```
Apply[Clear, Names["Global`*"]];
Off[General::spell];
Off[General::spell1];

(*Input data*)

n10 = 20;
fi = 20 N[Pi] / 180;
r1 = 1.;
r2 = 0.5;
mext = 400;

Print["sun gear speed n1 = ", n10, " rpm"];
Print["r1 = ", r1, " m"];
Print["r2 = ", r2, " m"];
Print["pressure angle fi = ", fi, " rad"];
Print["|Mext| = ", mext, " Nm"];

d1 = 2 r1;
d2 = 2 r2;


(* Position of joint B *)
xB = 0;
yB = r1;
rB = {xB, yB, 0};

(* Position of joint C *)
xC = 0;
yC = r1 + r2;
rC = {xC, yC, 0};

(* Position of joint E *)
xE = 0;
yE = r1 + 2 r2;
rE = {xE, yE, 0};

(* Position of joint D *)
rD = {xD, 0, 0};

" Contour 0-1-2-0 "

(*Relative velocities*)

n10v = {n10, 0, 0};
n21vSol = {n21Sol, 0, 0};
n02vSol = {n02Sol, 0, 0};

"n10 + n21 + n02 = 0"
"rB x n21 + rE x n02 = 0"
eqIk = (n10v + n21vSol + n02vSol) [[1]] == 0;
eqIi = (Cross[rB, n21vSol] + Cross[rE, n02vSol]) [[3]] == 0;
eqIk = (n10v + n21vSol + n02vSol) [[1]] == 0
eqIi = (Cross[rB, n21vSol] + Cross[rE, n02vSol]) [[3]] == 0;
solI = Solve[{eqIk, eqIi}, {n21Sol, n02Sol}];
n21v = n21vSol /. solI[[1]];
n02v = n02vSol /. solI[[1]];
```

```
Print["n21 = ", n21v, " rpm"];
Print["n02 = ", n02v, " rpm"];

(*Absolute velocities*)

n20v = -n02v;

Print["n20 = ", n20v, "rpm"];

" Contour 0-2-3-0 "

(*Relative velocities*)

n32vSol = {n32Sol, 0, 0};
n03vSol = {n03Sol, 0, 0};

"n20 + n32 + n03 = 0 "
"rE x n20 +  rC x n32 = 0"
eqIk = (n20v + n32vSol + n03vSol) [[1]] == 0;
eqIi = (Cross[rE, n20v] + Cross[rC, n32vSol]) [[3]] == 0;

solI = Solve[{eqIk, eqIi}, {n32Sol, n03Sol}];
n32v = n32vSol /. solI[[1]];
n03v = n03vSol /. solI[[1]];

Print["n32 = ", n32v, " rpm"];
Print["n03 = ", n03v, " rpm"];

(*Absolute velocities*)
n30v = -n03v;
Print["n30 = ", n30v, " rpm"];
Print["arm speed n3 = ", n30v[[1]], " rpm"];

Mext = -Sign[n30v[[1]]] {mext, 0, 0};
Print["Mext = - Sign[n3] |Mext|=", Mext, " rpm"];

"arm 3"
"F23s={0,F23y,F23z}"
F23s = {0, F23ys, F23zs};
"arm 3: sumM_D = rDC x F23 + Mext = 0 =>"
MD = (Cross[rC - rD, F23s] + Mext) [[1]];
s3 = Solve[MD == 0, F23zs];
F23z = F23zs /. s3[[1]];
Print["F23z =", F23z, " N"];

"planet gear 2"
"F12={0,Fr12,Ft12}"
"F02={0,Fr02,Ft02}"
"gear 2: sumM_C = rCE x F02 + rCB x F12 = 0"
"=> r2 Ft02 -  r2 Ft12  = 0 => Ft02=Ft12"
"=> F02={0,-Fr12,Ft12}"
"gear 2: sumF = F12 - F23 + F02 = 0"
"y-axis: F23y = 0"
"z-axis: Ft12 = F23z/2"
"Fr12=Ft12 Tan[fi]"
F23 = {0, 0, F23z};
Ft12 = F23z / 2;
```

```
Fr12 = Ft12 Tan[fi];
F12 = {0, Fr12, Ft12};
F02 = {0, -Fr12, Ft12};
Print["F23 =", F23, " N"];
Print["F12 =", F12, " N"];
Print["F02 =", F02, " N"];
"arm 3: F03 = -F23 =>"
Print["F03 =", -F23, " N"];

"sun gear 1"
"gear 1: sumM_A = rAB x F21 + Mmot = 0 =>"
MA = (Cross[rB, -F12] + {Mmot, 0, 0})[[1]];
s1 = Solve[MA == 0, Mmot];
Mms = Mmot /. s1[[1]];
Mmos = {Mms, 0, 0};
Print["Mmot =", Mmos, " Nm"];
"gear 1: sumF = F21 + F01 = 0 =>"
Print["F01 = -F21 = F12= ", F12, " N"];
```

```
sun gear speed n1 = 20 rpm

r1 = 1. m

r2 = 0.5 m

pressure angle fi = 0.349066 rad

|Mext| = 400 Nm

 Contour 0-1-2-0

n10 + n21 + n02 = 0

rB x n21 + rE x n02 = 0

20 + n02Sol + n21Sol == 0

n21 = {-40., 0, 0} rpm

n02 = {20., 0, 0} rpm

n20 = {-20., 0, 0}rpm

 Contour 0-2-3-0

n20 + n32 + n03 = 0

rE x n20 +  rC x n32 = 0

n32 = {26.6667, 0, 0} rpm

n03 = {-6.66667, 0, 0} rpm

n30 = {6.66667, 0, 0} rpm

arm speed n3 = 6.66667 rpm

Mext = - Sign[n3] |Mext|={-400, 0, 0} rpm

arm 3
```

```
F23s={0,F23y,F23z}

arm 3: sumM_D = rDC x F23 + Mext = 0 =>

F23z =266.667 N

planet gear 2

F12={0,Fr12,Ft12}

F02={0,Fr02,Ft02}

gear 2: sumM_C = rCE x F02 + rCB x F12 = 0

=> r2 Ft02 -  r2 Ft12  = 0 => Ft02=Ft12

=> F02={0,-Fr12,Ft12}

gear 2: sumF = F12 - F23 + F02 = 0

y-axis: F23y = 0

z-axis: Ft12 = F23z/2

Fr12=Ft12 Tan[fi]

F23 ={0, 0, 266.667} N

F12 ={0, 48.5294, 133.333} N

F02 ={0, -48.5294, 133.333} N

arm 3: F03 = -F23 =>

F03 ={0, 0, -266.667} N

sun gear 1

gear 1: sumM_A = rAB x F21 + Mmot = 0 =>

Mmot ={133.333, 0, 0} Nm

gear 1: sumF = F21 + F01 = 0 =>

F01 = -F21 = F12= {0, 48.5294, 133.333} N
```

```
Apply[Clear, Names["Global`*"]];
Off[General::spell];
Off[General::spell1];

(*Input data*)
"stub involute profile"
(*pressure angle ϕ=20 deg stub involute*)
σ0p = 15000; (*endurance strength steel psi*)
σ0g = 8000; (*endurance strength cast iron psi*)
ϕ = 20. π / 180;
np = 1200.; (*rpm*)
ng = 300.; (*rpm*)
i = -np / ng;(*transmission ratio*)
H = 33.; (*hp*);
BHN = 250;

Print["endurance strength for pinion -  σ0p = ", σ0p, " psi"];
Print["endurance strength for gear - σ0g = ", σ0g, " psi"];
Print["transmission ratio i = ", i];
Print["pressure angle ϕ = ", ϕ, " rad = ", ϕ 180 / π, " deg"];
Print["speed of the pinion np = ", np, " rpm"];
Print["speed of the gear ng = ", ng, " rpm"];
Print["power trasmitted H = ", H, " hp"];
Print["Brinell Hardness Number BHN = ", BHN];

Np = 16;
Print["select Np = ", Np, " teeth" ];
Ng = -i Np;
Print["Ng = -i Np = ", Ng, " teeth" ];
γp = 0.115;
Print["table 1, Np = ", Np, " => form factor γp = ", γp ];
x1 = 60.; y1 = 0.154;
x2 = 75; y2 = 0.158;
m1 = (y2 - y1) / (x2 - x1);
b1 = y2 - m1 x2;
γg = m1 Ng + b1;
Print["table 1, Ng = ", Ng, " => form factor γg = ", γg ];
Fp = σ0p γp;
Print["load carrying capacity pinion: Fp = σ0p  γp = ", Fp, ""];
Fg = σ0g γg;
Print["load carrying capacity gear: Fg = σ0g  γg = ", Fg, ""];
If[Fp < Fg,
  Print["Fp<Fg => pinion is weaker  => design will be based on the pinion"];
  Nw = Np; γ = γp; σ0  = σ0p; n = np ,
  Print["Fp>Fg => gear is weaker => design will be based on the gear"];
  Nw = Ng; γ = γg; σ0  = σ0g; n = ng ];
Mt = 63000 H / n;
Print["moment trasmitted Mt=63000 H/n = ", Mt, " lb-in"]
k = 4;
Print["face width factor k = ", k]
σ = σ0 / 2.;
Print["assume allowable stress σ = σ0/2 = ", σ, " psi"];
"σ=(2 Mt Pd^3 )/(k π^2 γ N) =>"
eq1 = σ == (2 Mt Pd ^ 3 ) / (k π ^ 2 γ Nw)
Pds = Solve[eq1, Pd];
Print["Pd = ", Pd /. Pds[[3]]];
P = Input["Select diametral pitch Pd"];
```

```
Print["try Pd = ", P, " teeth per inch"];
dw = N[Nw / P];
Print["pitch diameter dg = Ng/Pd = ", dw, " in"];
V = dw π n / 12.;
Print["pitch line velocity V = d π n/12 = ", V, " ft/min" ];
If[V < 2000, σ = 600 σ0 / (600 + V);
  Print["allowable stress for V<2000 ft/min is σ = 600 σ0/(600+V) = ", σ, " psi"]
 ];
If[2000 < V < 4000, σ = 1200 σ0 / (1200 + V);
  Print[
   "allowable stress for 2000<V<4000 ft/min is σ = 1200 σ0/(1200+V) = ", σ, " psi"]
 ];
If[V > 4000, σ = 78 σ0 / (78 + V^0.5);
  Print["allowable stress for V>4000 ft/min is σ = 78 σ0/(78+V^0.5)  = ", σ, " psi"]
 ];

σi = (2 Mt P^3 ) / (k π^2 γ Nw);
Print["induced stress σi=(2 Mt Pd^3 )/(k π^2 γ N) = ", σi, " psi"];
While[σi > σ,
  Print[
   "σi>σ => weak because the induced stress is greater than the allowable stress"];
  P = Input["Select a stronger diametral pitch Pd"];
  Print["try a stronger tooth, Pd = ", P, " teeth per inch"];

  dw = N[Nw / P];
  Print["pitch diameter: d = N/Pd = ", dw , " in"];
  V = dw π n / 12.;
  Print["pitch line velocity V = d π n/12 = ", V, " ft/min" ];
  If[V < 2000, σ = 600 σ0 / (600 + V);
   Print["allowable stress for V<2000 ft/min is σ = 600 σ0/(600+V) = ", σ, " psi"]
  ];
  If[2000 < V < 4000, σ = 1200 σ0 / (1200 + V);
   Print[
    "allowable stress for 2000<V<4000 ft/min is σ = 1200 σ0/(1200+V) = ", σ, " psi"]
  ];
  If[V > 4000, σ = 78 σ0 / (78 + V^0.5);
   Print["allowable stress for V>4000 ft/min is σ = 78 σ0/(78+V^0.5)  = ", σ, " psi"]
  ];

  σi = (2 Mt P^3 ) / (k π^2 γ Nw);
  Print["induced stress σi=(2 Mt Pd^3 )/(k π^2 γ N) = ", σi, " psi"]
 ];
Print["σi<σ => strong because the induced stress is less than the allowable stress"]

"k is reduced from the maximum value k=4"
"σ=(2 Mt Pd^3 )/(k π^2 γ N) => k"
eq2 = σ == (2 Mt P^3 ) / (kp π^2 γ Nw);
kps = Solve[eq2, kp];
k = kp /. kps[[1]];
Print["reduce k to k = ", k];
p = N[π / P];
Print["circular pitch p = π/Pd = ", p, " in."];
B = k p;
Print["face width B = k  p = ", B, " in."];
B = 2.75;
Print["select face width B = ", B, " in."];
a = 0.8 / P;
Print["addendum a = 0.8/Pd = ", a, " in."];
```

```
b = 1. / P;
Print["min. dedendum b = 1/Pd = ", b, " in."];
rp = Np / (2. P);
Print["radius of pitch diameter for pinion: rp = Np/(2 Pd) = ", rp, " in."];
rg = Ng / (2. P);
Print["radius of pitch diameter for gear: rg = Ng/(2 Pd) = ", rg, " in."];
rbp = rp Cos[ϕ];
rbg = rg Cos[ϕ];
Print["radius of base diameter for pinion: rbp = rp Cos[ϕ] = ", rbp, " in."];
Print["radius of base diameter for gear: rbg = rg Cos[ϕ] = ", rbg, " in."];
c = rp + rg;
Print["center distance c=rp+rg= ", c, " in"];
"radius of maximum possible addendum circle ra(max)=(rb^2+(c Sin[ϕ])^2)^.5"
ramp = (rbp^2 + (c Sin[ϕ])^2)^.5;
ramg = (rbg^2 + (c Sin[ϕ])^2)^.5;
Print["ra(max)p = (rbp^2+(c Sin[ϕ])^2)^.5 = ", ramp, " in."];
Print["ra(max)g = (rbg^2+(c Sin[ϕ])^2)^.5 = ", ramg, " in."];
"radius of addendum circle ra = r + a"
rap = rp + a;
rag = rg + a;
Print["rap = rp + a = ", rap, " in."];
Print["rag = rg + a = ", rag, " in."];
If[ramp > rap && ramg > rag,
  Print["ra(max)>ra  => no interference "], Print["interference "]];

pb = p Cos[ϕ];
Print["base pitch pb = p Cos[ϕ] =  ", pb, " in."];
CR = ((rap^2 - rbp^2)^.5 + (rag^2 - rbg^2)^.5 - c Sin[ϕ]) / pb;
Print[
  "contact ratio CR = ((rap^2-rbp^2)^.5+(rag^2-rbg^2)^.5-c Sin[ϕ])/pb = ", CR];
"contact ratio CR > 1.2 "
F0 = σ0 B γ p;
Print["endurance load F0 = σ0 B  γ p = ", F0, " lb"];

σes = 400 BHN - 10000;
Print["endurance limit of the gear pair σes = 400 BHN - 10000 = ", σes, " psi"];
Q = 2 Ng / (Np + Ng);
Print["Q = 2 Ng/(Np+Ng) = ", Q];
Ep = 30. 10^6 (*psi*);
Eg = 19. 10^6 (*psi*);
K = σes^2 Sin[ϕ] (1 / Ep + 1 / Eg) / 1.4;
Print["stress fatigue factor K = σes^2 Sin[ϕ] (1/Ep+1/Eg)/1.4 = ", K];
dp = 2 rp;
Fw = dp B K Q;
Print["wear load Fw = dp B K Q = ", Fw, " lb"];
F = Mt / rg;
Print["F = Mt/rg = ", F, " lb"];
Print["for V = ", V, " ftm => from fig 13(a) permissible error = 0.00225 in. "];
Print["from fig 13(a) for first class comercial
    gears with Pd=4 => error e = 0.0025 in. > permissible error "];
Print["from fig 13(b) for carefully cut gears with Pd=4 => error e = 0.0012 in. "];
(*deformation factor C*)
x1 = 0.001; y1 = 1180; (*y1=1140;*)
x2 = 0.002; y2 = 2360; (*y2=2280;*)
m1 = (y2 - y1) / (x2 - x1);
b1 = y2 - m1 x2;
e = 0.0012;
Print["select tooth error e = ", e, " in."];
```

```
Cf = m1 e + b1;
Print[
  "from table 2 calculate the deformation factor for dynamic load check C = ", Cf];
Fd = 0.05 V (B Cf + F) / (0.05 V + Sqrt[ B Cf + F]) + F;
Print[
  "dynamic load Fd = 0.05 V (B C+F)/(0.05 V + Sqrt[ B C+F]) + F = ", Fd, " lb"];
If[Fd > F0, Print["F0 = ", F0, " approx Fd = ", Fd, " within ", (1 - F0 / Fd) 100, " %"],
  Print["Fd<F0"]];
If[Fd > Fw, Print["Fw = ", Fw, " approx Fd = ", Fd, " within ", (1 - Fw / Fd) 100, " %"],
  Print["Fd<Fw"]];

Print["from fig 13(b) for precision gears with Pd=4 => error e = 0.00051 in. "];
e = 0.00051;
Cf = m1 e + b1;
Print[
  "from table 2 calculate the deformation factor for dynamic load check C = ", Cf];
Fd = 0.05 V (B Cf + F) / (0.05 V + Sqrt[ B Cf + F]) + F;
Print[
  "dynamic load Fd = 0.05 V (B C+F)/(0.05 V + Sqrt[ B C+F]) + F = ", Fd, " lb"];
If[Fd > F0, Print["F0 = ", F0, " approx Fd = ", Fd, " within ", (1 - F0 / Fd) 100, " %"],
  Print["Fd<F0"]];
If[Fd > Fw, Print["Fw = ", Fw, " approx Fd = ", Fd, " within ", (1 - Fw / Fd) 100, " %"],
  Print["Fd<Fw"]];
```

stub involute profile

endurance strength for pinion - $\sigma 0p$ = 15000 psi

endurance strength for gear - $\sigma 0g$ = 8000 psi

transmission ratio i = -4.

pressure angle ϕ = 0.349066 rad = 20. deg

speed of the pinion np = 1200. rpm

speed of the gear ng = 300. rpm

power trasmitted H = 33. hp

Brinell Hardness Number BHN = 250

select Np = 16 teeth

Ng = -i Np = 64. teeth

table 1, Np = 16 => form factor γp = 0.115

table 1, Ng = 64. => form factor γg = 0.155067

load carrying capacity pinion: Fp = $\sigma 0p \ \gamma p$ = 1725.

load carrying capacity gear: Fg = $\sigma 0g \ \gamma g$ = 1240.53

Fp>Fg => gear is weaker => design will be based on the gear

moment trasmitted Mt=63000 H/n = 6930. lb-in face width factor k = 4

assume allowable stress σ = σ0/2 = 4000. psi

σ=(2 Mt Pd^3)/(k π^2 γ N) =>

$4000. == 35.3757\, Pd^3$

Pd = 4.83561

try Pd = 5 teeth per inch

pitch diameter dg = Ng/Pd = 12.8 in

pitch line velocity V = d π n/12 = 1005.31 ft/min

allowable stress for V<2000 ft/min is σ = 600 σ0/(600+V) = 2990.08 psi

induced stress σi=(2 Mt Pd^3)/(k π^2 γ N) = 4421.96 psi

σi>σ => weak because the induced stress is greater than the allowable stress

try a stronger tooth, Pd = 4 teeth per inch

pitch diameter: d = N/Pd = 16. in

pitch line velocity V = d π n/12 = 1256.64 ft/min

allowable stress for V<2000 ft/min is σ = 600 σ0/(600+V) = 2585.32 psi

induced stress σi=(2 Mt Pd^3)/(k π^2 γ N) = 2264.04 psi

σi<σ => strong because the induced stress is less than the allowable stress

k is reduced from the maximum value k=4

σ=(2 Mt Pd^3)/(k π^2 γ N) => k

reduce k to k = 3.50292

circular pitch p = π/Pd = 0.785398 in.

face width B = k p = 2.75119 in.

select face width B = 2.75 in.

addendum a = 0.8/Pd = 0.2 in.

min. dedendum b = 1/Pd = 0.25 in.

radius of pitch diameter for pinion: rp = Np/(2 Pd) = 2. in.

radius of pitch diameter for gear: rg = Ng/(2 Pd) = 8. in.

radius of base diameter for pinion: rbp = rp Cos[φ] = 1.87939 in.

radius of base diameter for gear: rbg = rg Cos[φ] = 7.51754 in.

center distance c=rp+rg= 10. in

radius of maximum possible addendum circle ra(max)=(rb^2+(c Sin[φ])^2)^.5

ra(max)p = (rbp^2+(c Sin[φ])^2)^.5 = 3.90255 in.

ra(max)g = (rbg^2+(c Sin[φ])^2)^.5 = 8.25901 in.

radius of addendum circle ra = r + a

rap = rp + a = 2.2 in.

rag = rg + a = 8.2 in.

ra(max)>ra => no interference

base pitch pb = p Cos[φ] = 0.738033 in.

contact ratio CR = ((rap^2-rbp^2)^.5+(rag^2-rbg^2)^.5-c Sin[φ])/pb = 1.35303

contact ratio CR > 1.2

endurance load F0 = σ0 B γ p = 2679.36 lb

endurance limit of the gear pair σes = 400 BHN - 10000 = 90000 psi

Q = 2 Ng/(Np+Ng) = 1.6

stress fatigue factor K = σes^2 Sin[φ] (1/Ep+1/Eg)/1.4 = 170.11

wear load Fw = dp B K Q = 2993.94 lb

F = Mt/rg = 866.25 lb

for V = 1256.64 ftm => from fig 13(a) permissible error = 0.00225 in.

from fig 13(a) for first class comercial
gears with Pd=4 => error e = 0.0025 in. > permissible error

from fig 13(b) for carefully cut gears with Pd=4 => error e = 0.0012 in.

select tooth error e = 0.0012 in.

from table 2 calculate the deformation factor for dynamic load check C = 1416.

dynamic load Fd = 0.05 V (B C+F)/(0.05 V + Sqrt[B C+F]) + F = 3135.11 lb

F0 = 2679.36 approx Fd = 3135.11 within 14.5369 %

Fw = 2993.94 approx Fd = 3135.11 within 4.50291 %

from fig 13(b) for precision gears with Pd=4 => error e = 0.00051 in.

from table 2 calculate the deformation factor for dynamic load check C = 601.8

dynamic load Fd = 0.05 V (B C+F)/(0.05 V + Sqrt[B C+F]) + F = 2267.58 lb

Fd<F0

Fd<Fw