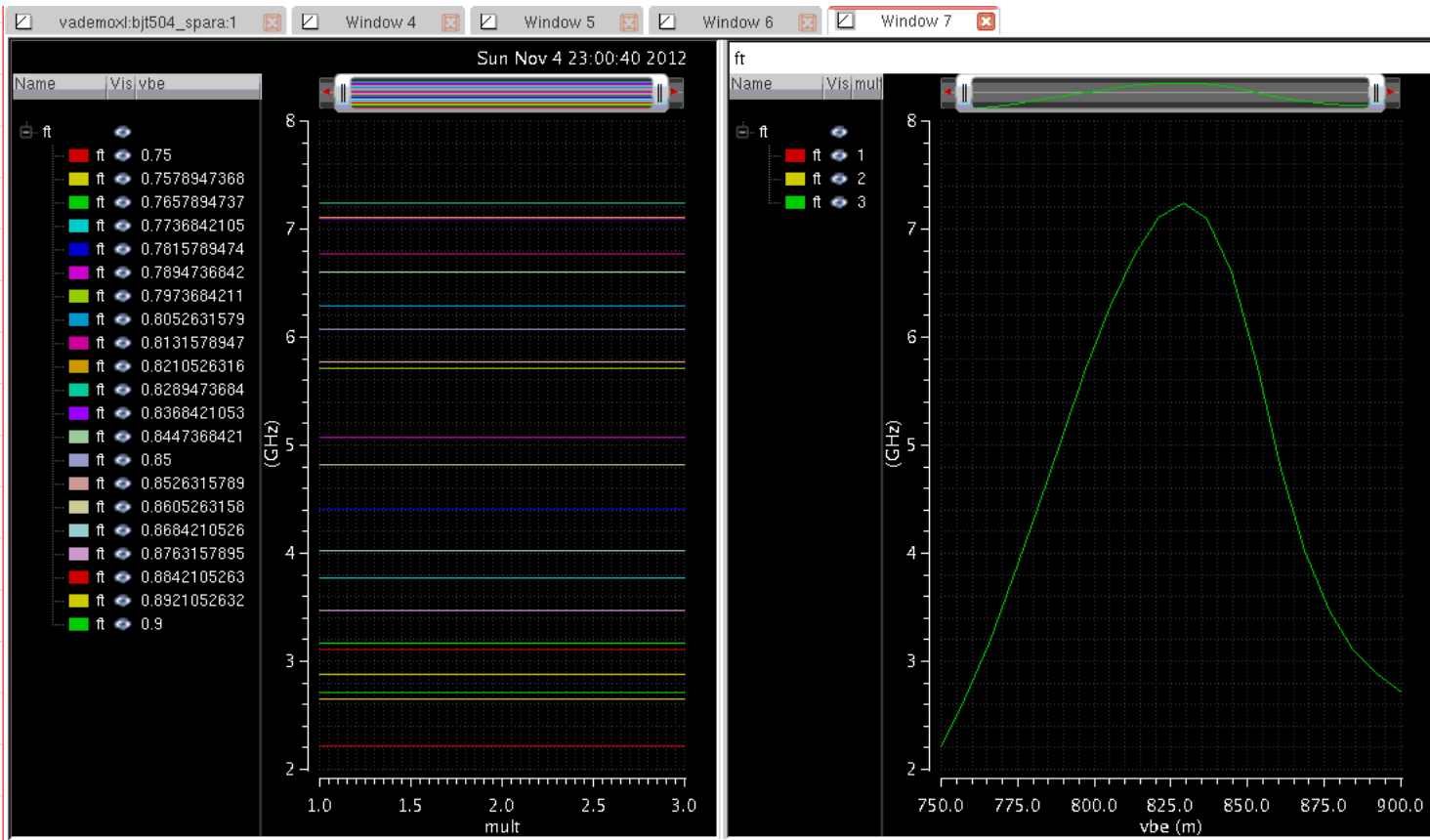
ft.

Sunday, November 04, 2012 11:06 PM

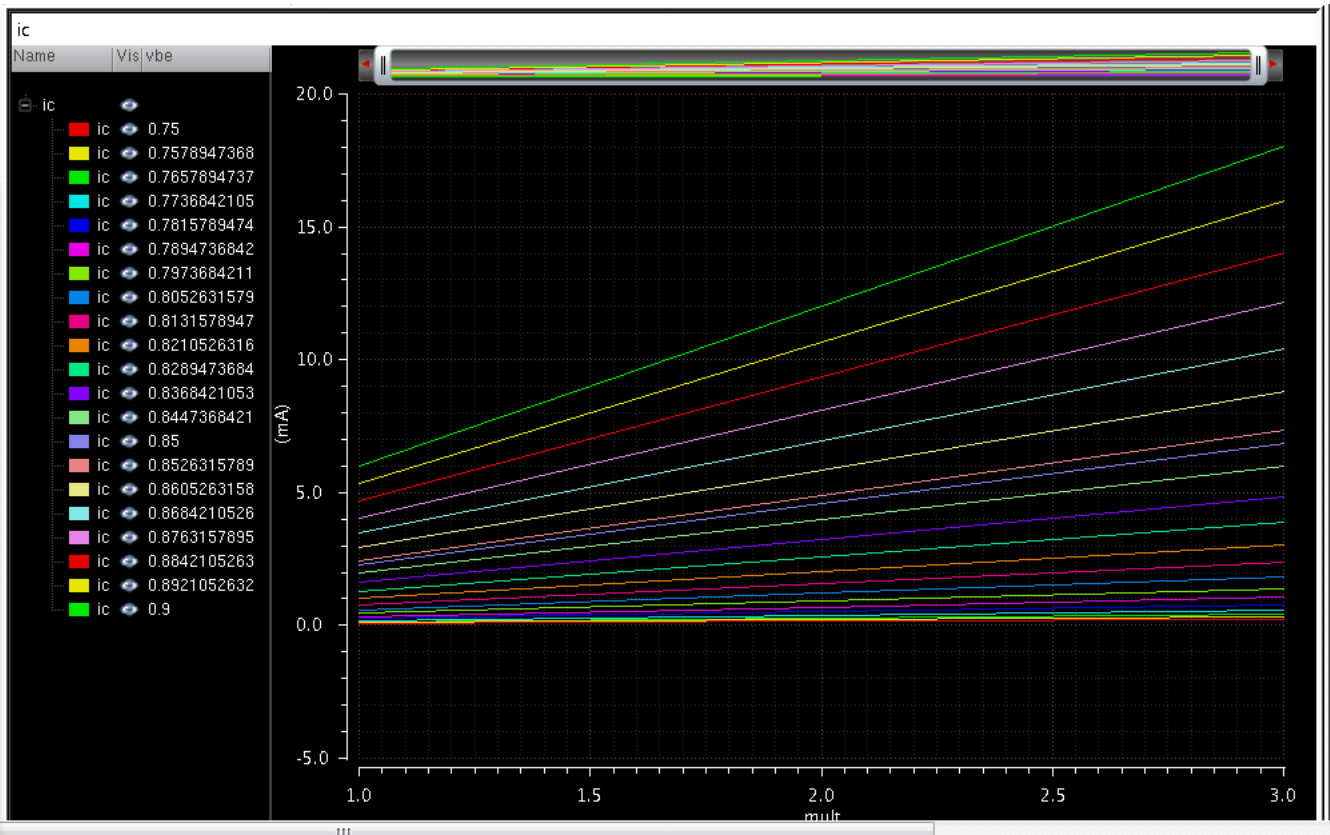
We can repeat this for many other VBE's:



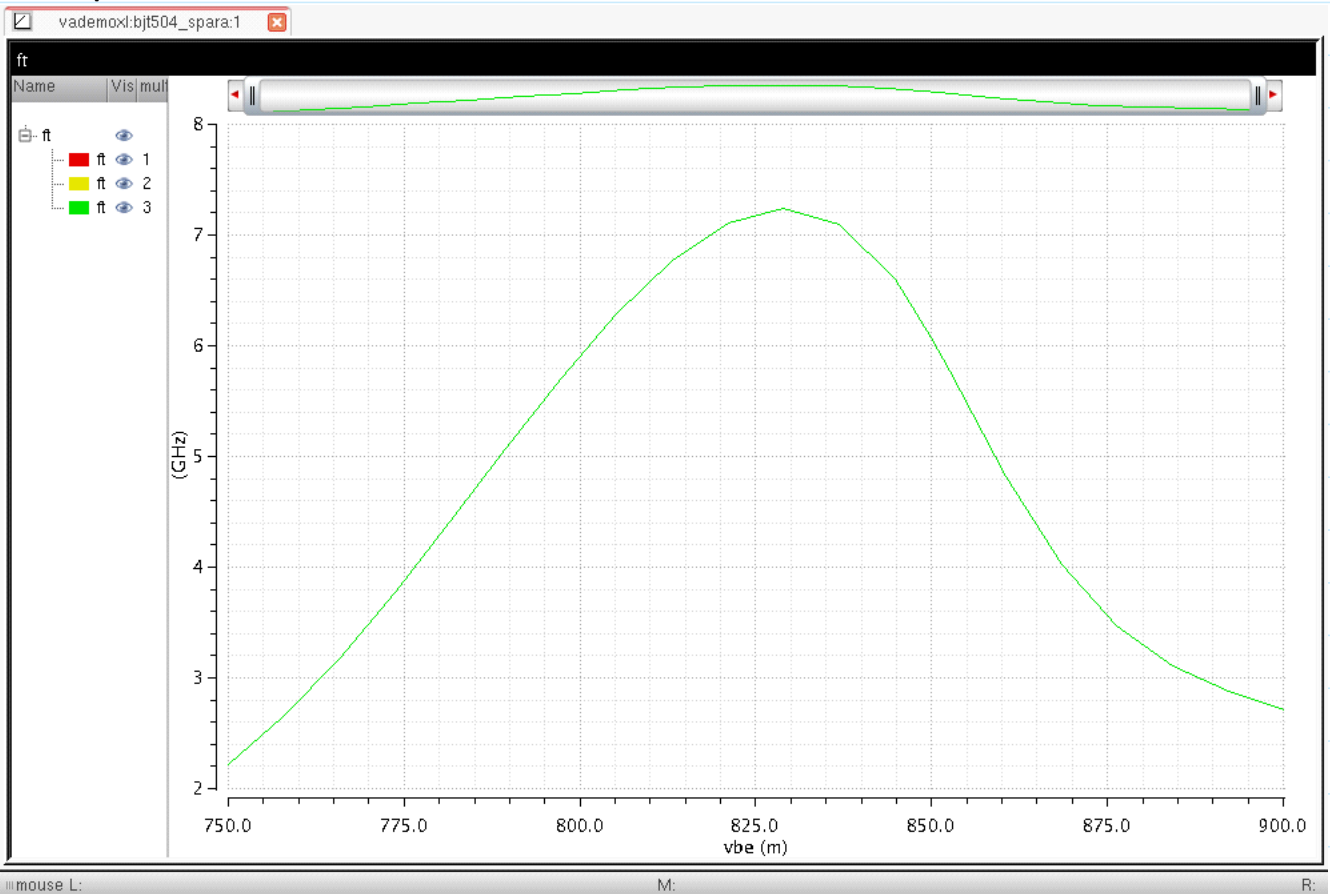
If we plot out f_t -VBE for the three mult values:



We must realize that for the same VBE, all of the currents increase linearly with multi.



Now plot f_t - V_{be} for the 3 mult values:

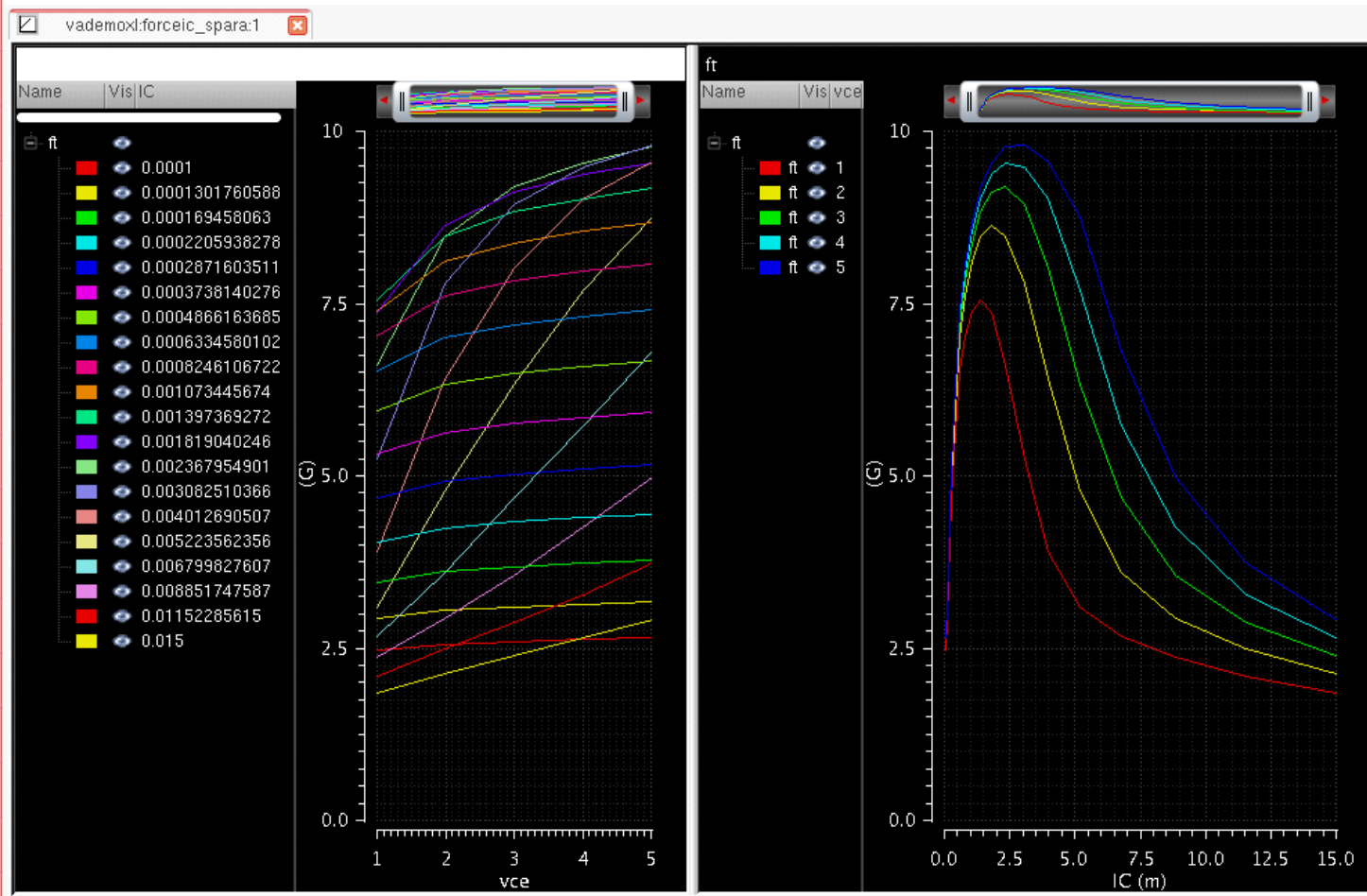


Ft-IC curve for multiple VCE

Sunday, November 04, 2012 9:58 PM

In general, for the same IC, a higher VCE leads to a higher ft in high injection.

This is primarily due to suppression of high injection effect.



How to use IC instead of VBE as design variable

Monday, November 05, 2012 9:52 PM

Often you want to use current as design variable.
How can we achieve this in circuit simulation?

What we have done is to use "YvsY" in plotting.
This does not always work.

Say you have to fix current at 3mA, and want to find an optimal size that will give say lowest minimum noise figure.

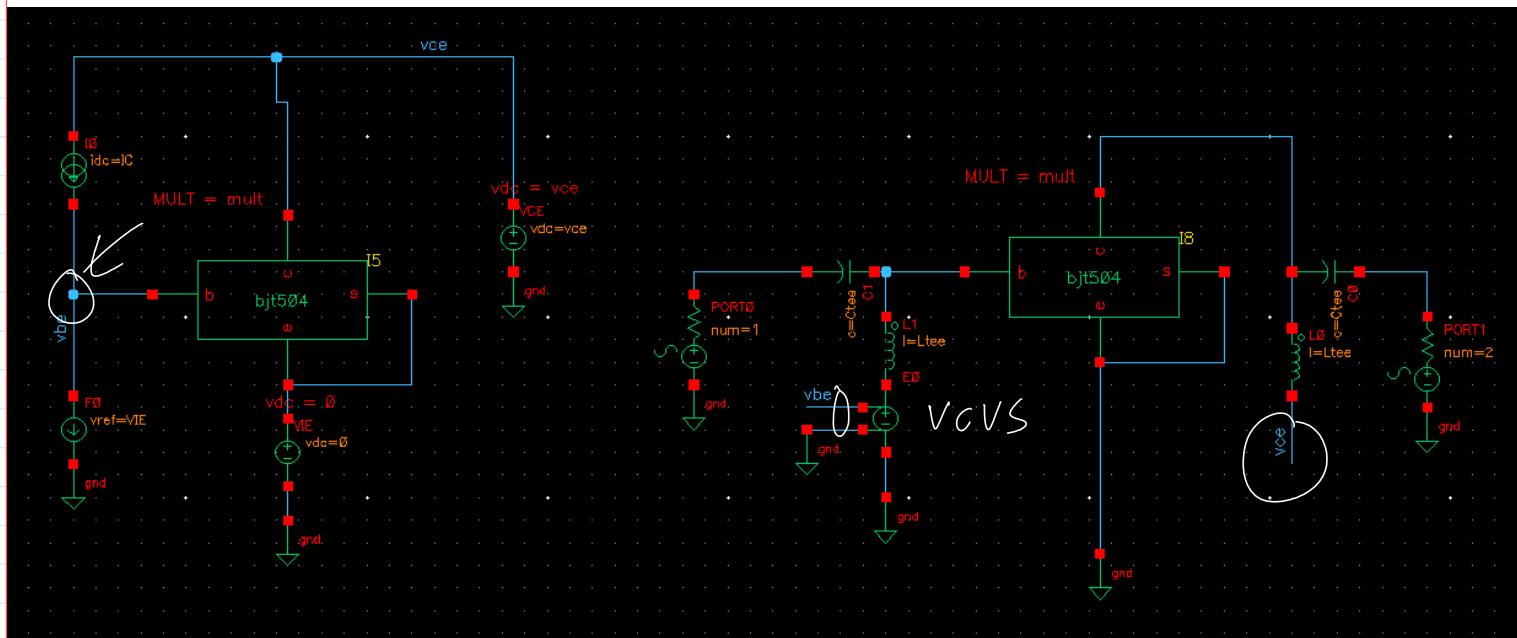
You may / can then need to sweep this current.

The circuit below allows you to do just that.

On the left is a circuit that produces the VBE required for a given IC - which is specified using a current source (I0).

Think about how this circuit works.

Then this VBE is applied to our s-parameter measurement circuit's base-emitter voltage dc bias, via a voltage controlled voltage source (E0).



This can be a very hard to understand circuit.

A famous circuit in analog is the "Wilson" current source. It is a result of a competition between two analog designers, Gilbert (yes the same person in Gilbert mixer) and Wilson, in the 60s.

It was about trying to come up with a better current source using only 3 transistors.

Wilson won.

Google "Wilson current source" for more info.

We cannot directly use that source as it still does not allow us to set VCE.

So instead of using a real current mirror at the bottom, we here use just an ideal current mirror using CCCS - current controlled current source, this way, we can use "zero" voltage for our current "sensor".

There are a few other ways to make the circuit precisely producing an IC that is the same as our IC input current source, but I have had a very hard time getting convergence.

I once got it to converge, but then lost it quickly.

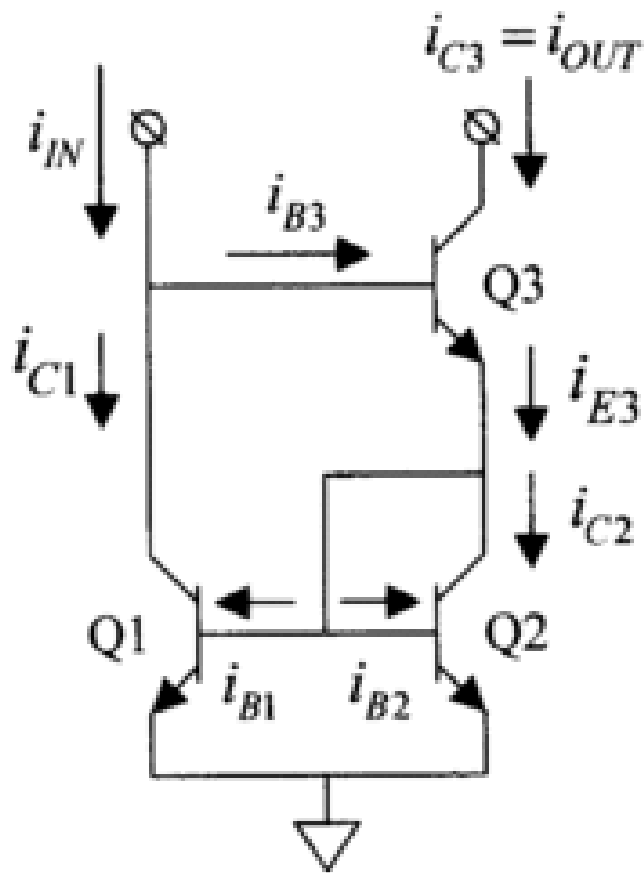
But for our purpose, this works almost perfectly. There is a small error of $2 \cdot I_B$, which means $2/\beta$ in percentage.

Fixing VCB and IC at the same time, however, is much easier.

Wilson current mirror

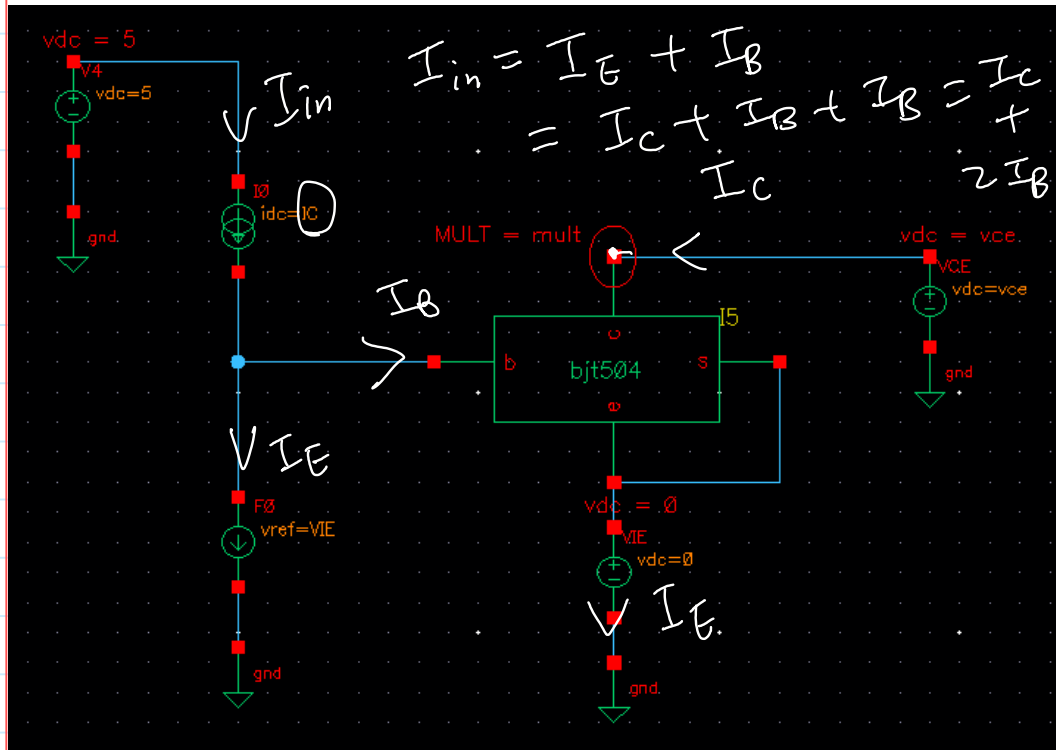
Wednesday, November 07, 2012 9:29 PM

http://en.wikipedia.org/wiki/Wilson_current_mirror



Circuit that produces the right amount of IC at given VCE

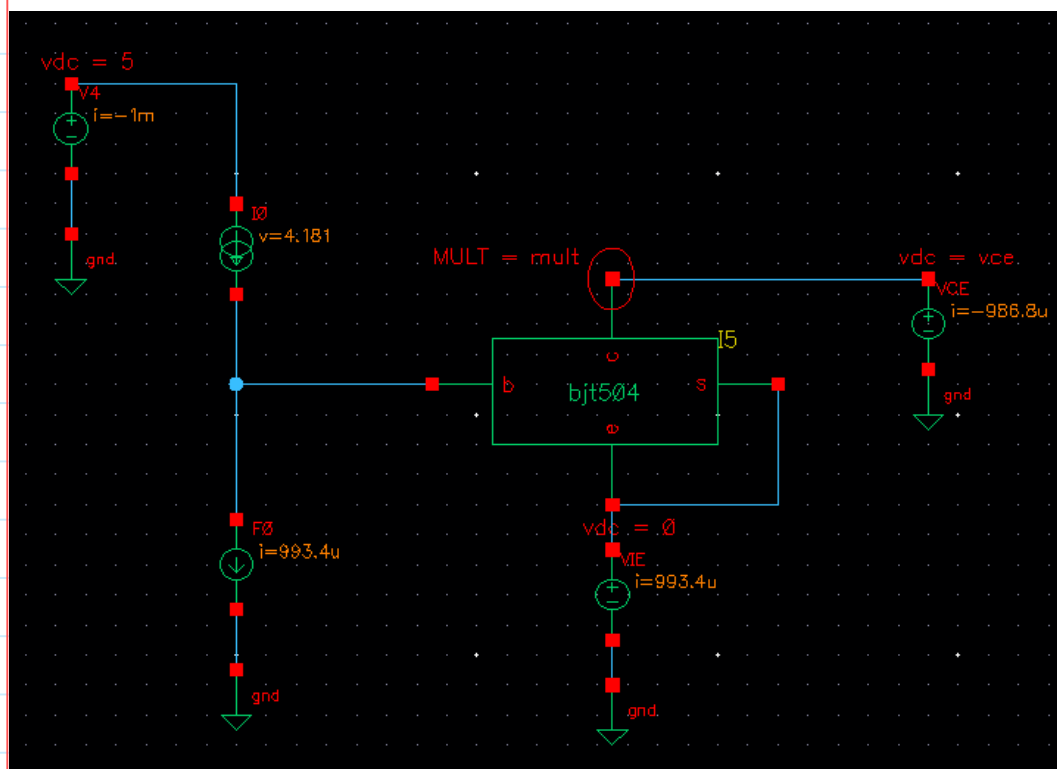
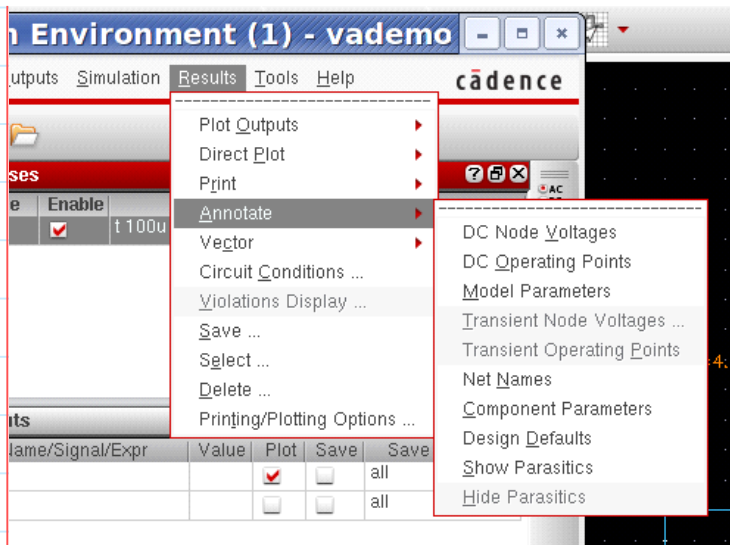
Wednesday, November 07, 2012 9:15 PM



Design Variables

	Name	Value
1	IC	1m
2	mult	1
3	vce	3

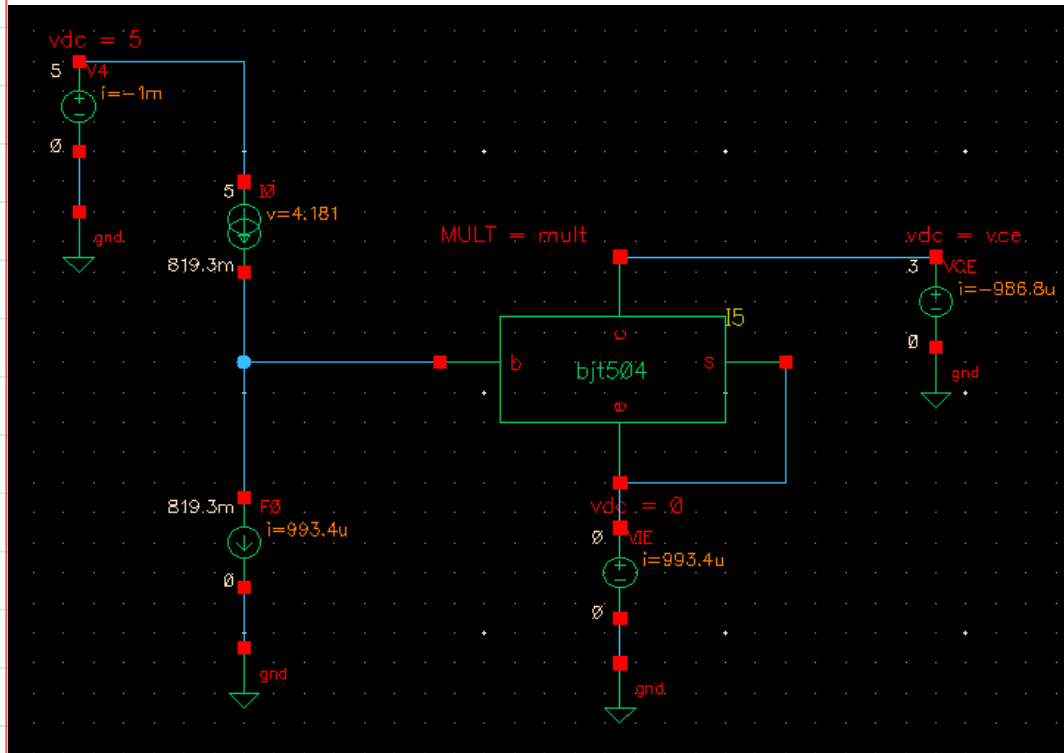
Dc solution :



You will find that for an input "IC" (I0 current) of 1mA - (V4 current), and VCE of 3V, we obtain an actual IC (flowing through VCE) of 986.8uA.

For voltage source, if the current shown is **negative**, it means current flows **out of the "+" terminal**.

Now add dc voltages:

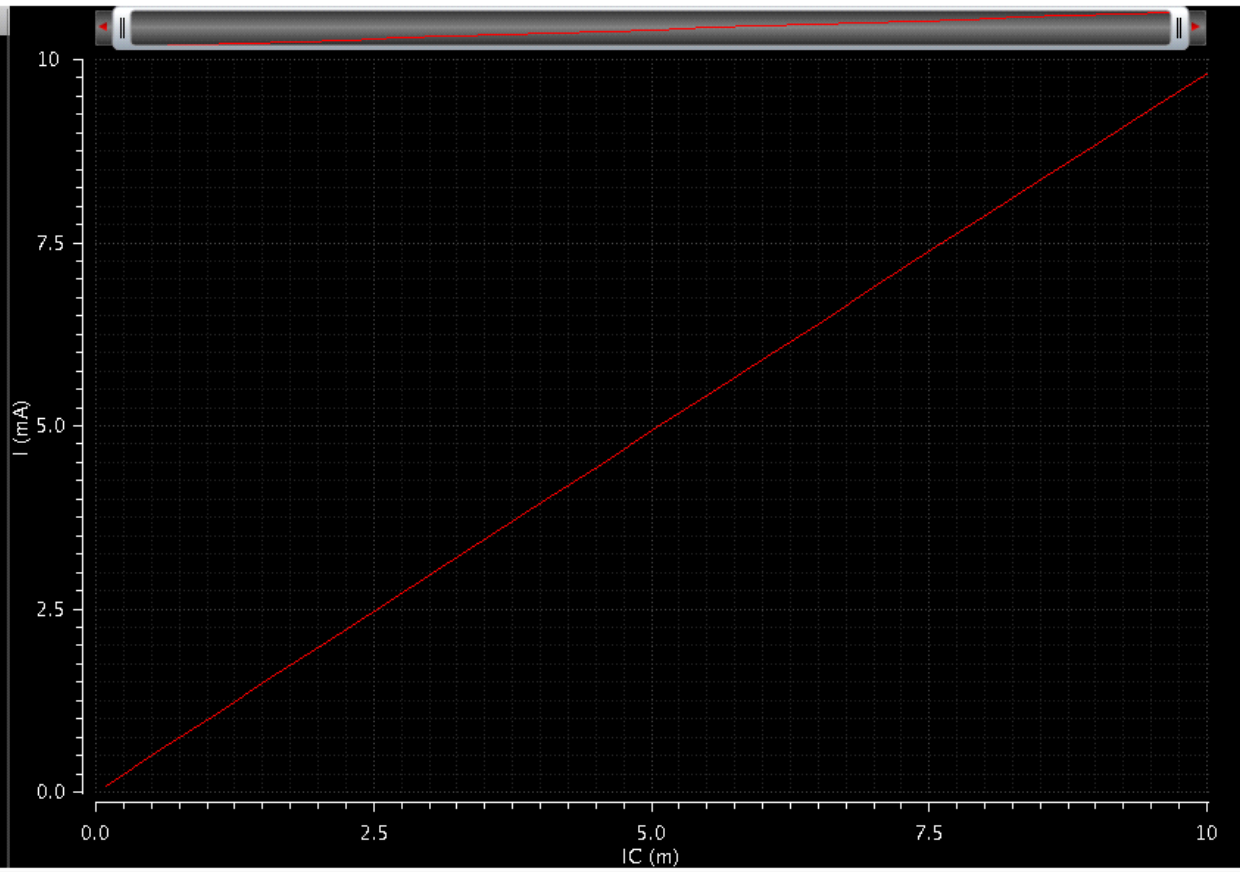


Now let us sweep the "IC" design variable, and plot out the actual collector current at the "c" terminal of bjt504.

DC Response

Name Vis

/15/c



Transistor noise parameters and LNA design - the short version

Monday, November 12, 2012 5:18 PM

I'll present here a short version of this complex topic.

We will use a fixed biasing current optimization as an example and show how to do design using cadence.

Noise parameters - short version

Monday, November 12, 2012 5:12 PM

Noise parameters

noise figure



$$NF = 10 \log_{10} F$$

F : noise factor

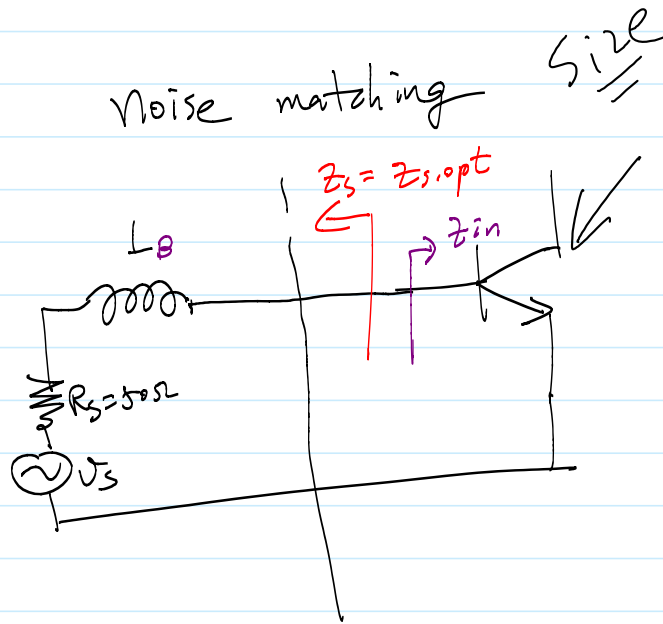
$$Y_s = G_s + jB_s$$

$$F = F_{min} + \frac{R_n}{G_s} |Y_s - Y_{opt}|^2$$

- ① $F \rightarrow$ minimum F_{min} when $Y_s = Y_{s,opt}$
- ② $|F - F_{min}|$ the deviation depends on R_n
 R_n determines sensitivity to mismatch

Noise matching thru transistor sizing

Monday, November 12, 2012 5:16 PM



$$z_{s,opt} \neq z_{in} !$$

noise matching \neq impedance matching

① scale device size such that $R_{s,opt} = R_s$

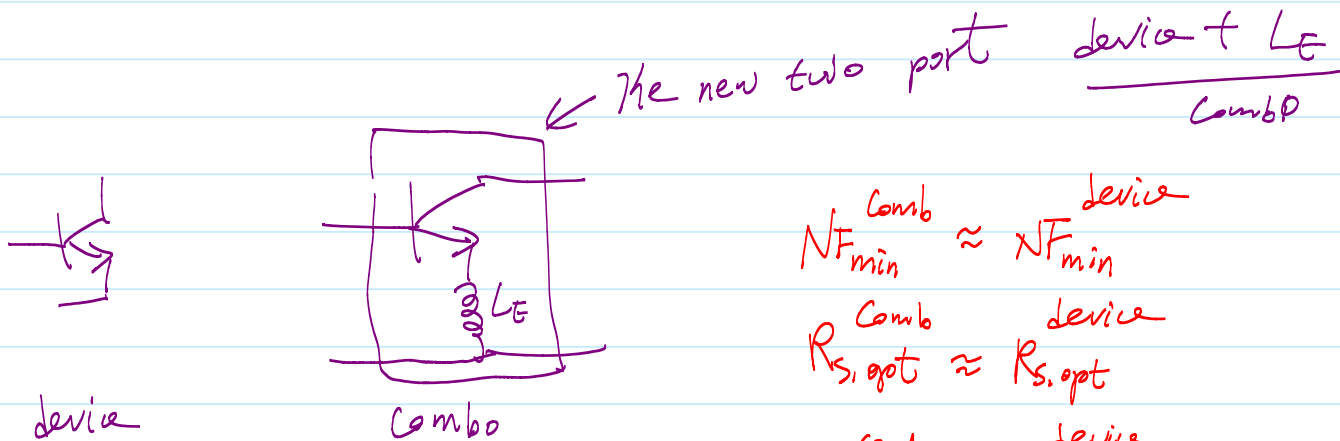
② choose L for $\omega L = X_{s,opt}$.

$$z_{s,opt} = R_{s,opt} + jX_{s,opt}$$
$$= \frac{1}{Y_{s,opt}}$$

Adding emitter inductor's impact

Monday, November 12, 2012 5:12 PM

Now consider adding L_E - emitter inductor



through linear circuit analysis \Rightarrow

$$NF_{min}^{Combo} \approx NF_{min}^{device}$$

$$R_{s,opt}^{Combo} \approx R_{s,opt}^{device}$$

$$R_n^{Combo} \approx R_n^{device}$$

$$X_{s,opt}^{Combo} \approx X_{s,opt}^{device} - \omega L_E$$

L_E can be chosen to produce real part Z_{in}

$$\text{Re}(Z_{in}) = \omega_T \cdot L_E = R_s$$

Simultaneous noise and impedance matching

Monday, November 12, 2012 5:20 PM

Can we possibly achieve noise matching and impedance matching at the same time without increasing noise figure? - Yes.

Now consider adding L_E - emitter inductor

device

The new two port $\frac{\text{device} + L_E}{\text{Combo}}$

Through linear circuit analysis \Rightarrow

$$NF_{\min}^{\text{Combo}} \approx NF_{\min}^{\text{device}}$$

$$R_{s,\text{opt}}^{\text{Combo}} \approx R_{s,\text{opt}}^{\text{device}}$$

$$R_n^{\text{Combo}} \approx R_n^{\text{device}}$$

$$X_{s,\text{opt}}^{\text{Combo}} \approx X_{s,\text{opt}}^{\text{device}} - \omega L_E$$

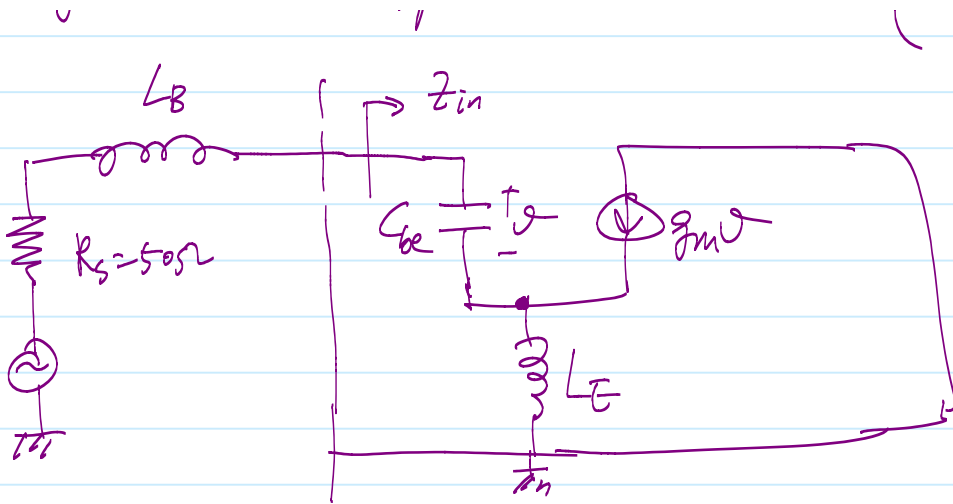
L_E can be chosen to produce real part Z_{in}

$$\text{Re}(Z_{in}) = \omega_T \cdot L_E = R_s$$

So the real part is impedance matched (Z -match)

luckily, the $X_{s,\text{opt}}^{\text{Combo}}$ is also the $(X_{in}^{\text{Combo}})^*$!!

$L_R \quad r \approx Z_{in}$



$$Z_{in} = \frac{1}{j\omega C_{be}} + (1 + \beta_{RF}) j\omega L_E$$

$$\beta_{RF} = \frac{g_m v}{j\omega C_{be} v} = -j \frac{g_m}{C_{be} \omega} = -j \frac{\omega_T}{\omega}$$

$$\omega_T = \frac{g_m}{C_{be}}$$

Thus: $Z_{in} = \frac{1}{j\omega C_{be}} + j\omega L_E + \omega_T L_E$

$$L_E = \frac{R_S}{2\pi f_T}$$

choose L_E
for
 $\omega_T L_E = R_S$

So now $\text{Re}(Z_{in}) = \omega_T L_E$ through L_E choosing

$\text{Re}(Z_{opt}^{comb}) = R_S$ through sizing

$$\text{Im}(Z_{in}) = -\frac{1}{\omega C_{be}} + \omega L_E$$

combo

device

$$I_m(z_{s,opt}) = I_m(z_{s,opt}) - \omega L_E$$

$$\begin{array}{c} X_{in}^{comb} \\ \downarrow \\ I_m(z_{in}) = -\frac{1}{\omega C_{be}} + \omega L_E \end{array}$$

$$I_m(z_{s,opt})^{comb} = I_m(z_{s,opt})^{device} - \omega L_E$$

$$\begin{array}{c} X_{s,opt}^{comb} \\ \downarrow \\ = \frac{1}{\omega C_{be}} - \omega L_E \end{array}$$

$$\text{We have proven } X_{in}^{comb} = \left(X_{s,opt}^{comb} \right)^*$$

a very fortunate case

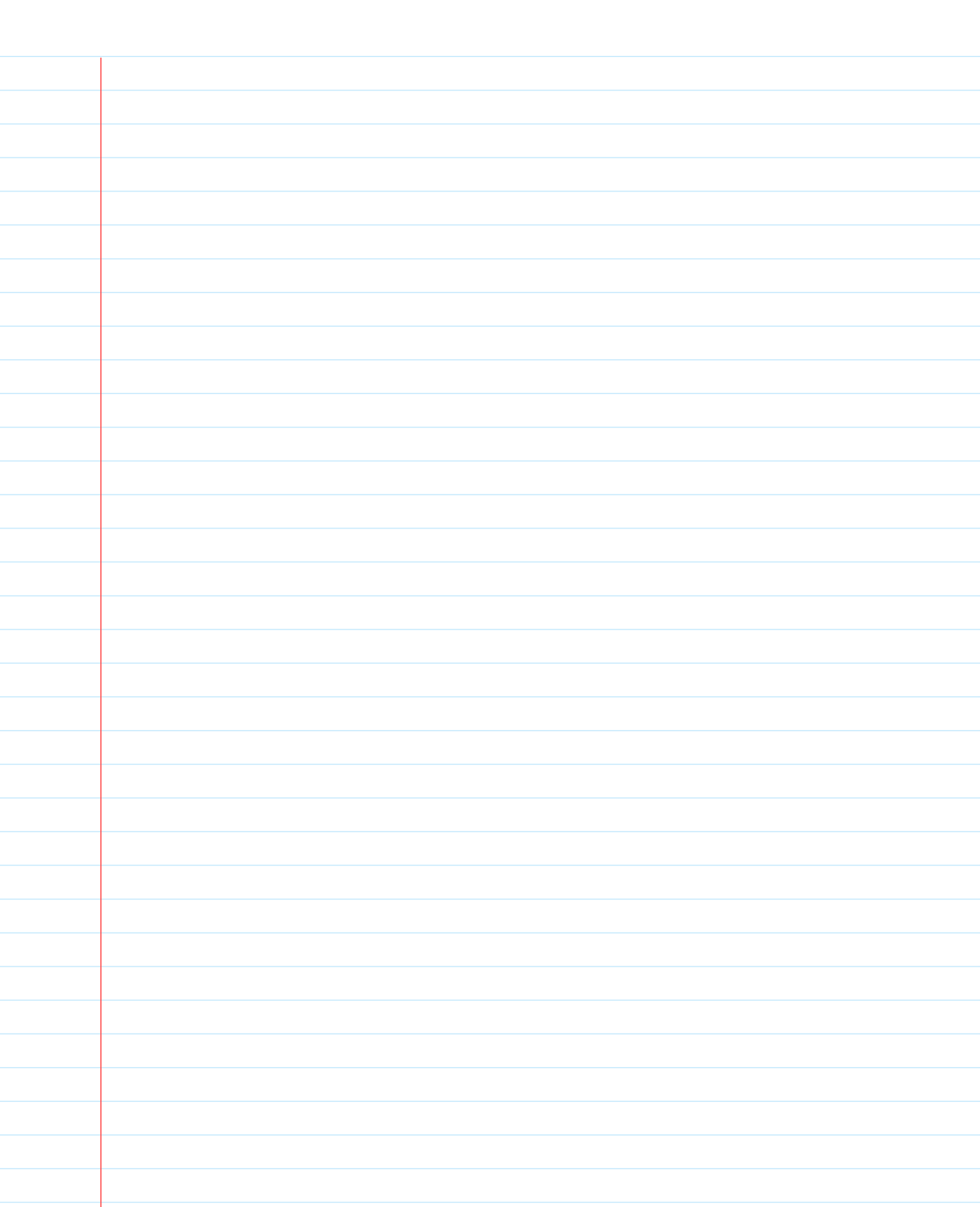
* Even if these assumptions are not always valid,

We can tolerate some noise mis-match as long as

it is not too far off and R_n is small

* The insight can still be used at least to

noise match the real part, + impedance matching.



<http://sdrv.ms/PGtOun>

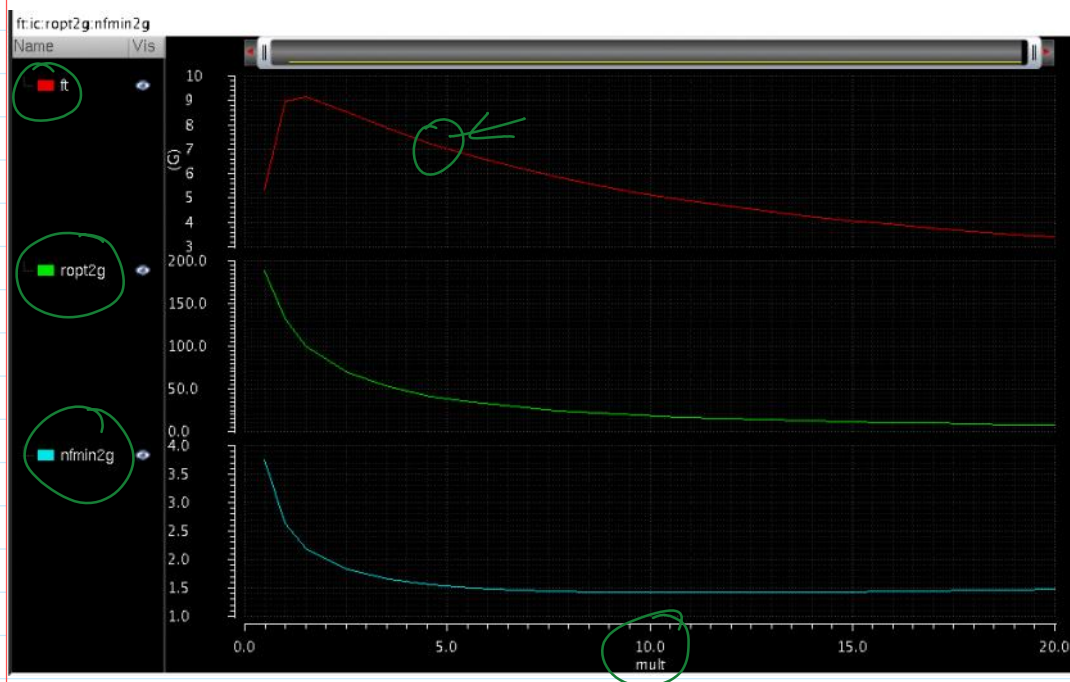
Case Study of Device Circuit Interaction - Optimal Sizing Under Fixed current (power consumption) for RF Low-Noise Amplifier Design

Sunday, November 11, 2012 1:19 PM

A very important issue in IC design is to optimize size of transistors.

For instance, in RFIC design, e.g. the LNA, amplifier transistor's size can be optimized.

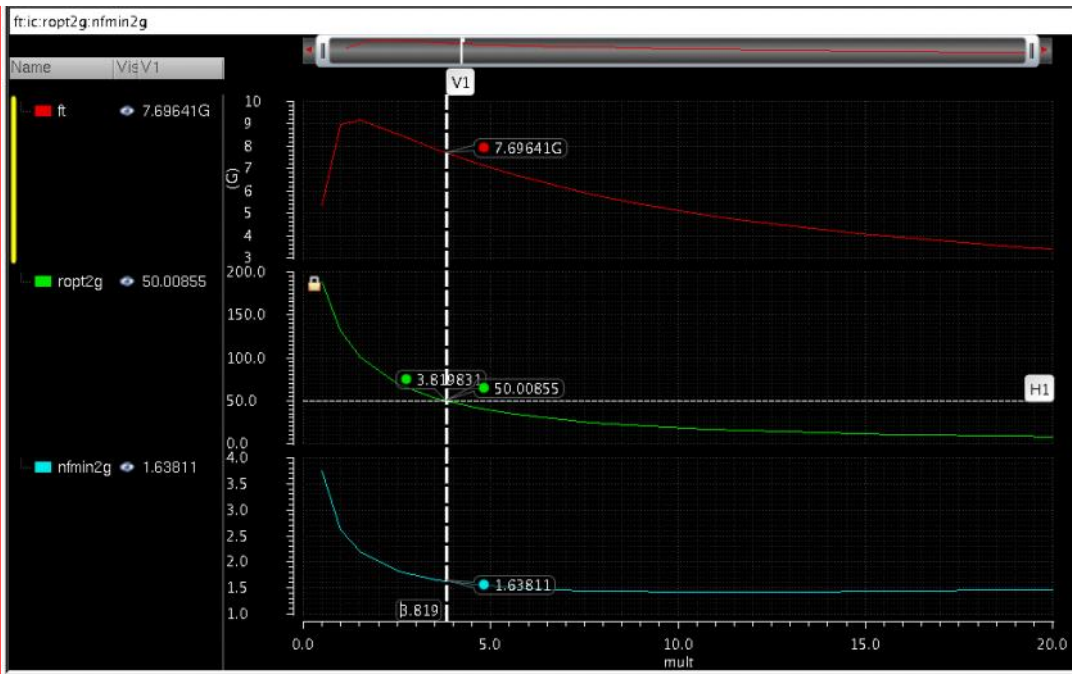
Consider 3mA, 3V VCE, for the same bjt we have been using, at 2GHz,



We sweep "mult" and plot out ft and ropt2g as well as NFmin2G versus Mult.

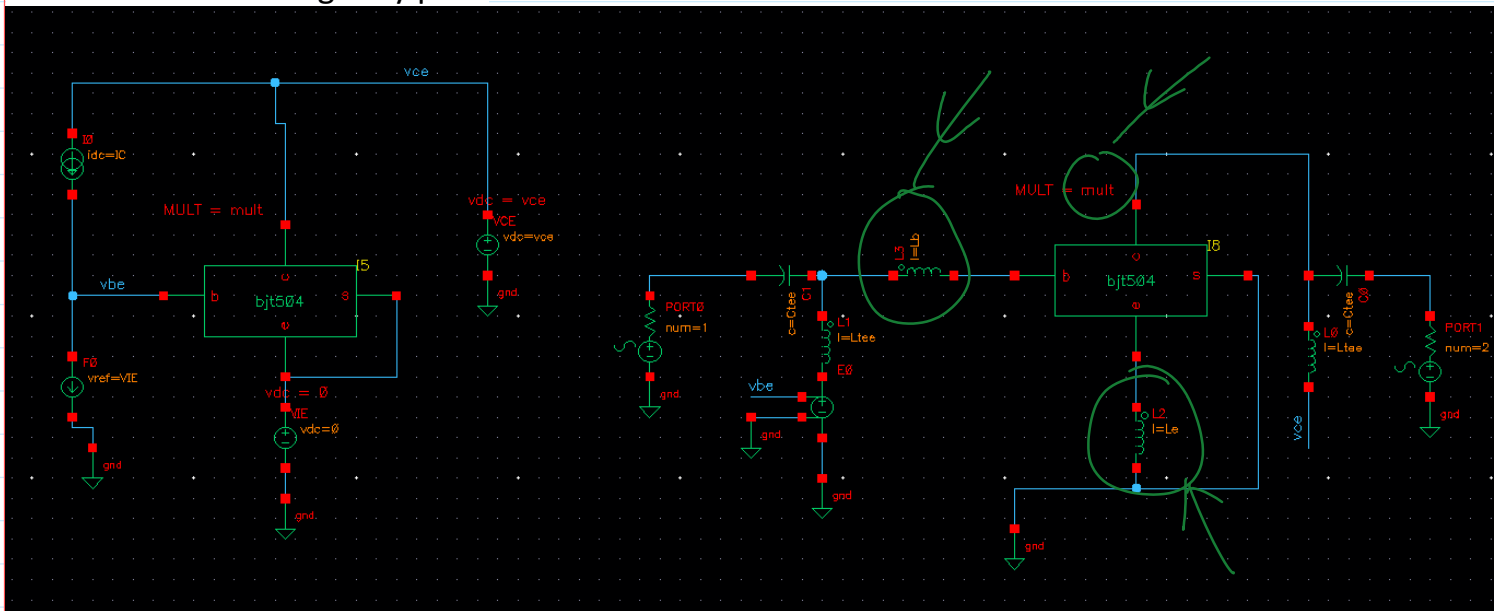
Let us find the "mult" that gives us a 50ohm ropt2G. The ft. is still

7.7GHz, not too bad, NFmin2g is 1.64dB, almost near the minimum one can have for this current value.



So we set "mult=3.82",

Next, we need to produce an input impedance equal to source impedance (source side impedance matching) for several reasons in RF LNA design. This can be achieved by placing an emitter degeneration inductor L_e , which produces a real part (50 Ohm), and modifies the imaginary part.

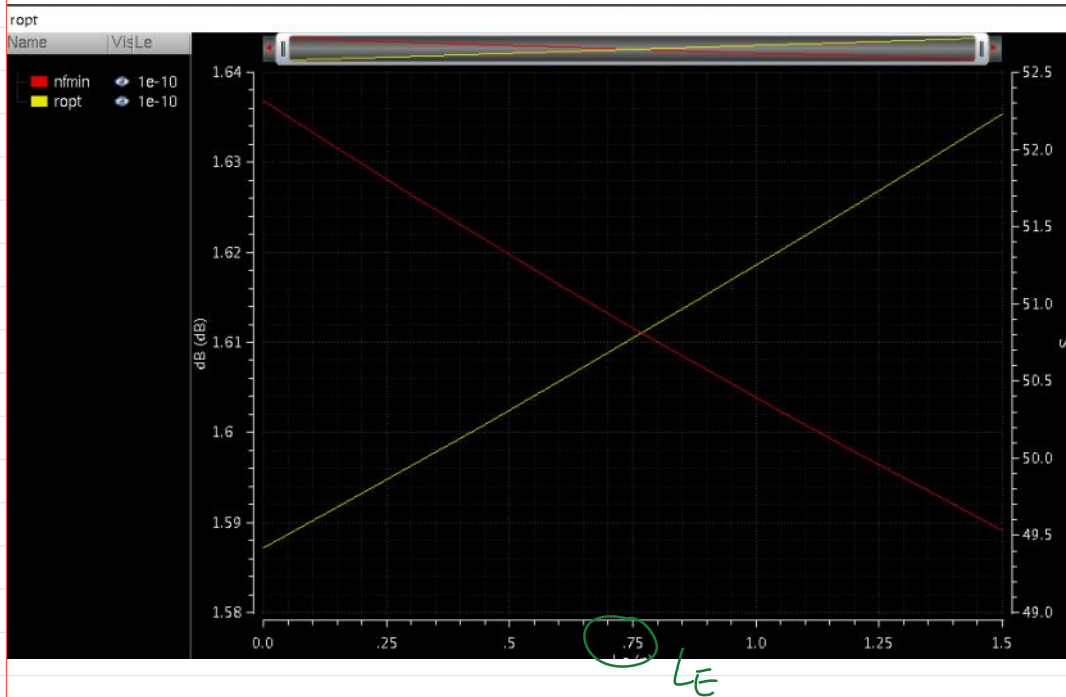


Then we base inductor L_b can be adjusted to make the total

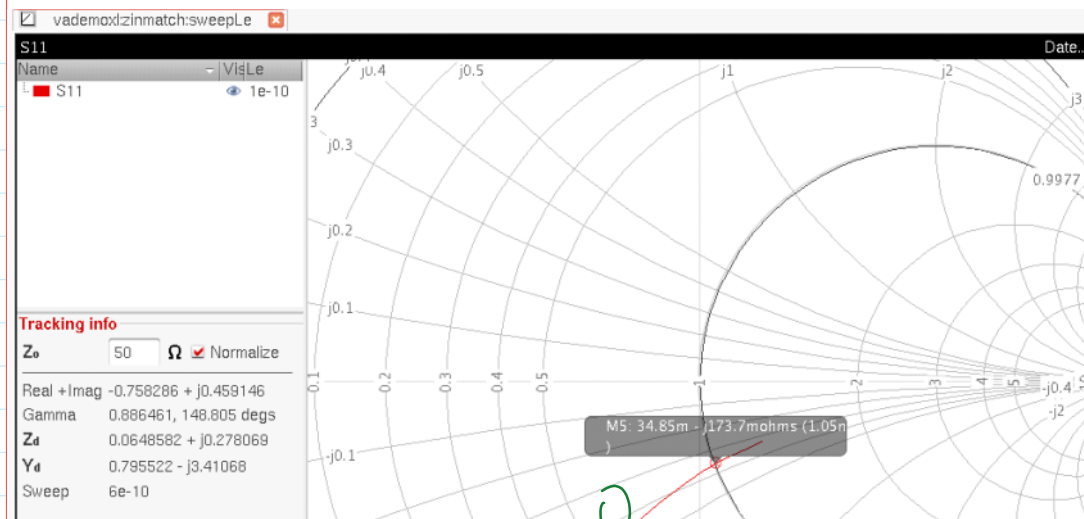
Then an base inductor L_b can be adjusted to make the total imaginary part zero.

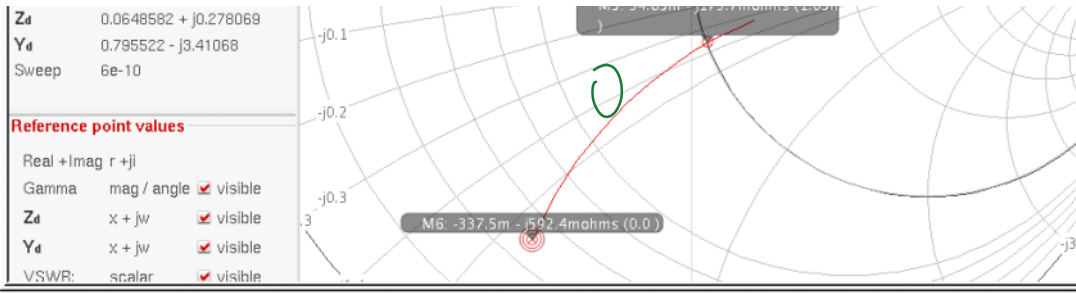
With the "mult" (size) determined above, we sweep emitter inductance L_e from 0 to 2nH,

We plot out S_{11} and see that as we sweep L_e , N_{fmin} and R_{opt} do not change for all practical purposes (tiny bit),



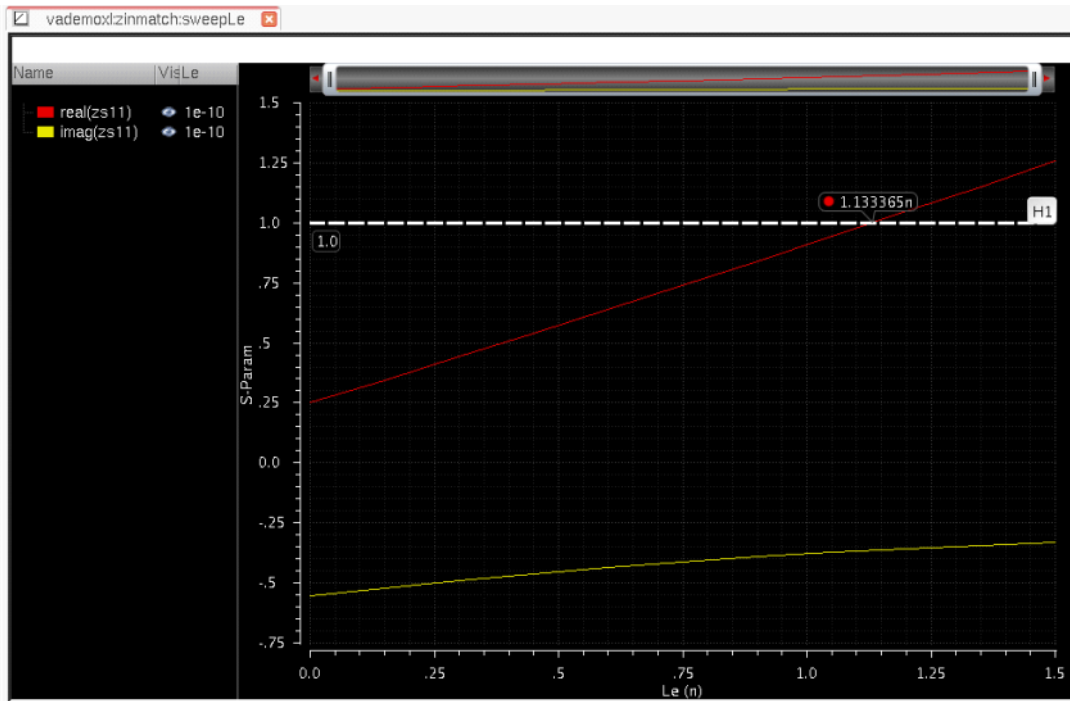
However, the real part of z_{s11} (or Z_{in} for 50ohm load) increases, of course the imaginary part increases some too,





Better yet, we can do a transform from s_{11} to z_{s11} (see previous notes if doubts), and plot out the real and imag of z_{s11} versus

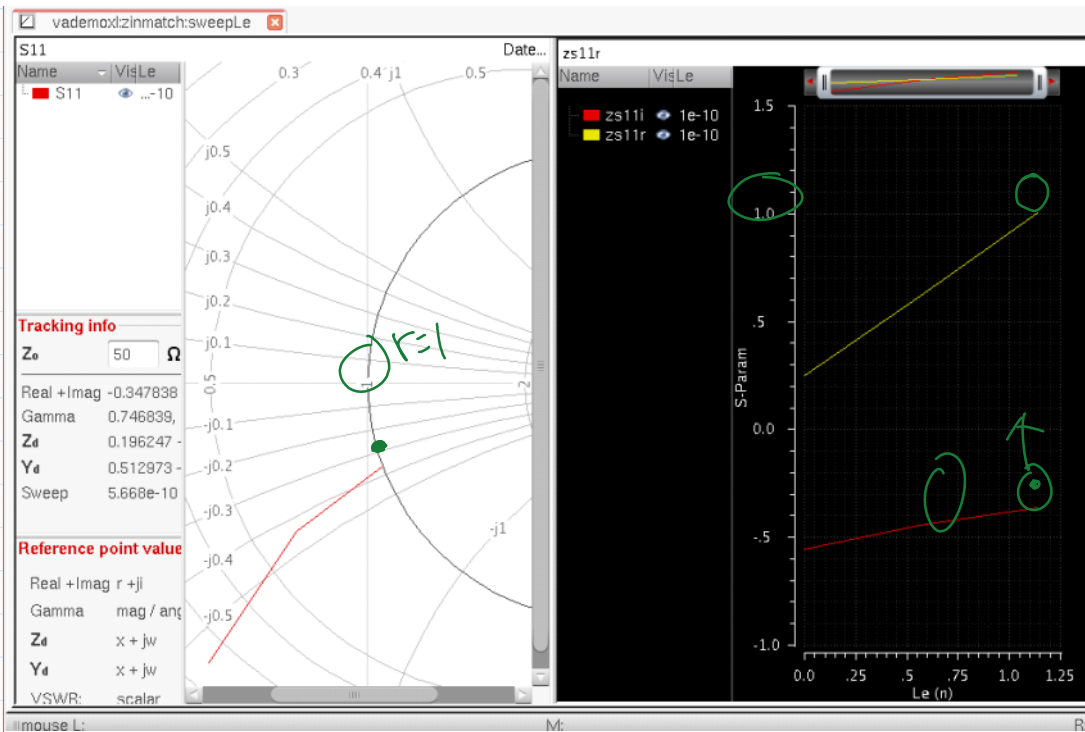
Le:



We can label the $Le=0$ and $Le=1.136$ nH points to see the difference - **this can be proven from circuit analysis, that is, adding emitter inductor can produce a real part in the input impedance at RF.**

This applies to MOSFET too, and is used in both RF SiGe, III-V and CMOS LNA designs.

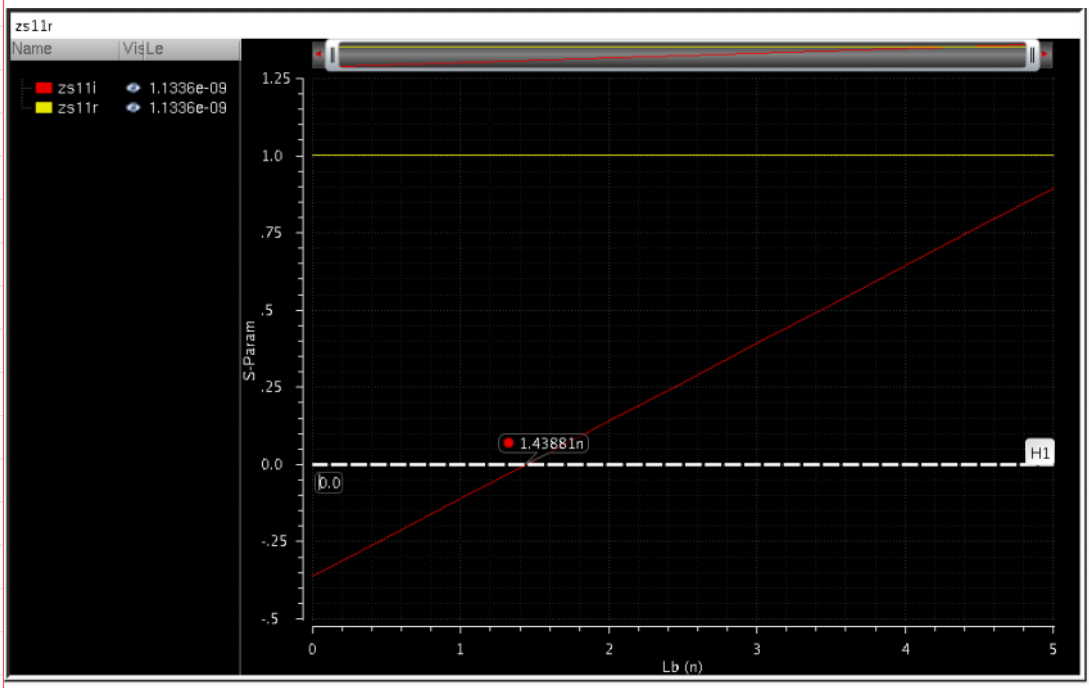
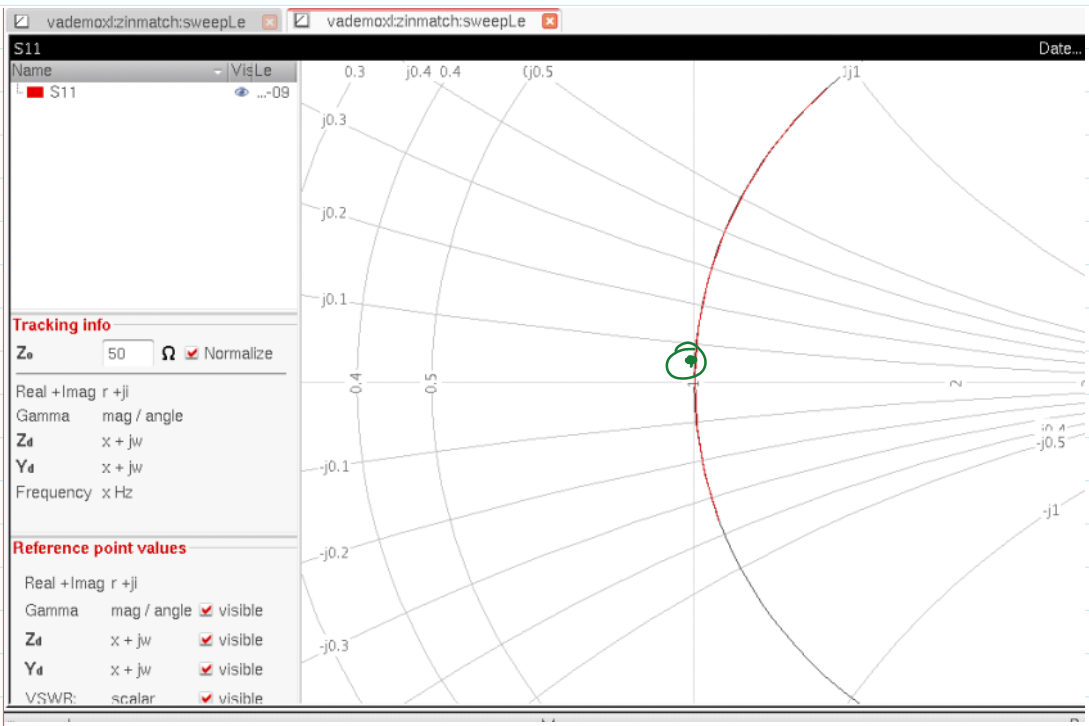
Modify your Le upper limit in sweep to 1.1336nH as found above:



We see from both Smith and zs11r plots that **we can indeed produce a 50ohm real part of input impedance by adding proper amount of emitter inductance.**

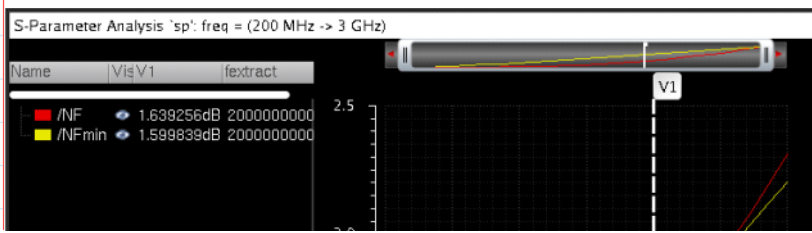
Next let us set $Le = 1.1336\text{nH}$ and **sweep Lb to move S_{11} to center of Smith, or to make the imaginary part of Z_{s11} zero.**

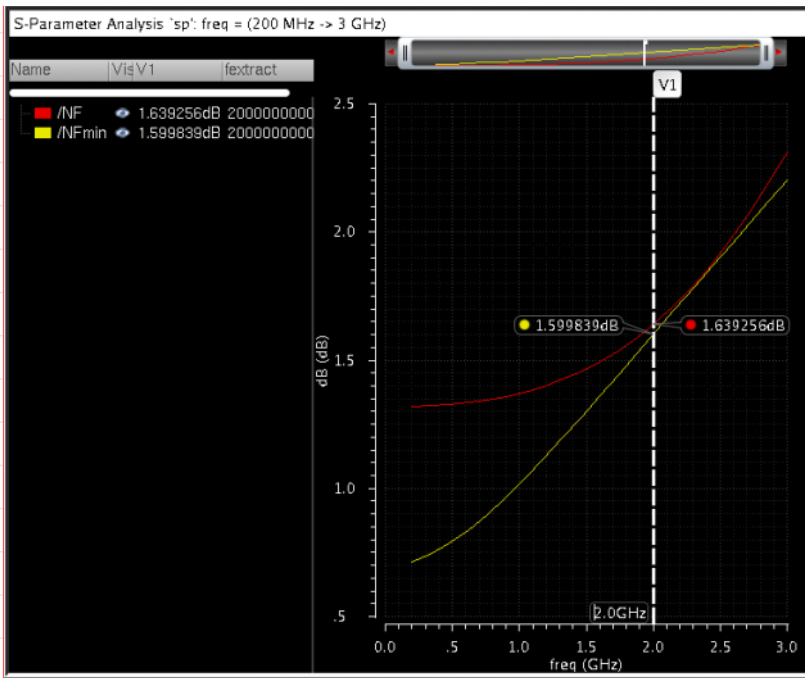
Now sweep Lb , observe that $zs11r$ (real part) does NOT change, while $zs11i$ (imag prt) changes.



The L_b value required is 1.438 nH. Now we have determined the size (mult), base inductance L_b and emitter inductance L_e .

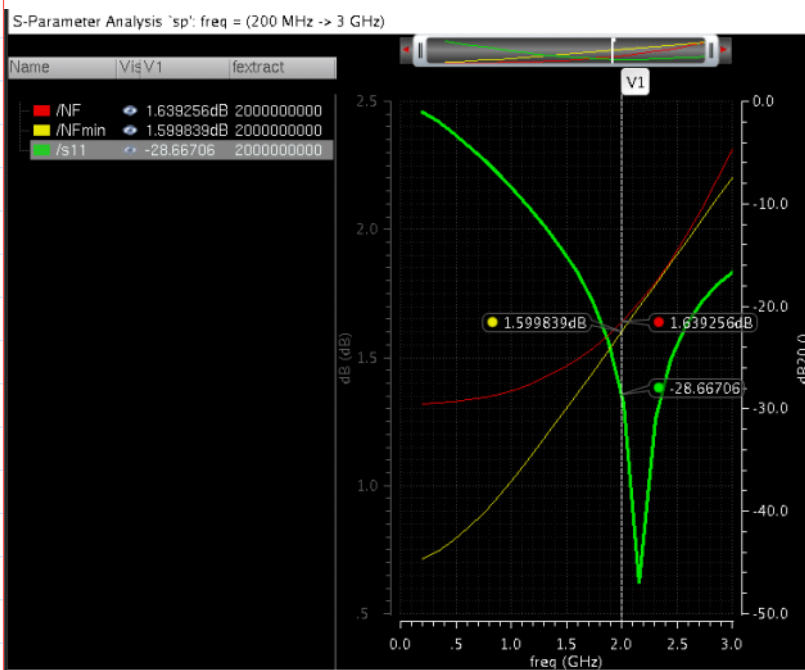
Now put all the values in, do a freq sweep from 0.2 to 3GHz,





We see that the noise figure is very close to Nfmin at 2GHz,
 and
 this is also very close to Nfmin of the transistor alone at the
 same
 transistor size and bias.

Now add S11,



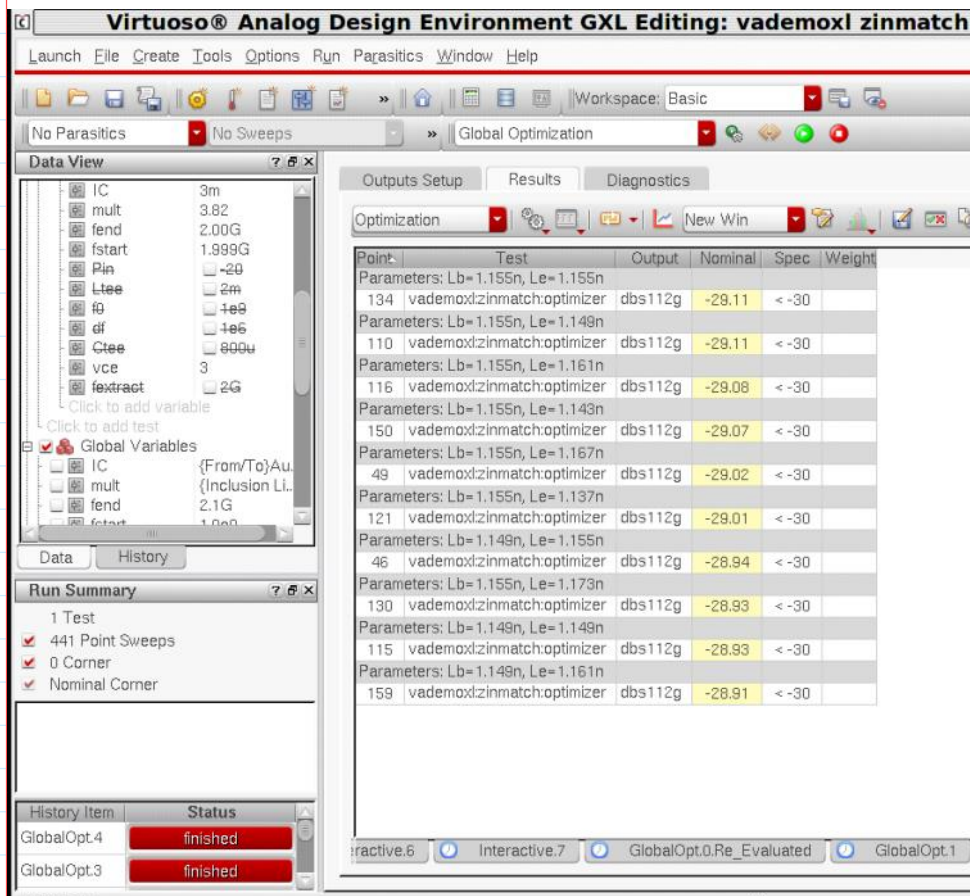
The s11 dip does not happen precisely at 2GHz, which is
 normal

considering that we did not use optimizer. We can of course make the dip center at 2Ghz in design. The L's and their changes will eventually make the measured S11 worse than simulated / designed.

Optimization in Cadence for impedance matching is not easy to do compared to say in ADS.

One thing I do is after I have found a solution using my design procedure, I'll run the optimizer near the solution. Without that, optimization simply fails.

Below is an optimization done using ADE-GXL:

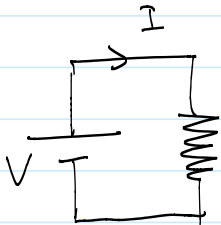


GlobalOpt.4 stopped because ADE GXL cannot find a better design point.
Number of points completed: 225
Number of simulation errors: 0
GlobalOpt.4 completed.
Current time: Sun Nov 11 16:35:23 2012

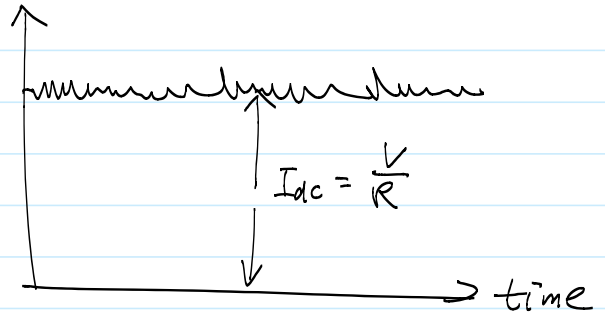
noise

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noise



$$I = \frac{V}{R} + \text{noise}$$



* actually only the dc component is $\frac{V}{R}$.

* There is "noisy" current even if $V=0$

$$i = I_{dc} + \bar{i}_n(t)$$

* $\overline{\bar{i}_n(t)} = 0$ average is zero

* instant value is unpredictable

* In noise work, we talk about the rms value

(root mean square). $\bar{i}_{n,rms} \triangleq \sqrt{\overline{i_n^2}}$

for voltage $\bar{v}_{n,rms} \triangleq \sqrt{\overline{v_n^2}}$

* If there are two sources of noise

e.g. \bar{i}_1, \bar{i}_2 . $\overline{i_n^2(t)} = \overline{i_1^2(t)} + \overline{i_2^2(t)} + 2\overline{i_1(t)i_2(t)}$

$$+ \overline{i_{n1}(t) i_{n2}(t)}$$

* In measuring noise, the amount of noise depends on the bandwidth of the measuring system

a very narrow bandwidth Δf is typically involved

centered at f , as $\Delta f \rightarrow 0$ $\frac{\overline{i_n^2(t)}}{\Delta f} \rightarrow \underline{\underline{S_I(f)}}$

$$\left. \begin{array}{l} \overline{i_n^2} \quad A^2 \\ \Delta f \quad Hz \end{array} \right\} \rightarrow S_I \quad A^2/Hz$$

PSD
Power Spectral Density

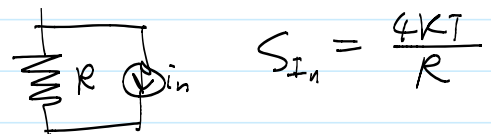
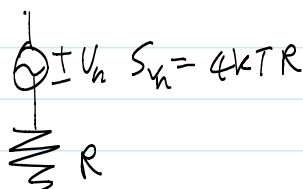
for voltage $S_V \cdot V^2/Hz$

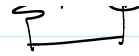
The sqrt of the PSD is also often used.

* The total mean square current from f_1 to f_2 is

$$\overline{i_n^2}(f_1 \rightarrow f_2) = \int_{f_1}^{f_2} S_I(f) df$$

* Thermal noise





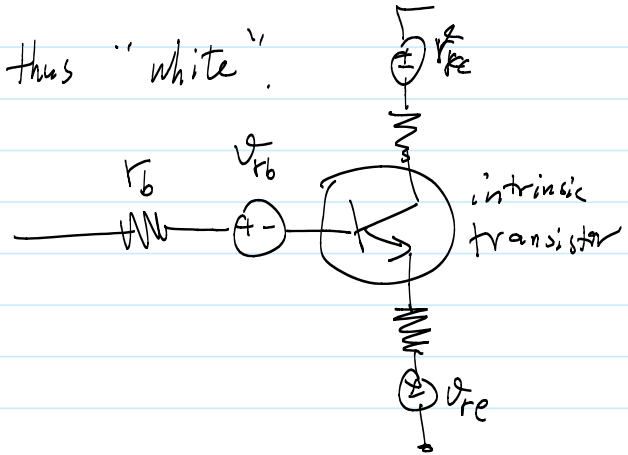
S_{V_n}
 S_{I_n} are independent of f . thus "white".

thermal noise in SiGe HBT

$$S_{V_{R_b}} = 4kT R_b$$

$$S_{V_{R_e}} = 4kT R_e$$

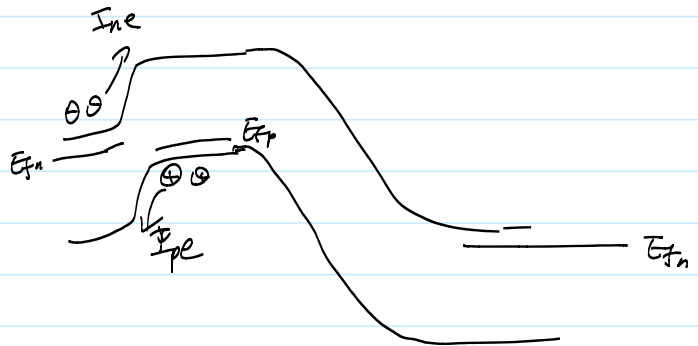
$$S_{V_{R_c}} = 4kT R_c$$



R_b is at the input, and most important

SiGe HBT allows high base doping due to base E_g reduction, \Rightarrow less R_b than Si BJT for the same β
 \Rightarrow less $4kT R_b$ thermal noise

* Shot noise



Carriers crossing a barrier independently

each passing of e or h delivers a q charge

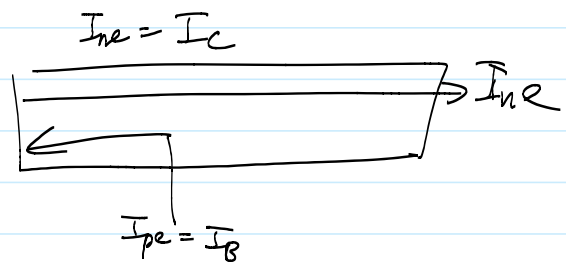
The flow of "q" obeys Poisson statistics, \Rightarrow

math / statistics yields $\Rightarrow S_{I_{ne}} = 2q I_{ne} = 2q I_c$

$S_{I_{pe}} = 2q I_{pe} = 2q I_B$

if $S_I = 2q I$

it is called "shot" noise



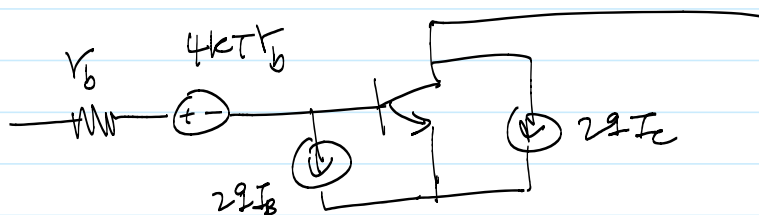
I_c current has $2q I_c$ noise PSD

I_B - — $2q I_B$ noise PSD

often PSD is left out

$S_{I_B} = 2q I_B = \frac{2q I_c}{\beta}$ * since increases β
 so since HBTs have less S_{I_B}

A good first order white noise ckt



* Keep in mind that the $2qI_B$ or $2qI_E$ shot noise assumes that V_{BE} and V_{CE} are "fixed" ideal voltage sources, i.e. ac short circuit condition

If we have finite source/load resistances, the S_{ib} S_{ic} measured would differ

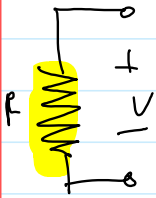
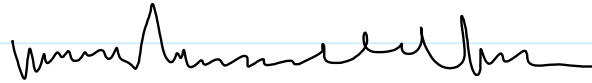
* In noise work, we are mostly interested in the "noise source",

* Short circuit noise currents are such noise sources

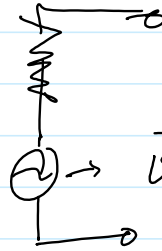
Thermal noise (Johnson noise)

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① Thermal noise

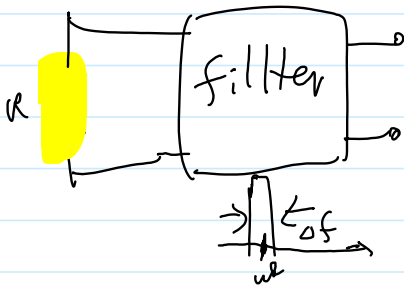


$$\begin{aligned} \overline{V} &= 0 \\ \overline{V^2} &\neq 0 \end{aligned} \Rightarrow$$



$$\overline{V^2} = 4kTR\Delta f$$

mean square value



$$\overline{V^2} = 4kTR\Delta f$$

Δf
filter bandwidth

white noise.

The power of the noise output voltage is

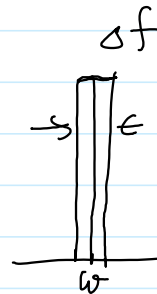
the same as that of a sin wave

with



$$\frac{A_n^2}{2} = 4kTR\Delta f$$

$$V_n = A_n \cos \omega t$$



ω being center freq. of the filter

* A_n is zero to peak.

$$* \overline{V_n^2}_{rms} = \frac{A_n^2}{2} = 4kTR\Delta f$$

$$* \quad V_{n,rms}^2 = \frac{A_n^2}{2} = 4kTR \Delta f,$$

It is very important to keep this point in mind as we proceed, because it simplifies noise analysis to regular ac circuit analysis.

That is, **we can treat noise at a given frequency, for a small band width Δf , as a sine wave as far as noise power is concerned.**

* often, $\frac{V_{n,rms}^2}{\Delta f}$ or $\frac{V_{n,rms}}{\sqrt{\Delta f}}$ is used in calculations.

$$S_V \triangleq \frac{V_{n,rms}^2}{\Delta f} = 4kTR$$

$$R = 1k\Omega$$

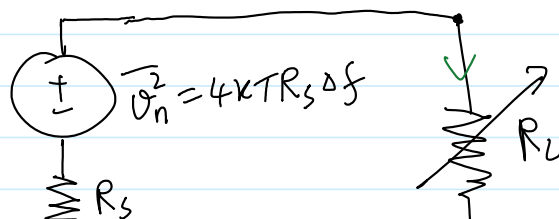
$$T = 290K$$

$$S_V \triangleq \frac{V_{n,rms}}{\sqrt{\Delta f}} = \sqrt{4kTR}$$

at $T = 290K$, $S_V = 4nV \cdot \sqrt{Hz}$ for $R = 1k\Omega$

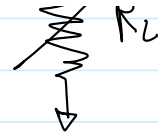
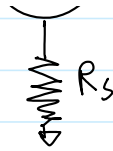
$$S_V^2 = 1.6 \times 10^{-17} V^2/Hz$$

What is S_V for 50Ω at $290K$?



$$R_L = R_s$$

Example 1.



$$R_L = R_s$$

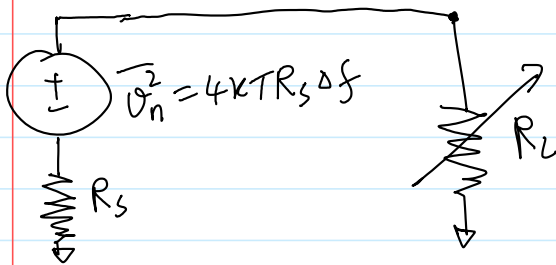
Example 1:

Assume that $k =$ **Boltzmann constant** = 1.38066×10^{-23} J/K. Bandwidth Δf is **1Hz**.

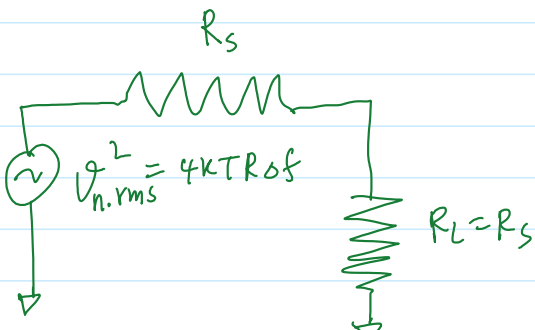
$$T = 290K$$

Apply the equivalence you just learned above to find out the maximum amount of thermal noise power that can be delivered to the noiseless load for any possible R_L values. Then convert that number to dBm.

Is your result dependent on the value of R_s ?

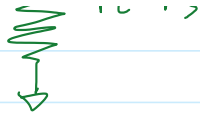


R_s is fixed. when $R_L = R_s$. P_{R_L} is maximized



$$P_{\text{noise available}} = \frac{V_{n,rms}^2}{R_s + R_s} \cdot \frac{1}{2}$$

$$= \frac{4kTR_s \Delta f}{2} = kT \Delta f$$



$$= \frac{4kTR_s \Delta f}{4R_s} = kT\Delta f$$

$$= kT \quad \text{for } \Delta f = 1\text{Hz}$$

tnoise = 290 #k

kb = 1.38066e-23 #k

```
def tnoise_floor(bw=1):
```

```
    kt = kb * tnoise
```

```
    power = kt * bw
```

```
    return dbm(power)
```

-174 dBm/Hz

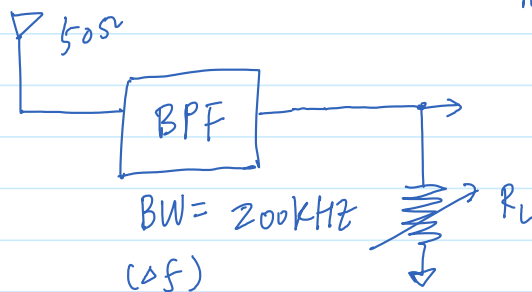
$$-173.975152593 \text{ dBm} \approx -174 \text{ dBm} \quad \Delta f = 1\text{Hz}$$

Remember this important result & its meaning.

Quiz:

Find P_{noise} available for 200 kHz bandwidth.

$$\Delta f = 200 \text{ kHz}$$



Solution: $P_{n.a} = -174 \text{ dBm/Hz} \times 200 \text{ kHz}$

$$= -174 * 200e3 = -34,800,000 \text{ dBm}$$

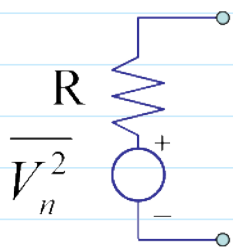
* solution $P_{\text{noise available}} = kT \cdot \Delta f$

$$\frac{P_{n.a.}}{1mW} = \frac{kT}{1mW} \cdot \Delta f$$

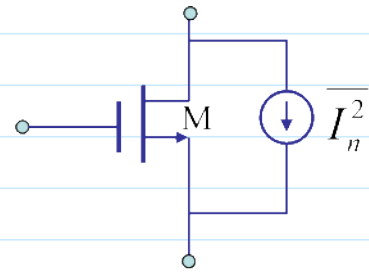
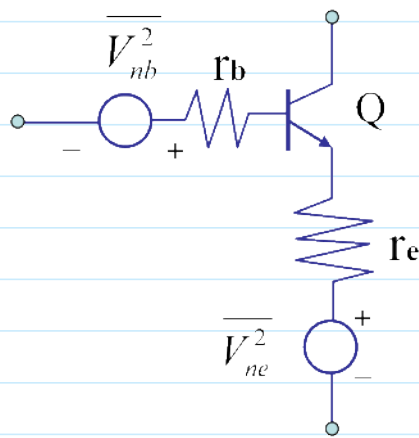
$$10 \log_{10} \frac{P_{na}}{1mW} = 10 \log_{10} \frac{kT}{1mW} + 10 \log_{10} \Delta f$$

$$P_{na} (dbm) = -174 + 10 \log_{10} \cdot 200 \cdot 10^3$$

$$-174 + 10 * \log_{10}(200e3) = -120.9897000433602$$



$$\overline{V_n^2} = 4kTR\Delta f$$



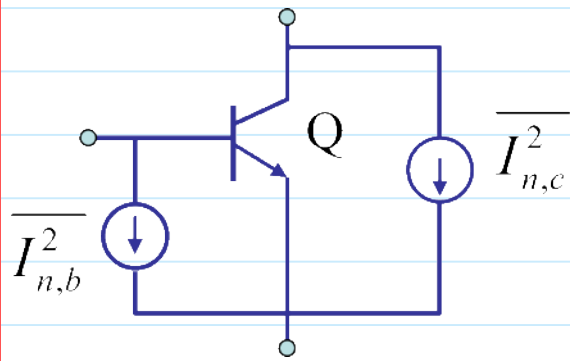
$$\overline{I_n^2} = 4kT \left(\frac{2}{3} g_m \right) \Delta f$$

>2/3 for submicro MOS

1 2

Bipolar transistor shot noise

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$$i_{bn} = \sqrt{2qI_B}$$

$$i_{cn} = \sqrt{2qI_C}$$

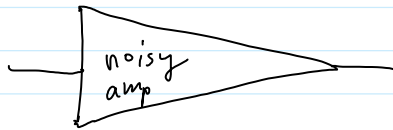
$$\overline{I_n^2} = 2qI\Delta f$$

Noise figure

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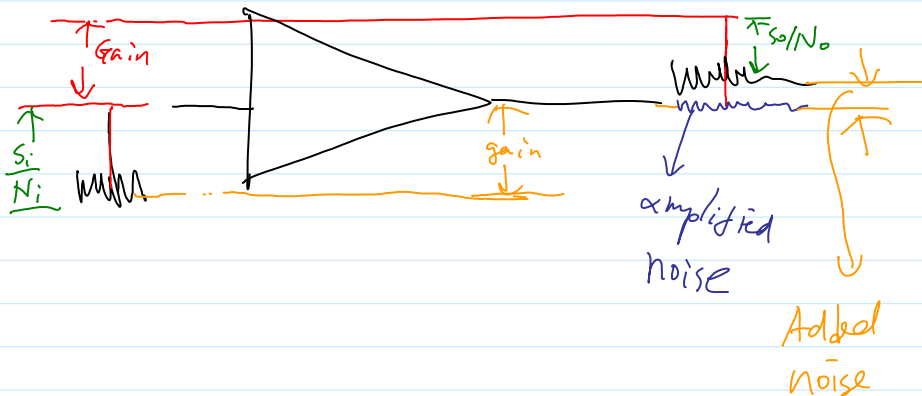
Noise figure:

purpose: describe how noisy amplifier is



how much noise is added by amp?

Definition



At the input of your amplifier you always have some noise (say from its 50ohm source), and some signal.

You amplifier has no knowledge of what is noise and what is signal, and will amplify both.

If the amp does not add any noise, SNR at input S_{in}/N_{in} will be the same as SNR at output S_{out}/N_{out} ,

That is a noiseless amplifier - that does NOT exist in practice.

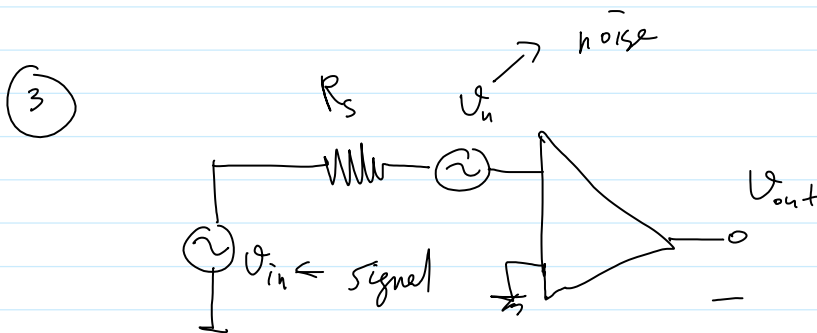
Amplifier adds some noise, so S_{out}/N_{out} will be **smaller**

than S_{in}/N_{in} .

Note ① we are talking about power

$$F \triangleq \frac{S_{in}/N_{in}}{S_{out}/N_{out}}$$

$$NF \triangleq 10 \log_{10} F = \text{dB}$$



in terms of power

$$S_{out} = S_{in} \cdot \text{Gain}$$

$$N_{out} = N_{in} \cdot \text{Gain} + N_{added}$$

$$F \triangleq \frac{S_{in}}{N_{in}} \cdot \frac{S_{out}}{N_{out}} = \frac{S_{in}}{N_{in}} \frac{N_{in} \cdot \text{Gain} + N_{added}}{S_{in} \cdot \text{Gain}}$$

$$= 1 + \frac{N_{added}}{N_{in} \cdot \text{Gain}}$$

So once we know N_{added} , F is known

N_{added} : output noise generated by Amp

$N_{in, gain}$: output noise generated by the source

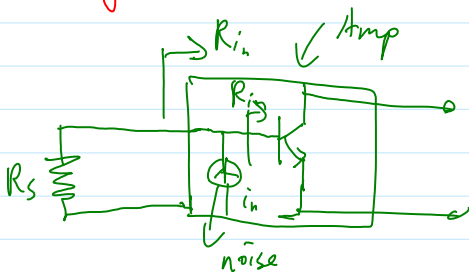
So F can also be written as

$$F = 1 + \frac{\text{Output noise due to Amp}}{\text{Output noise due to source}}$$

* R_S has a lot to do with

N_{added} .

easy to understand



if $R_S = 0$
 i_{in} will be
shorted.
 \Rightarrow no output

due to Amp.

but no output due to R_S

either ↓↓

$$F = 1 + \frac{N_{added}}{N_{in} \cdot Gain}$$

if $R_s = \infty$.

$$N_{added} = \langle i_n^2 \rangle \cdot \beta^2 \cdot R_{load}^2$$

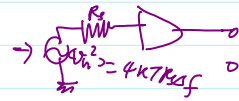
$$N_{in} \cdot Gain = 0$$

$$F = \infty$$

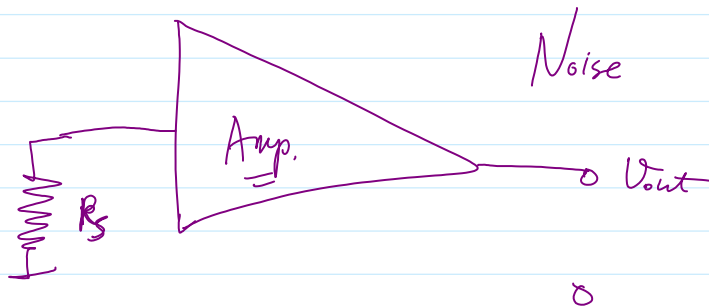
So noise factor depends on a lot R_s

$$F = 1 + \frac{N_{added}}{N_{in} \cdot Gain}$$

depends on R_s

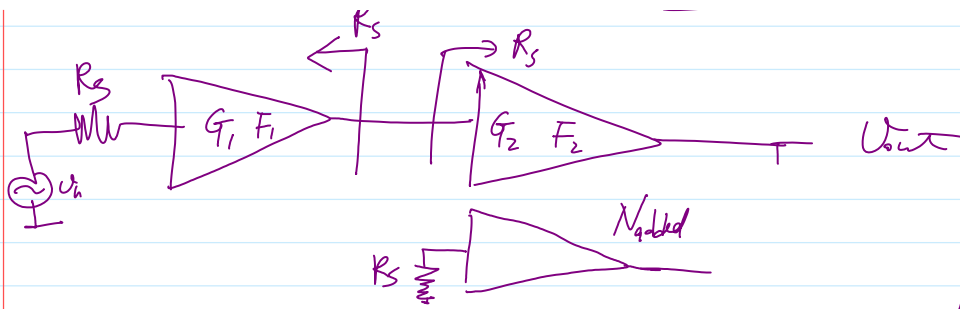


$$N_{added} = (F - 1) N_{in} \cdot Gain$$



$$N_{out} = N_{in} \cdot Gain + \underline{\underline{N_{added}}}$$





$$N_{\text{added}}|_2 = (F_2 - 1) \cdot N_{\text{in}}|_{R_S} \cdot G_2$$

$$F = 1 + \frac{N_{\text{added}}}{N_{\text{due to source}}}$$

$$= 1 + \frac{N_{\text{added}}|_{1st} \times G_2 + N_{\text{added}}|_{2nd}}{N_{\text{in}} \cdot G_1 \cdot G_2}$$

Key: Noise added by 1st stage will be amplified by the 2nd stage!

$$(F-1)|_{\text{total}} = \frac{N_{\text{added}}|_{1st}}{N_{\text{in}} \cdot G_1} + \frac{N_{\text{added}}|_{2nd}}{(N_{\text{in}} \cdot G_2) \cdot G_1}$$

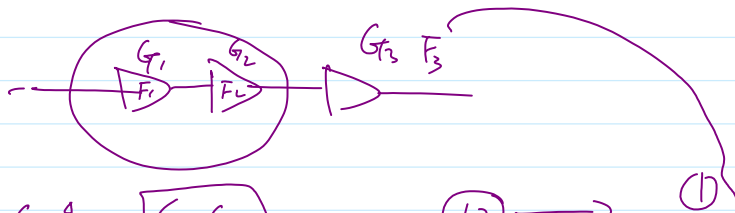
↑↑

$$N_{\text{added}}|_{1st} = (F_1 - 1) \cdot N_{\text{in}} \cdot G_1$$

$$N_{\text{added}}|_{2nd} = (F_2 - 1) \cdot N_{\text{in}} \cdot G_2$$

What about 3 stages? Just combine the first 2 stages with a gain of $G_1 \cdot G_2$, then apply the above procedure again:

$$(F-1)|_{\text{total}} = (F_1 - 1) + \frac{F_2 - 1}{G_1}$$

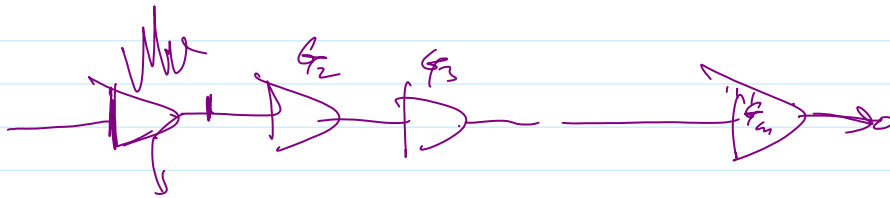


$$G_{1,2} = \boxed{G_1 G_2} \leftarrow (12) \rightarrow \begin{matrix} (1) \\ (2) \end{matrix}$$

$$(F-1)_{12}^0 = \boxed{(F_1 - 1) + \frac{F_2 - 1}{G_1}}$$

$$(F-1)_{total} = (F_1 - 1) + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{\boxed{G_1 G_2}}$$

① Overall NF is dominated by early stages close to source



② $NF \iff R_s$

$$4kTR_s \Delta f$$

③ $290K \Rightarrow 1888$ source temperature

④ Δf = narrowest BW of all filters

★ Question:

What is the noise figure of a perfect amplifier that has no noise inside?

$$F_{tot} = 1 + (F_1 - 1) + \frac{F_2 - 1}{G_{p1}} + \frac{F_3 - 1}{G_{p1} G_{p2}} + \dots + \frac{F_n - 1}{G_{p1} G_{p2} \dots G_{p(n-1)}}$$

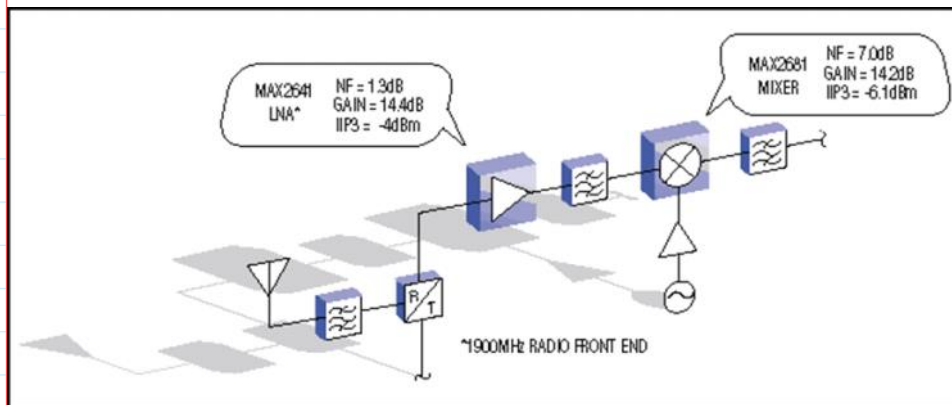
F_{tot} – total equivalent Noise Factor

F_m – Noise Factor of m^{th} stage

G_{pm} – Available power gain of m^{th} stage

Design Example: Typical RF front end circuitry

Implemented with Maxim GST-3 SiGe processing, (f_T)= 35GHz



Long and Comprehensive Version

Monday, November 12, 2012 5:17 PM

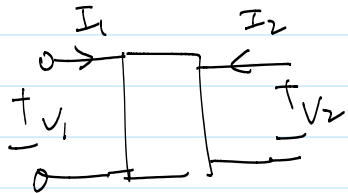
This will show the long version, with no power constraint.

This section also has more device relevant material.

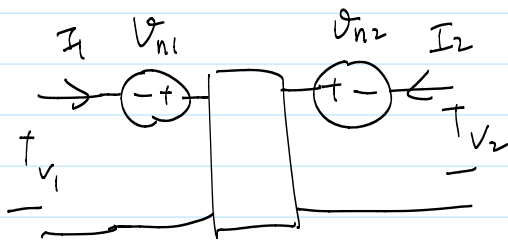
Equivalent noise representations

Thursday, September 06, 2012 2:54 PM

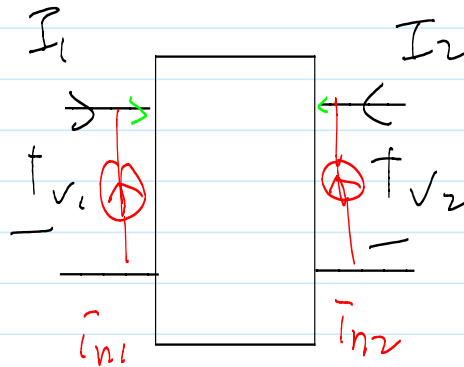
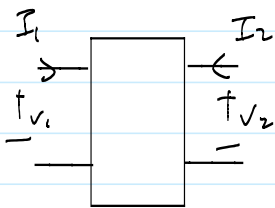
* There are always equivalent ways of circuit representations



$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$



$$\begin{pmatrix} V_1 + V_{n1} \\ V_2 + V_{n2} \end{pmatrix} = \begin{pmatrix} z \\ z \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$



$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$$\begin{pmatrix} I_{n1} \\ I_{n2} \end{pmatrix} + \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

or

$$I_{n1} = 2gI_B$$

for the intrinsic transistor
without any "r"

or

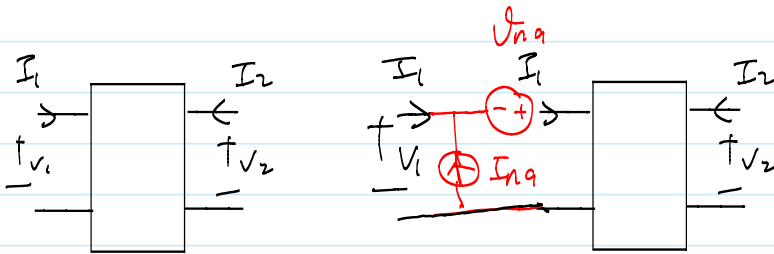
$$\begin{pmatrix} I_{n1} \\ I_{n2} \end{pmatrix} + \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = Y \cdot \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$\rightarrow I_{n1} = \dots$

$$S_{Inr} = 2g I_c$$

for ...
without any ...

Chain representation, input V and I noise definitions



$$V_i \rightarrow V_i + V_{na}$$

$$I_i \rightarrow I_i + I_{na}$$

$$\begin{pmatrix} I_1 + I_{na} \\ I_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_1 + V_{na} \\ V_2 \end{pmatrix} \Rightarrow \begin{pmatrix} I_{na} \\ 0 \end{pmatrix} + \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = Y \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} + Y \cdot \begin{pmatrix} V_{na} \\ 0 \end{pmatrix}$$

\Rightarrow

Noise parameters

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Noise parameters

noise figure

$$NF = 10 \log_{10} F$$



F : noise factor

$$Y_s = G_s + jB_s$$

$$F = F_{min} + \frac{R_n}{G_s} |Y_s - Y_{opt}|^2$$

① $F \rightarrow$ minimum F_{min} when $Y_s = Y_{s,opt}$

② $|F - F_{min}|$ the deviation depends on R_n
 R_n determines sensitivity to mismatch

often Γ_{opt} is used in measurement

$$\Gamma = \frac{1 - YZ_0}{1 + YZ_0}$$

$$Z_0 = 50 \Omega$$

$$R_n, Y_{s,opt} = G_{s,opt} + jB_{s,opt}$$

$$R_n = \frac{S_{Vn}}{4kT}$$

$$G_{s,opt} = \sqrt{\frac{S_{In}}{S_{Vn}} - \left[\frac{\text{Im}(S_{In} V_n^*)}{S_{Vn}} \right]^2}$$

from $R_n, NF_{min}, Y_{s,opt}$

S_{Vn}, S_{In} and $S_{In} V_n^*$

can also be calculated.

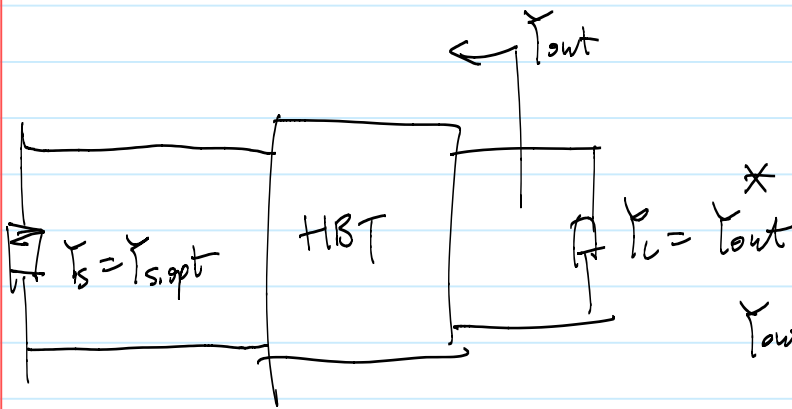
$$G_{s,opt} = \sqrt{\frac{S_{I_n}}{S_{V_n}} - \left| \frac{S_{I_n}}{S_{V_n}} \right|}$$

$$B_{s,opt} = - \frac{\text{Im}(S_{I_n} V_n^*)}{S_{V_n}}$$

$$F_{min} = 1 + 2R_n \left(G_{s,opt} + \frac{\text{Re}(S_{I_n} V_n^*)}{S_{V_n}} \right)$$

$G_A = G_T$ | noise matching for I_s
conjugate matching for Y_L

$$= \left| \frac{Y_{21}}{Y_{11} + Y_{s,opt}} \right|^2 \frac{G_{s,opt}}{G_{out}}$$



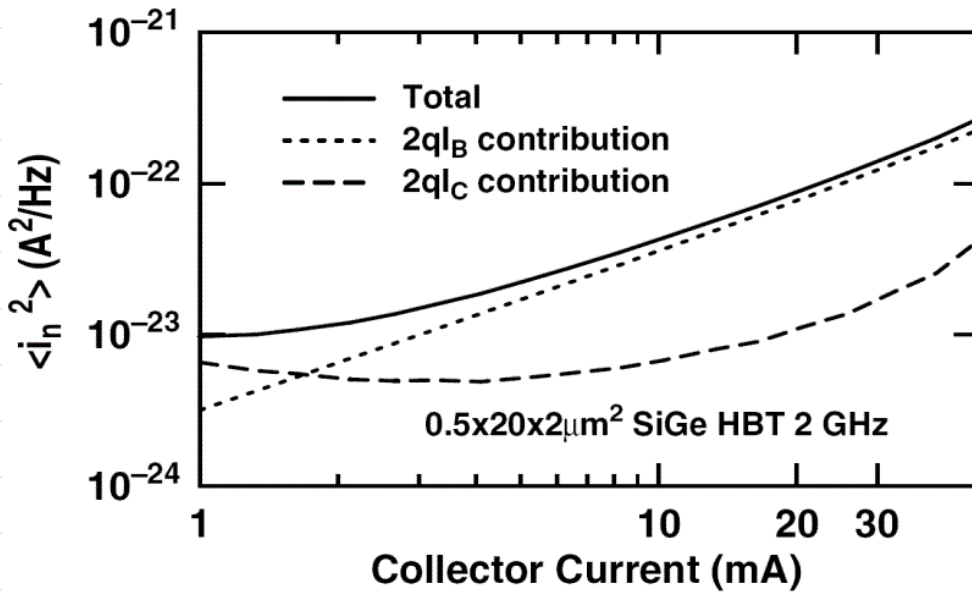
$$G_{out} = \text{Re}(Y_{out})$$

$$Y_{out} = Y_{22} - \frac{Y_{12} Y_{21}}{Y_{11} + Y_{s,opt}}$$

Input noise current limitation in siGe HBTs

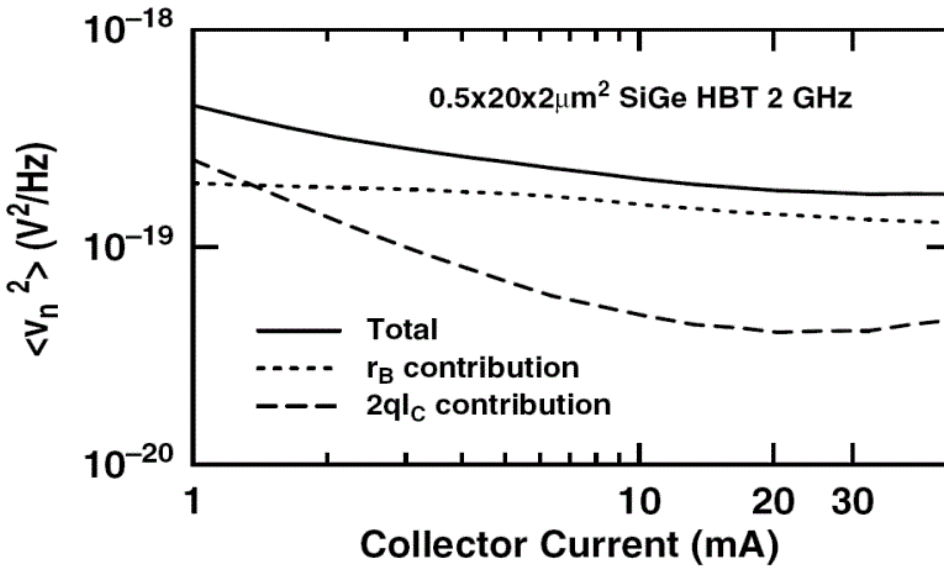
$$S_{I_n} = 2qI_B + \frac{2qI_C}{|H_{21}|^2}$$

$$= 2qI_C/\beta + \dots$$



Input noise voltage limitation

$$S_{V_n} = 4kTR_b + \frac{2qI_C}{|Y_{21}|^2}$$



$$Y_{21} = g_m = \frac{I_C}{V_T}$$

Approaches to noise Improvement

• β Ge content in base

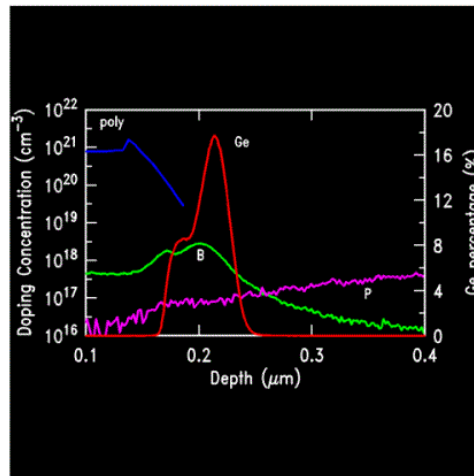
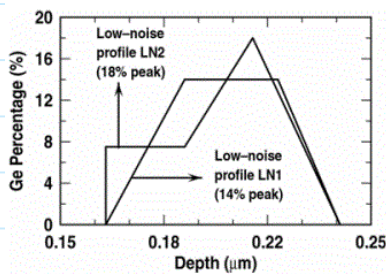
② h_{21} Ge grading. profile optimization

③ V_b thermal cycle limit, total # of boron dose kept in base after fabrication

Having noise equations allows us to identify dominant performance limits, is it β , $h_{21}(f_T)$ or V_b ???

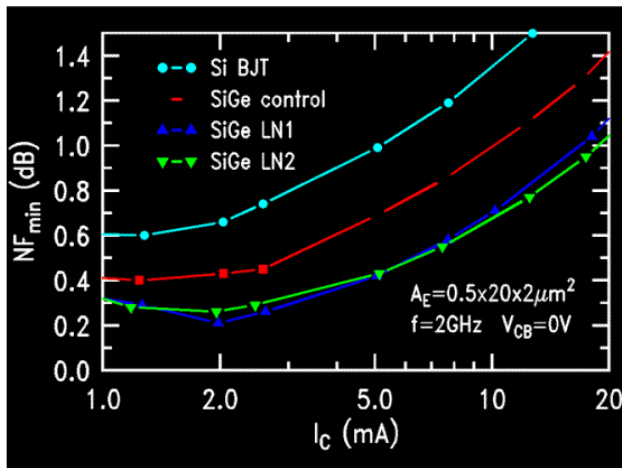
Optimum Digital Ge Profiles

- More Ge in the base is needed for higher beta
- Ge edge must be pushed towards the surface



NF_{min} (2GHz) vs Profile

- Higher beta and f_T are translated into lower RF noise



Conversion between representations

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Conversion Between Different noise formats.

Example. I_{n1} I_{n2} to V_{na} I_{na} conversion

$$\begin{aligned}
 \text{(a)} \quad \begin{pmatrix} I_{na} \\ 0 \end{pmatrix} + \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} &= (Y) \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} + (Y) \begin{pmatrix} V_{na} \\ 0 \end{pmatrix} \\
 \text{(b)} \quad \begin{pmatrix} I_{n1} \\ I_{n2} \end{pmatrix} + \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} &= Y \cdot \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}
 \end{aligned}
 \Rightarrow \begin{pmatrix} I_{n1} \\ 0 \end{pmatrix} - \begin{pmatrix} y_{11} V_{na} \\ y_{21} V_{na} \end{pmatrix} = \begin{pmatrix} I_{n1} \\ I_{n2} \end{pmatrix}$$

$$\begin{aligned}
 V_{na} &= - \frac{I_{n2}}{Y_{21}} \\
 I_{na} &= I_{n1} + Y_{11} \cdot V_{na} \\
 &= I_{n1} - \frac{Y_{11}}{Y_{21}} \cdot I_{n2}
 \end{aligned}$$

Input noise voltage and current, chain representation,

without thermal noise. $I_{n1} \rightarrow 2qI_B$ $S_{In1} = 2qI_B$
 $I_{n2} \rightarrow 2qI_C$ $S_{In2} = 2qI_C$
 $I_{n1} I_{n2}^* = 0$

$$V_{na} = - \frac{I_{n2}}{Y_{21}}$$

$$S_{Vn} = S_{Vna} = \frac{V_{na} V_{na}^*}{\Delta f} = \frac{S_{I_{n2}}}{|Y_{21}|^2}$$

$$\begin{aligned}
 I_{na} &= I_{n1} + Y_{11} \cdot V_{na} \\
 &= I_{n1} - \frac{Y_{11}}{Y_{21}} \cdot I_{n2} \\
 &= I_{n1} - \frac{I_{n2}}{11}
 \end{aligned}$$

$$\begin{aligned}
 S_{In} = S_{Ina} &= \frac{I_{na} I_{na}^*}{\Delta f} = S_{In1} + \frac{S_{In2}}{|H_{21}|^2} \\
 &= 2qI_B + \frac{2qI_C}{|H_{21}|^2} \quad \text{similarly}
 \end{aligned}$$

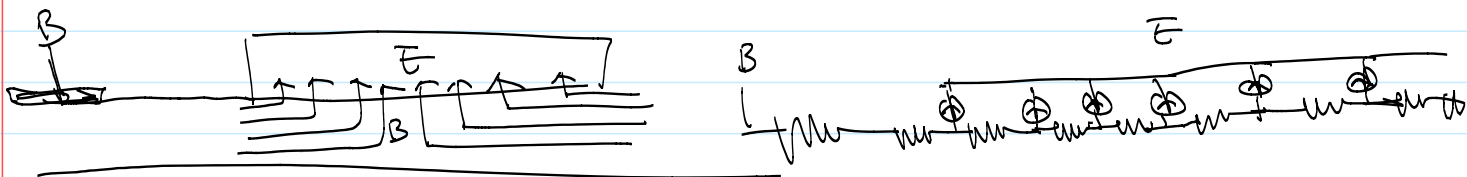
$$= I_{n1} - \frac{I_{n2}}{h_{21}}$$

$$H_{21}^o = \frac{Y_{21}}{Y_{11}}$$

$$S_{v_{in}}^* = 2g_{IC} \cdot \frac{Y_{11}^*}{|H_{21}|^2}$$

$$S_{in v_n}^* = [S_{v_{in}}^*]^* = 2g_{IC} \frac{Y_{11}}{|H_{21}|^2}$$

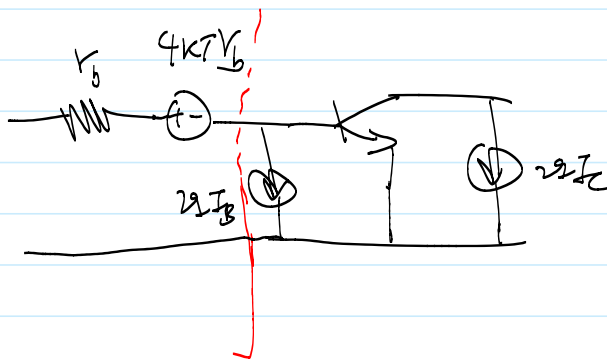
Similarly



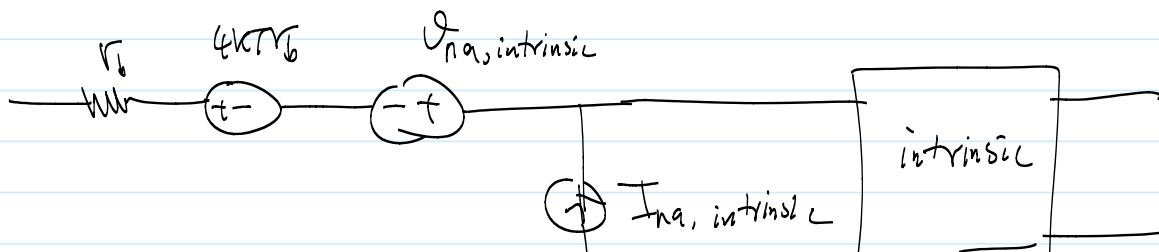
* r_b is
distributive
in nature

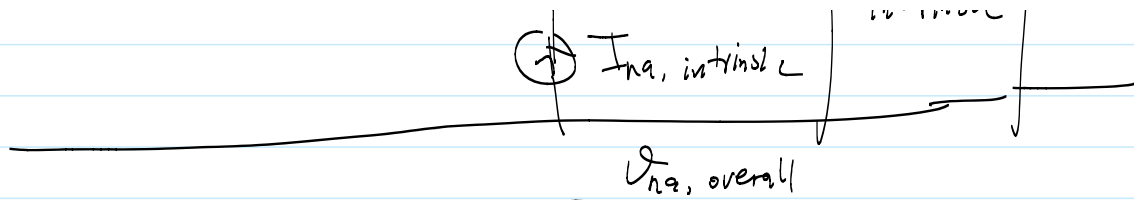
* It is a lumped
approximation

* We assume the noise
of distributive r_b is
also "lumped" as
 $4kT r_b$



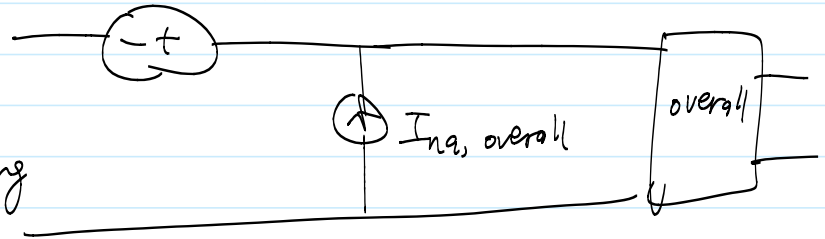
If we further look at chain representation





We can derive the

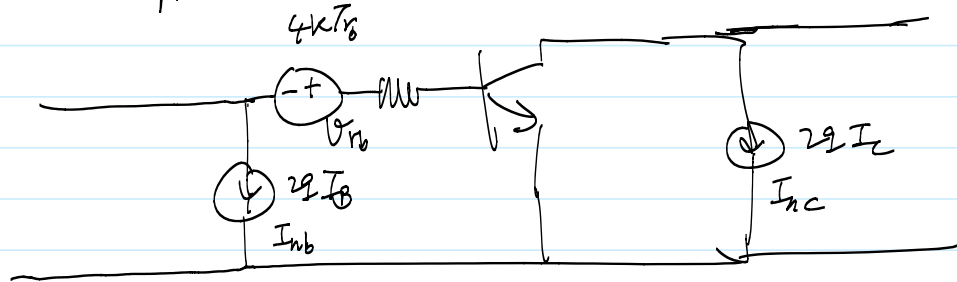
$V_{na, overall}$
 $I_{na, overall}$
 expressions using



$V_{na, intrinsic}$ $I_{na, intrinsic}$

As first order approximation, we may simply

assume



Then only S_{v_n} needs to be modified

$$S_{v_n} = \frac{29I_c}{|Z_{in}|^2} + 4kTr_b$$

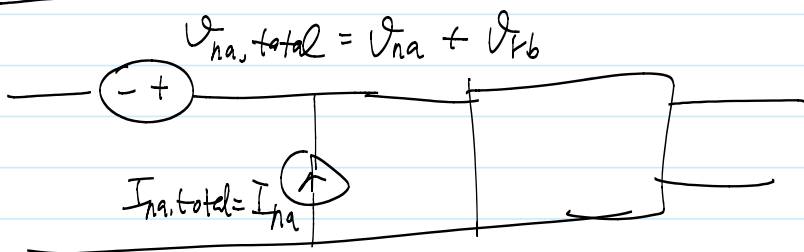
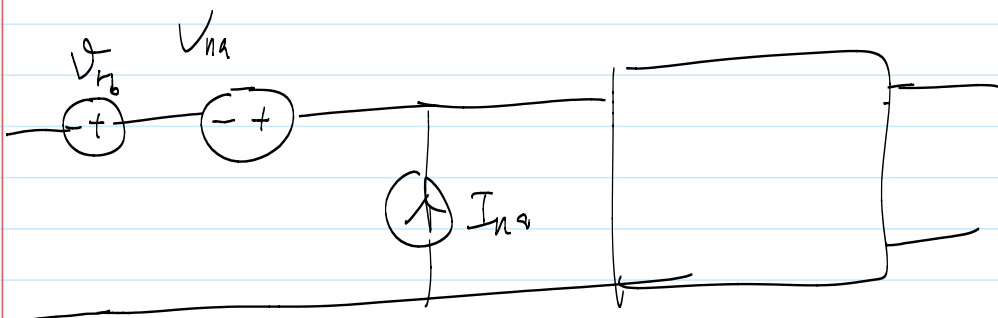
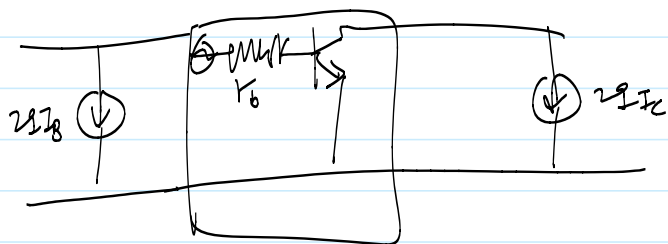
as ① I_{na} is not affected by V_{rb}

② $V_{na} = V_{na \text{ without } V_{rb}} + V_{rb}$

Graphics explanation is :



Graphics explanation is:



Summary of S_{Vn} S_{In} $S_{In} V_n^*$ in single HBTs

$$S_{Vn} = 4kTR_b + \frac{29I_C}{|Y_{21}|^2}$$

$$S_{In} = 29I_B + \frac{29I_C}{|H_{21}|^2} = 29I_C/\beta + \dots$$

$$S_{In} V_n^* = 29I_C \frac{r_n}{|Y_{21}|^2}$$

$$|Y_{21}| \approx g_m \approx \frac{I_C}{V_T}$$

$$|H_{21}| \times f \approx f_T \quad (|H_{21}| \approx \dots)$$

$$I_B = \frac{I_C}{\beta}$$

(1) Site allows high β and

low r_b at the same time \Rightarrow low S_{Vn} (4kTR_b) . low S_{In} ($29\frac{I_C}{\beta}$)

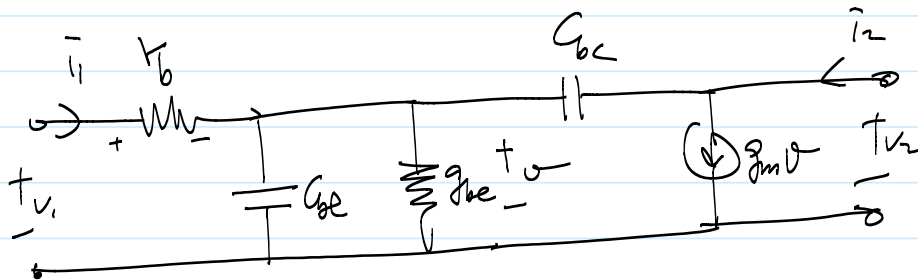
(2) Grading Ge \Rightarrow high $f_T \Rightarrow$ high $|h_{21}| \Rightarrow$ low S_{In} ($\frac{2I_E}{|h_{21}|^2}$)

Transistor noise parameters expressions

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Analytical models for noise parameters

- 1) express y-parameters using gm, beta, C, fT - variables one can associate with biasing, sizing, and device design (such as fT)
- 2) Calculate Svn, Sin and Sinvn*
- 3) Calculate noise parameters



"ignore" V_{be} voltage drop \rightarrow

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} g_{be} + j\omega C & -j\omega C_{bc} \\ g_m - j\omega C_{bc} & j\omega C_{bc} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$Y_{11} = \frac{g_m}{\beta} + j\omega C_i$$

$$C_i \triangleq C_{be} + C_{bc}$$

$$Y_{12} = -j\omega C_{bc}$$

$$C_{be} = g_m \tau_f + C_{te}$$

$$Y_{21} \approx g_m$$

$$g_m \approx \frac{I_c}{V_t} \quad \text{or replace } I_c \text{ with } g_m \cdot V_t$$

$$Y_{22} = j\omega C_{bc}$$

Recall $f_T \approx \frac{g_m}{2\pi C_i}$

Now we can express S_{in} S_{Vn} $S_{in} V_n^*$ using

familiar variables such as g_m , β , T_f , G_{te} , etc.

$\Rightarrow R_n$, $G_{s,opt}$, $B_{s,opt}$, F_{min} are known

① R_n

$$R_n = \frac{S_{Vn}}{4KT} = r_b + \frac{1}{2g_m}$$

low r_b is good for low sensitivity

derivation:

$$S_{Vn} = 4kTr_b + \frac{2qI_c}{|Y_{21}|^2}$$

$$I_c = g_m \cdot \frac{KT}{q}$$

$$Y_{21} = g_m$$

$$\frac{2q \cdot g_m \cdot \frac{KT}{q}}{g_m^2} = \frac{2KT}{g_m}$$

② $G_{s,opt}$

$$G_{s,opt} = \sqrt{\frac{S_{in}}{S_{Vn}} - \left[\frac{\text{Im}(S_{in} V_n^*)}{S_{Vn}} \right]^2}$$

derive it yourself

$$= \sqrt{\frac{g_m}{2R_n} \frac{1}{\beta} + \frac{(\omega C_i)^2}{2g_m R_n} \left(1 - \frac{1}{2g_m R_n}\right)}$$

$$B_{s,opt} = - \frac{\text{Im}(S_{in} V_n^*)}{S_{in}} = - \frac{\omega C_i}{2g_m R_n}$$

derivation example:

S_{Vn} $g_m R_n$

example.

$$S_{in V_n^*} = \frac{2g_m I_c}{\left| \frac{I_c}{V_t} \right|^2} \cdot (g_{re} + j\omega C_i)$$

$$\text{Im}(S_{in V_n^*}) = \omega C_i \cdot \frac{2g_m I_c (KT)^2}{I_c^2 g^2} = \omega C_i \frac{2 \cdot (KT)^2}{I_c g}$$

$$S_{Vn} = 4KT R_n$$

$$B_{opt} = - \frac{\omega C_i \cdot \frac{2}{I_c} \cdot \frac{(KT)^2}{g}}{4KT R_n} = - \omega C_i \cdot \frac{1}{2 R_n} \cdot \frac{1}{I_c} \cdot \frac{KT}{g}$$

$$= - \omega C_i \cdot \frac{1}{2g_m R_n}$$

⊗

$B_{opt} < 0$. inductive source needed to
noise match imaginary part

$$F_{min} = 1 + \frac{1}{\beta} + \sqrt{\frac{2g_m R_n}{\beta} + \frac{2R_n (\omega C_i)^2}{g_m} \left(1 - \frac{1}{2g_m R_n}\right)}$$

$$\approx 1 + \frac{1}{\beta} + \sqrt{2g_m R_b} \sqrt{\frac{1}{\beta} + \left(\frac{f}{f_T}\right)^2}$$

* high β

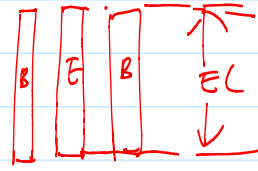
* low R_b are needed for low NF_{min}

* high f_T

Sizing for noise match

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device sizing.



① increasing emitter length (EL)

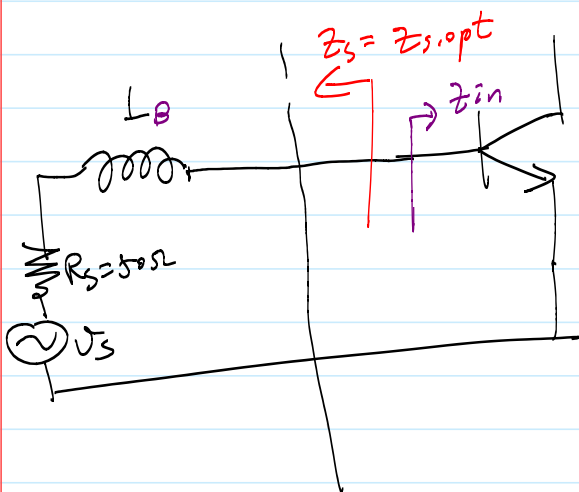
$$Z_{s,opt} \triangleq \frac{1}{Y_{s,opt}} \propto \frac{1}{EL} \quad (\text{why?})$$

just like $V_b \propto \frac{1}{EL}$.

② for the same V_{BE} , NF_{min} is independent of EL

just like f_T . it is a property of the technology

Noise matching



① scale device size such that

$$R_{s,opt} = R_s$$

② choose L for

$$\omega L = X_{s,opt}$$

$$Z_{s,opt} = R_{s,opt} + jX_{s,opt} = \frac{1}{Y_{in}}$$

$$Z_{s,opt} \neq Z_{in} !$$

$$= \frac{1}{Y_{opt}}$$

noise matching \neq impedance matching

$$R_{s,opt} \approx \frac{f_T}{f} \sqrt{\frac{2I_b}{g_m}}$$

with certain approximations (see text)

$$\text{if } f \gg f_T / \sqrt{\beta}$$

under various assumptions, \Rightarrow

$$X_{s,opt} \approx \frac{1}{\omega C_i} \approx \frac{1}{\omega C_{be}}$$

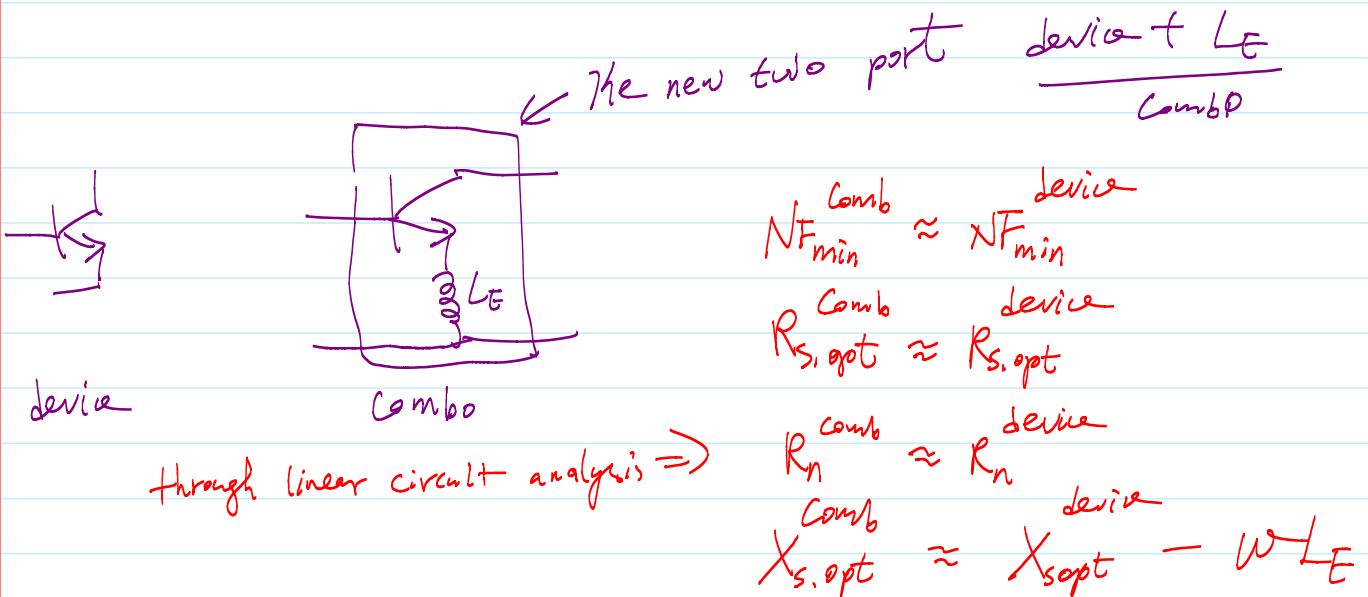
We would choose $\omega L_B = \frac{1}{\omega C_{be}}$ $L_B = \frac{1}{\omega^2 C_{be}}$

Simultaneous noise and impedance match

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Can we possibly achieve noise matching and impedance matching at the same time without increasing noise figure? - Yes.

Now consider adding L_E - emitter inductor



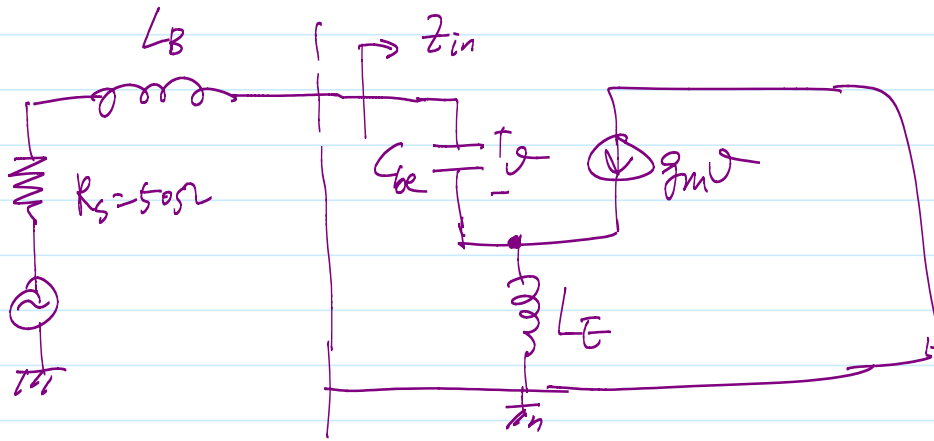
L_E can be chosen to produce real part Z_{in}

$$\text{Re}(Z_{in}) = \omega_T \cdot L_E = R_s$$

So the real part is impedance matched (Z -match)

luckily, the $X_{s,opt}^{Combo}$ is also the $(X_{in}^{Combo})^*$

$L_E \rightarrow Z_{in}$



$$Z_{in} = \frac{1}{j\omega C_{be}} + (1 + \beta_{RF}) j\omega L_E$$

$$\beta_{RF} = \frac{g_m v}{j\omega C_{be} v} = -j \frac{g_m}{C_{be} \omega} = -j \frac{\omega_T}{\omega}$$

$$\omega_T = \frac{g_m}{C_{be}}$$

Thus:
$$Z_{in} = \frac{1}{j\omega C_{be}} + j\omega L_E + \underbrace{\omega_T L_E}_{\substack{\text{choose } L_E \\ \text{for} \\ \omega_T L_E = R_s}}$$

So now
$$\text{Re}(Z_{in}) = \omega_T L_E \quad \text{through } L_E \text{ choosing}$$

$$\text{Re}(Z_{s, \text{opt}}^{\text{comb}}) = R_s \quad \text{through sizing}$$

$$\text{Im}(Z_{in}) = -\frac{1}{\omega C_{be}} + \omega L_E$$

$$I_{in}(Z_{s, \text{opt}}^{\text{comb}}) = I_{in}(Z_{s, \text{opt}}^{\text{device}}) - \omega L_E$$

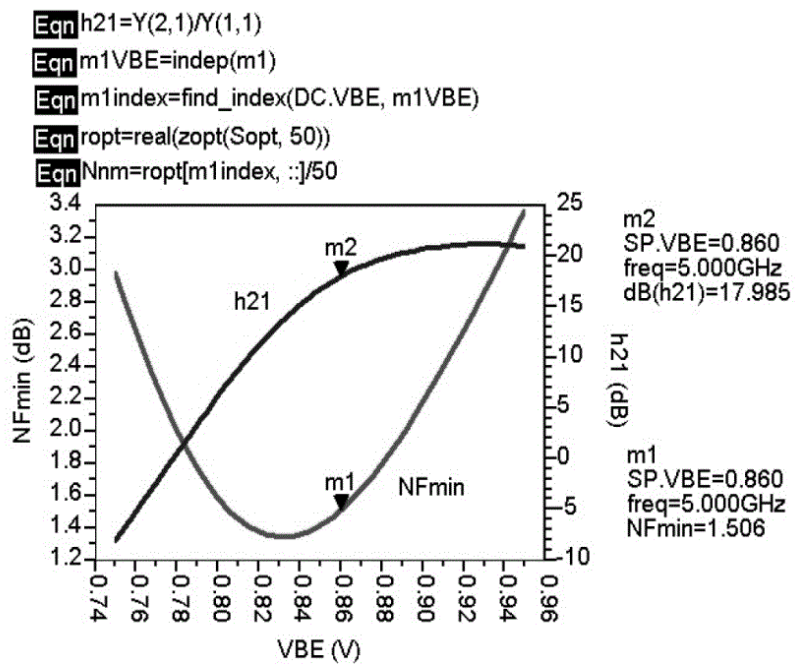
$$\begin{aligned}
 & \downarrow X_{in}^{comb} \\
 I_{in}(Z_{in}) &= -\frac{1}{\omega C_{be}} + \omega L_E \\
 I_{in}(Z_{s,opt})^{comb} &= I_{in}(Z_{s,opt})^{device} - \omega L_E \\
 & \downarrow \\
 &= \frac{1}{\omega C_{be}} - \omega L_E \\
 \text{We have proven } X_{in}^{comb} &= \left(X_{s,opt}^{comb} \right)^*
 \end{aligned}$$

a very fortunate case

- * Even if these assumptions are not always valid, we can tolerate some noise mis-match as long as it is not too far off and R_n is small
- * The insight can still be used at least to noise match the real part, + impedance matching.
- * f_c requirement determines J_c needed. L_E fixed then

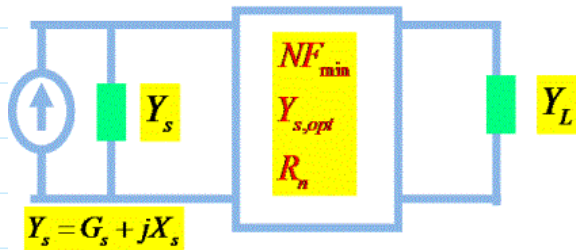
R_s (50Ω) then determines EL needed.

biasing/sizing example



$$NF = NF_{min} + \frac{R_n}{G_s} |Y_s - Y_{s,opt}|^2$$

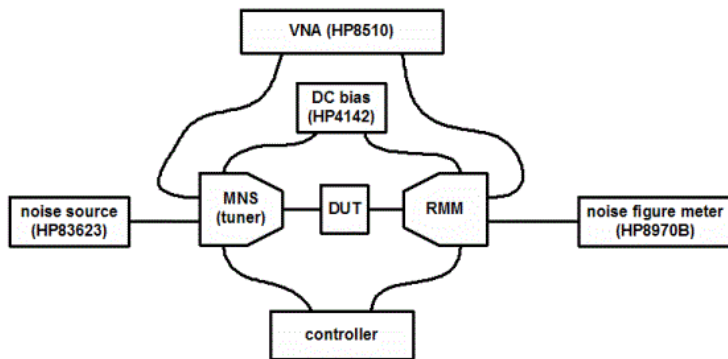
- ◆ NF is determined by noise parameters + source
- ◆ NF => NF_{min} when $Y_s = Y_{s,opt}$ (noise matching)
- ◆ R_n determines sensitivity to deviation from $Y_{s,opt}$



All of the noise parameters are important!

Noise Parameter Measurement Setup

ATN-NP5 system: solid-state source tuner
 Measure both ac and noise parameters)



ATN-NP5 noise measurement system setup

<http://sdrv.ms/PGtOun>