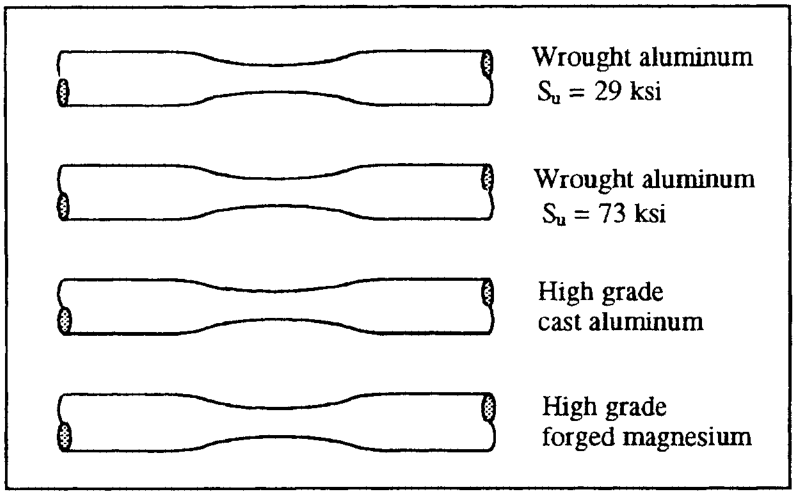
SOLUTION (8.16)
Known: Four standard R.R. Moore specimens of known materials are given.

Find: Estimate the long-life fatigue strength for reversed torsional loading. (State whether it is for 10^8 or 5×10^8 cycles.)

Schematic and Given Data:



Assumption: Figs. 8.8, 8.9, and 8.10 can be used to estimate long life fatigue for reversed loading.

Analysis:

1. From Fig. 8.9, for the wrought aluminum having S_u = 29 ksi, the rotating bending fatigue strength at 5×10^8 cycles is S_n' = 12 ksi. Since, for reversed torsional loading $S_n = 0.58\, S_n'$, $S_n = 0.58(12) = 7$ ksi ■
2. From Fig. 8.9, for the wrought aluminum having S_u = 73 ksi, the rotating bending fatigue strength at 5×10^8 cycles is S_n' = 19 ksi. Thus, for reversed torsional loading, $S_n = 0.58(130) = 75$ MPa ■
3. From Fig. 8.8, for high grade cast aluminum, the rotating bending fatigue strength at 5×10^8 cycles is 11 ksi for sand cast and 15 ksi for permanent mold cast. Thus, for reversed torsional loading,
   $S_n = 0.58(11) = 6.48$ ksi for sand cast
   $S_n = 0.58(15) = 8.7$ ksi for permanent mold cast ■
4. From Fig. 8.10, for high grade forged magnesium, the rotating bending fatigue strength at 10^8 cycles is 22 ksi. Thus, for reversed torsional loading, $S_n = 0.58(22) = 12.76$ ksi

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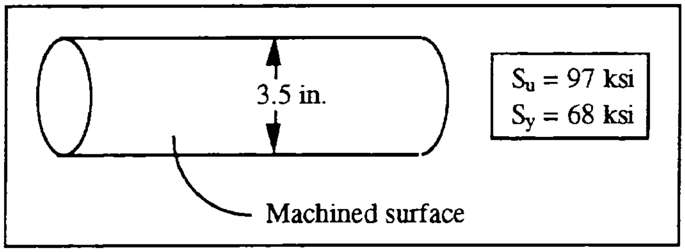
SOLUTION (8.17)
Known: A steel bar having known S_u and S_y has average machined surfaces.

Find: Plot on log-log coordinates estimated S-N curves for (a) bending, (b) axial, and (c) torsional loading. For each of the three types of loading, determine the fatigue strength corresponding to (1) 10^6 or more cycles and (2) 5×10^4 cycles.

Schematic and Given Data:



Assumptions:
1. Actual fatigue data is not available for this material.
2. The estimated S-N curves constructed using Table 8.1 are adequate.
3. Fig. 8.13 can be used to estimate surface factor, C_S.
4. The gradient factor, $C_G = 0.9$, for axial and torsional loading.

Analysis:
1. Endurance limits: (10^6 cycle strength)

   $S_n = S_n' C_L C_G C_s$

   For bending,

   $S_n' = 0.5\ S_u = 0.5(97) = 48.5$ ksi (Fig. 8.5)
   $C_L = 1$ (Table 8.1)
   $C_G = 0.9$ (Table 8.1)
   $C_s = 0.76$ (Fig. 8.13)
   $S_n = (48.5)(1)(0.9)(0.76) = 33.2$ ksi ■

   For axial,

   $S_n' = 48.5$ ksi
   $C_L = 1$
   $C_G = 0.8$ (between 0.7 and 0.9)
   $C_s = 0.76$
   $S_n = 48.5(1)(0.8)(0.76) = 29.5$ ksi ■

   For torsion,

   $S_n' = 48.5$ ksi
   $C_L = 0.58$
   $C_G = 0.9$
   $C_s = 0.76$
   $S_n = 48.5(0.58)(0.9)(0.76) = 19.2$ ksi ■

2. 10^3 cycle strength

   For bending,

   $0.9S_u = 0.9(97) = 87.3$ ksi (Table 8.1)

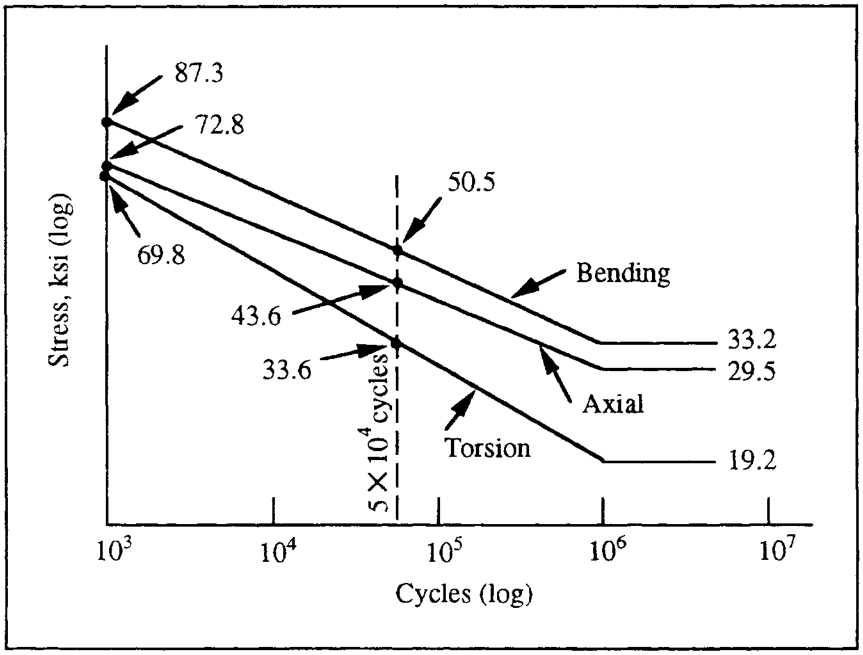
   For axial,

   $0.75S_u = 0.75(97) = 72.8$ ksi

   For torsion,

   $0.9S_{us} = 0.9(0.8)(97) = 69.8$ ksi

3. S-N curves



4. 5×10^4 cycle strength

Bending: 50.5 ksi ■

Axial: 43.6 ksi ■

Torsion: 33.6 ksi ■

Comments:

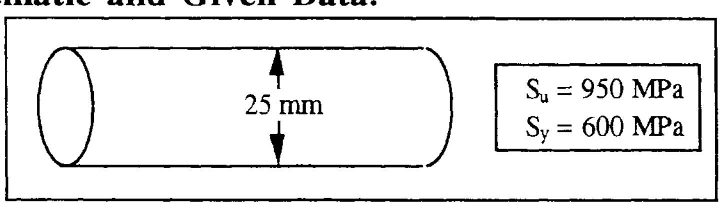
1. The surface factor, C_S is not used for correcting the 10^3-cycle strength because for ductile parts the 10^3 strength which is close to the static strength, is unaffected by surface finish.
2. For critical designs, pertinent test data should be used rather than the preceding rough approximation.

SOLUTION (8.18)

Known: A steel bar having known S_u and S_y has a hot rolled surface finish.

Find: Determine the fatigue strength at 2×10^5 cycles for reversed axial loading.

Schematic and Given Data:



Assumptions:

1. Actual fatigue data is not available for this material.
2. The estimated S-N curves constructed using Table 8.1 are adequate.
3. Fig. 8.13 can be used to estimate surface factor, C_S.

Analysis:

1. Endurance limit (10^6 cycle strength)

   $S_n = S_n' C_L C_G C_s$

   For axial,

   $S_n' = 0.5S_u = 0.5(950) = 475$ MPa

   $C_L = 1$

   $C_G = 0.8$ (between 0.7 and 0.9)

   $C_s = 0.475$

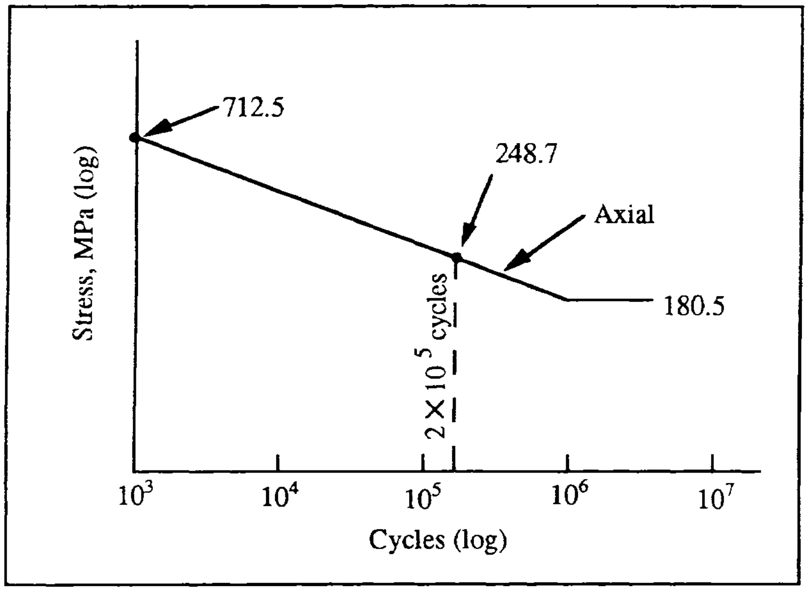
   $S_n = (475)(1)(0.8)(0.475) = 180.5$ MPa ■

2. 10^3 cycle strength

   For axial,

   $0.75S_u = 0.75(950) = 712.5$ MPa

3. S-N curves



4. 2×10^5 cycle strength

   Axial: 248.7 MPa ■

Comments:

1. The surface factor, C_s is not used for correcting the 10^3-cycle strength because for ductile parts the 10^3 strength is relatively unaffected by surface finish.
2. For critical designs, pertinent test data should be used rather than the preceding rough approximation.
3. Analytically the 200,000 cycle strength for reverse axial loading may be determined by solving

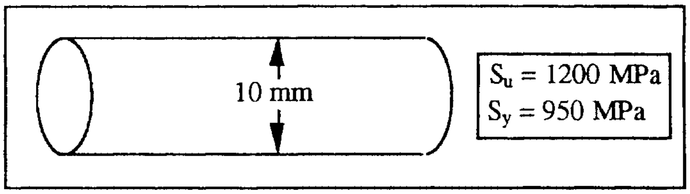
   [log (712.5) - log (180.5)]/(6 - 3) = [log (S) - log (180.5)]/(6 - log (200,000)).

SOLUTION (8.19)

Known: A steel bar having known S_u and S_y has a fine ground surface.

Find: Determine the fatigue strength for bending corresponding to (1) 10^6 or more cycles and (2) 2×10^5 cycles.

Schematic and Given Data:

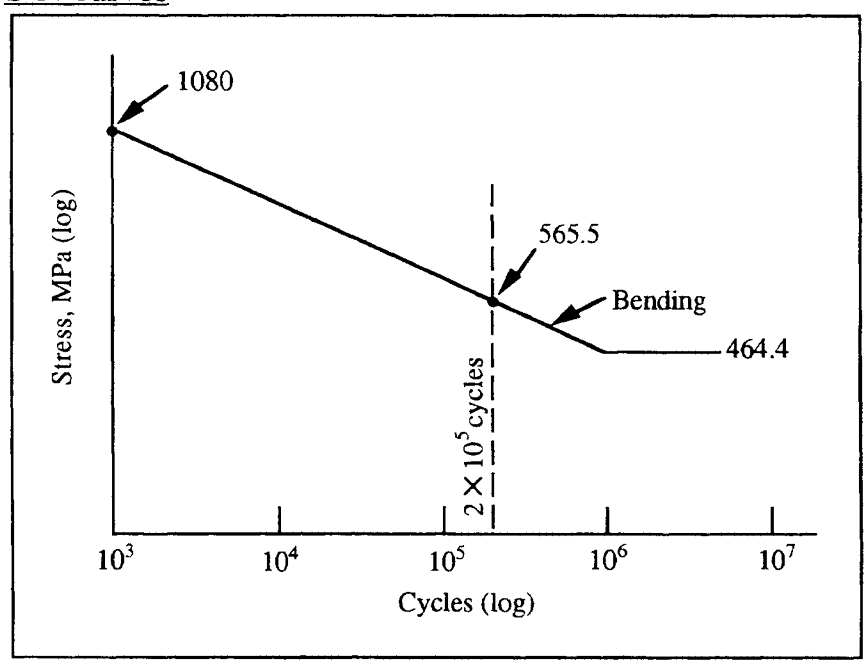


Assumptions:

1. Actual fatigue data is not available for this material.
2. The estimated S-N curves constructed using Table 8.1 are adequate.
3. Fig. 8.13 can be used to estimate surface factor, C_S.
4. The gradient factor, $C_G = 0.9$.

Analysis:

1. Endurance limits: (10^6 cycle strength)
   $S_n = S_n' C_L C_G C_S$
   For bending,
   $S_n' = 0.5\ S_u = 0.5(1200) = 600$ MPa (Fig. 8.5)
   $C_L = 1$ (Table 8.1)
   $C_G = 0.9$ (Table 8.1)
   $C_S = 0.86$ (Fig. 8.13)
   $S_n = (600)(1)(0.9)(0.86) = 464.4$ MPa ■
2. 10^3 cycle strength
   For bending,
   $0.9S_u = 0.9(1200) = 1080$ MPa (Table 8.1)
3. S-N curves



4. 2×10^5 cycle strength
   Bending: 565.5 MPa ■

Comments:

1. The surface factor, C_s is not used for correcting the 10^3-cycle strength because for ductile parts the 10^3 strength is relatively unaffected by surface finish.
2. For critical designs, pertinent test data should be used rather than the preceding rough approximation.
3. Analytically the 200,000 cycle fatigue strength for bending may be determined by solving

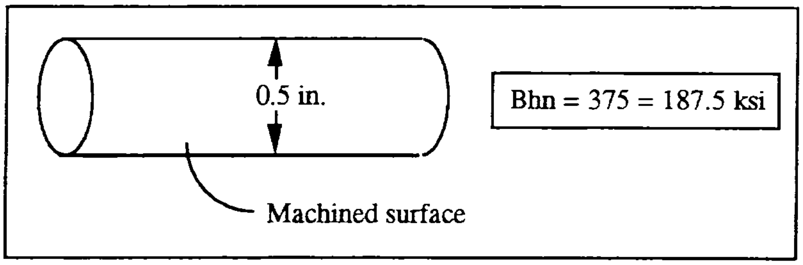
   $[\log (1080) - \log (565.5)]/(6 - 3) = [\log (S) - \log (565.5)]/(6 - \log (200{,}000))$.

SOLUTION (8.20)

Known: A steel bar having known Brinell hardness has average machined surfaces.

Find: Determine the fatigue strength for bending corresponding to (1) 10^6 or more cycles and (2) 2×10^5 cycles.

Schematic and Given Data:



Assumptions:

1. Actual fatigue data is not available for this material.
2. The estimated S-N curves constructed using Table 8.1 are adequate.
3. Fig. 8.13 can be used to estimate surface factor, C_s.
4. The gradient factor, $C_G = 0.9$.

Analysis:

1. Endurance limits: (10^6 cycle strength)

   $S_n = S_n' C_L C_G C_s$

   For bending,

   $S_n' = 0.25$ Bhn $= 0.25(375) = 93.75$ ksi (Fig. 8.5)

   $C_L = 1$ (Table 8.1)

   $C_G = 0.9$ (Table 8.1)

   $C_s = 0.64$ (Fig. 8.13)

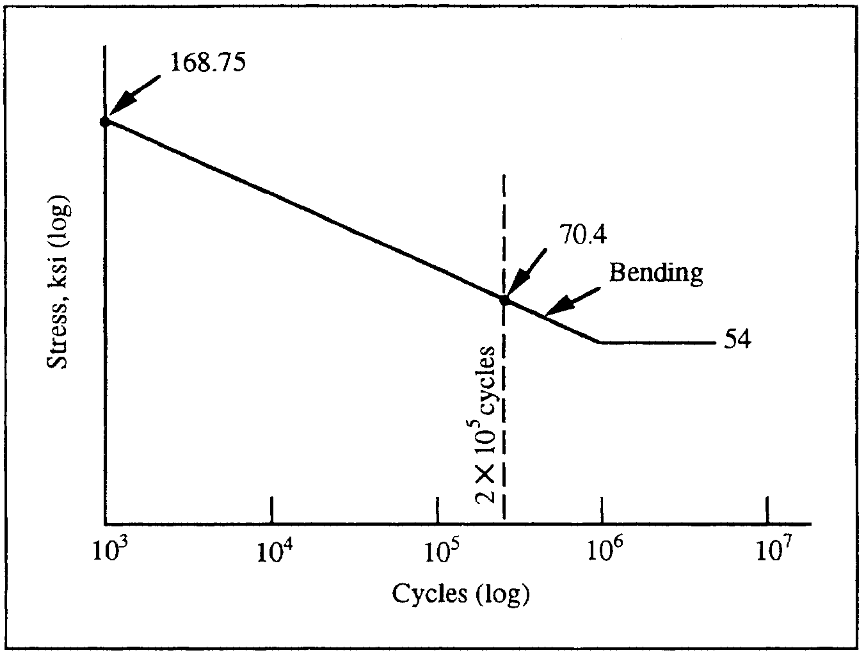
   $S_n = (93.75)(1)(0.9)(0.64) = 54$ ksi ■

2. 10^3 cycle strength

   For bending,

   $S \approx 0.45$Bhn $= 0.45(375) = 168.75$ ksi (Table 8.1)

3. S-N curves



4. 2×10^5 cycle strength
   Bending: 70.4 ksi ■

Comments:

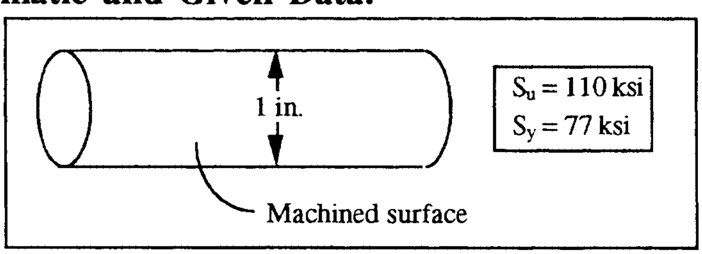
1. C_S is not used for correcting 10^3-cycle strength because for ductile parts this is close to static strength, which is unaffected by surface finish.
2. For critical designs pertinent test data should be used rather than the preceding rough approximation.

SOLUTION (8.21)

Known: A steel bar having known S_u and S_y has average machined surfaces.

Find: Plot on log-log coordinates estimated S-N curves for (a) bending, (b) axial, and (c) torsional loading. For each of the three types of loading, determine the fatigue strength corresponding to (1) 10^6 or more cycles and (2) 6×10^4 cycles.

Schematic and Given Data:



Assumptions:

1. Actual fatigue data is not available for this material.
2. The estimated S-N curves constructed using Table 8.1 are adequate.
3. Fig. 8.13 can be used to estimate surface factor, C_S.

Analysis:

1. Endurance limits: (10^6 cycle strength)

$S_n = S_n' C_L C_G C_s$

For bending,

$S_n' = 0.5\ S_u = 0.5(110) = 55$ ksi (Fig. 8.5)

$C_L = 1$ (Table 8.1)

$C_G = 0.9$ (Table 8.1)

$C_s = 0.74$ (Fig. 8.13)

$S_n = (55)(1)(0.9)(0.74) = 36.6$ ksi ■

For axial,

$S_n' = 55$ ksi

$C_L = 1$

$C_G = 0.8$ (between 0.7 and 0.9)

$C_s = 0.74$

$S_n = 55(1)(0.8)(0.74) = 32.6$ ksi ■

For torsion,

$S_n' = 55$ ksi

$C_L = 0.58$

$C_G = 0.9$

$C_s = 0.74$

$S_n = 55(0.58)(0.9)(0.74) = 21.2$ ksi ■

2. 10^3 cycle strength

For bending,

$0.9S_u = 0.9(110) = 99.0$ ksi (Table 8.1)

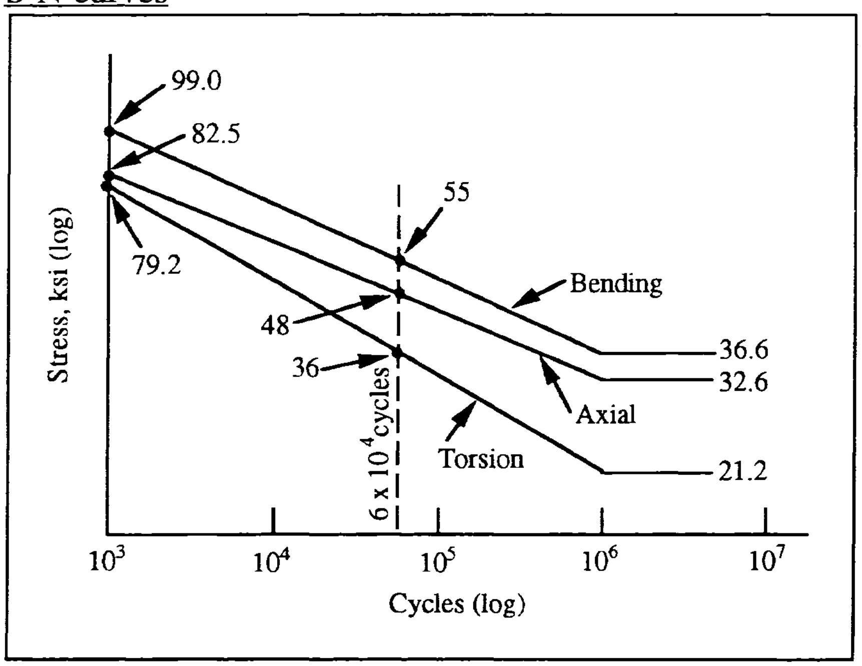
For axial,

$0.75S_u = 0.75(110) = 82.5$ ksi

For torsion,

$0.9S_{us} = 0.9(0.8)(110) = 79.2$ ksi

3. S-N curves



4. 6 × 10^4 cycle strength
   Bending: 55 ksi ■
   Axial: 48 ksi ■
   Torsion: 36 ksi ■

Comments:

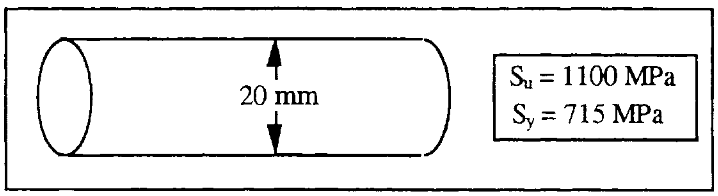
1. C_s is not used for correcting 10^3-cycle strength because for ductile parts this is close to static strength, which is unaffected by surface finish.
2. For critical designs pertinent test data should be used rather than the preceding rough approximation.

SOLUTION (8.22)

Known: A steel bar having known S_u and S_y has average ground surfaces.

Find: Plot on log-log coordinates estimated S-N curves for (a) bending, (b) axial, and (c) torsional loading. For each of the three types of loading, what is the fatigue strength corresponding to (1) 10^6 or more cycles and (2) 6×10^4 cycles?

Schematic and Given Data:



Assumptions:

1. Actual fatigue data is not available for this material.
2. The estimated S-N curves constructed using Table 8.1 are adequate.
3. Fig. 8.13 can be used to estimate surface factor, C_S.

Analysis:

Fine Ground Surface:

1. Endurance limits (10^6 cycle strength)

$S_n = S_n' C_L C_G C_s$

For bending,

$S_n' = 0.5S_u = 0.5(1100) = 550$ MPa (Fig. 8.5)
$C_L = 1$ (Table 8.1)
$C_G = 0.9$ (Table 8.1)
$C_s = 0.89$ (Fig. 8.13)
$S_n = (550)(1)(0.9)(0.89) = 440.6$ MPa ■

For axial,

$S_n' = 550$ MPa
$C_L = 1$
$C_G = 0.8$ (between 0.7 and 0.9)
$C_s = 0.89$
$S_n = (550)(1)(0.8)(0.89) = 392$ MPa ■

For torsion,

$S_n' = 550$ MPa

$C_L = 0.58$

$C_G = 0.9$

$C_s = 0.89$

$S_n = (550)(0.58)(0.9)(0.89) = 255.5$ MPa ■

2. 10^3 cycle strength

For bending,

$0.9S_u = 0.9(1100) = 990$ MPa (Table 8.1)

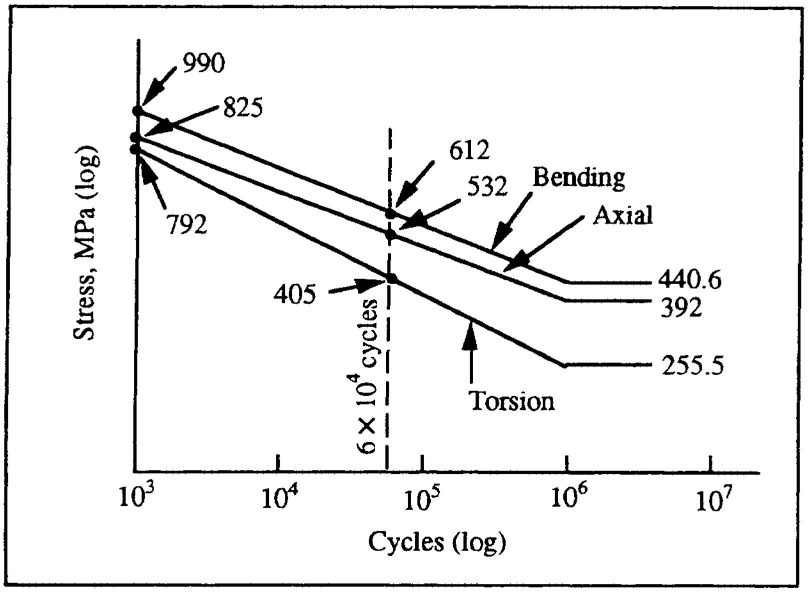
For axial,

$0.75S_u = 0.75(1100) = 825$ MPa

For torsion,

$0.9S_{us} = 0.9(0.8)(1100) = 792$ MPa

3. S-N curves



4. 6×10^4 cycle strength

Bending:	612 MPa ■
Axial:	532 MPa ■
Torsion:	405 MPa ■

Average machined surface:

1. Endurance limits (10^6 cycle strength)

$S_n = S_n' C_L C_G C_s$

For bending,

$S_n' = 0.5S_u = 0.5(1100) = 550$ MPa (Fig. 8.5)

$C_L = 1$ (Table 8.1)

$C_G = 0.9$ (Table 8.1)

$C_s = 0.68$ (Fig. 8.13)

$S_n = (550)(1)(0.9)(0.68) = 337$ MPa ■

For axial,

$S_n' = 550$ MPa
$C_L = 1$
$C_G = 0.8$ (between 0.7 and 0.9)
$C_s = 0.68$
$S_n = (550)(1)(0.8)(0.68) = 299$ MPa ■

For torsion,

$S_n' = 550$ MPa
$C_L = 0.58$
$C_G = 0.9$
$C_s = 0.68$
$S_n = (550)(0.58)(0.9)(0.68) = 195$ MPa ■

2. 10^3 cycle strength

For bending,

$0.9S_u = 0.9(1100) = 990$ MPa (Table 8.1)

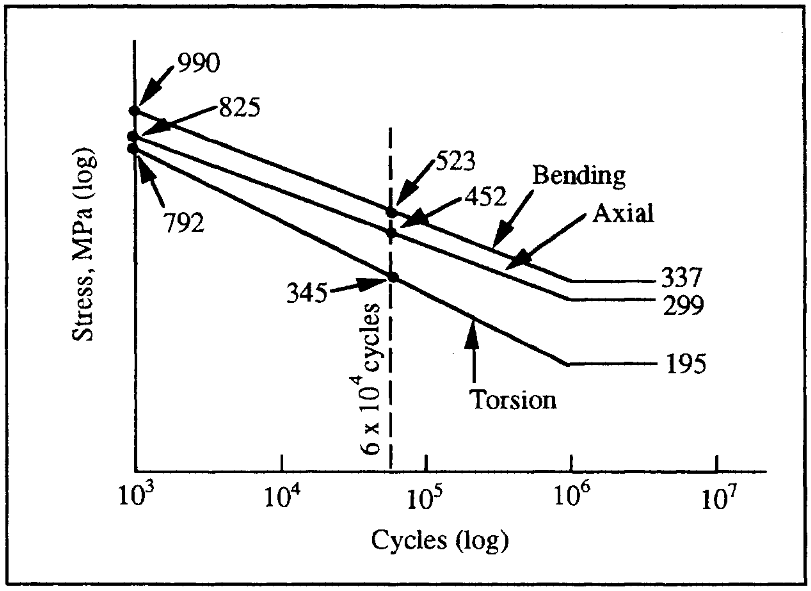
For axial,

$0.75S_u = 0.75(1100) = 825$ MPa

For torsion,

$0.9S_{us} = 0.9(0.8)(1100) = 792$ MPa

3. S-N curves



4. 6×10^4 cycle strength

Bending:	523 MPa ■
Axial:	452 MPa ■
Torsion:	345 MPa

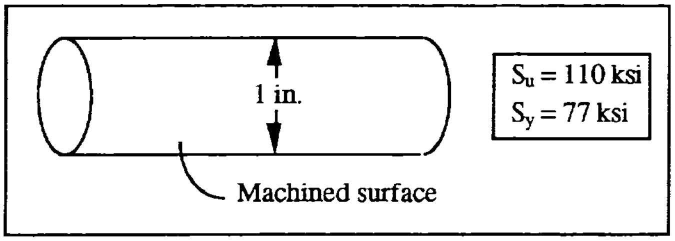
Comments:

1. The surface factor, C_s is not used for correcting the 10^3-cycle strength because for ductile parts the 10^3 strength is relatively unaffected by surface finish.
2. For critical designs, pertinent test data should be used rather than the preceding rough approximation.

SOLUTION (8.23)
Known: A steel bar having known S_u and S_y has average machined surfaces.

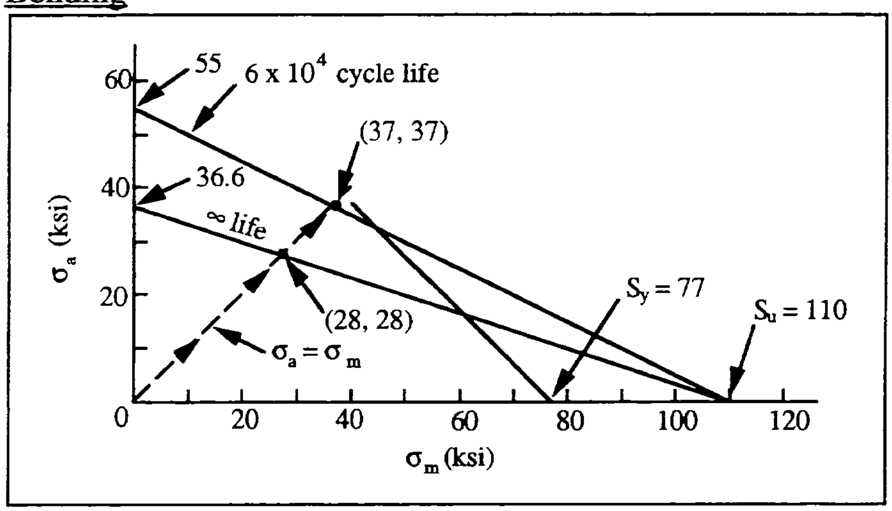
Find: Determine the fatigue strength corresponding to (1) 10^6 or more cycles and (2) 6×10^4 cycles for the case of zero-to-maximum (rather than completely reversed) load fluctuations for bending, axial, and torsional loading.

Schematic and Given Data:



Assumptions:
1. Actual fatigue data is not available for this material.
2. The estimated S-N curves constructed using Table 8.1 are adequate.
3. Fig. 8.13 can be used to estimate surface factor, C_S.

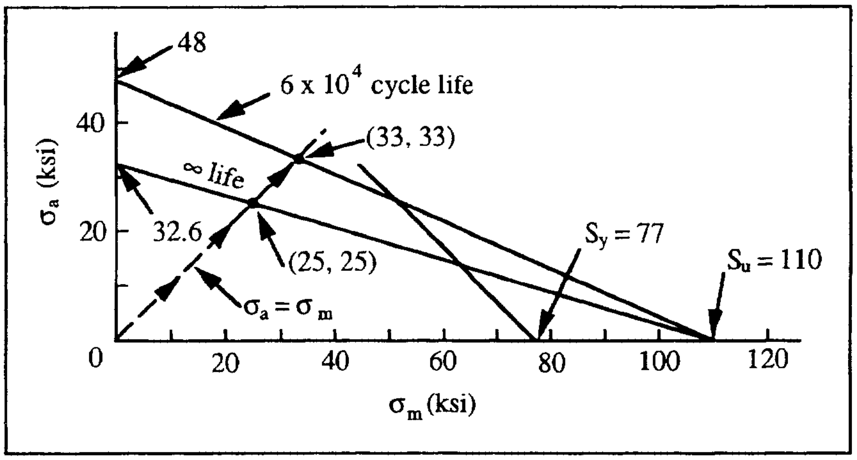
Analysis:
1. Bending



For ∞ life, σ_{max} = 56 ksi ■

For 6×10^4 cycles, σ_{max} = 74 ksi ■

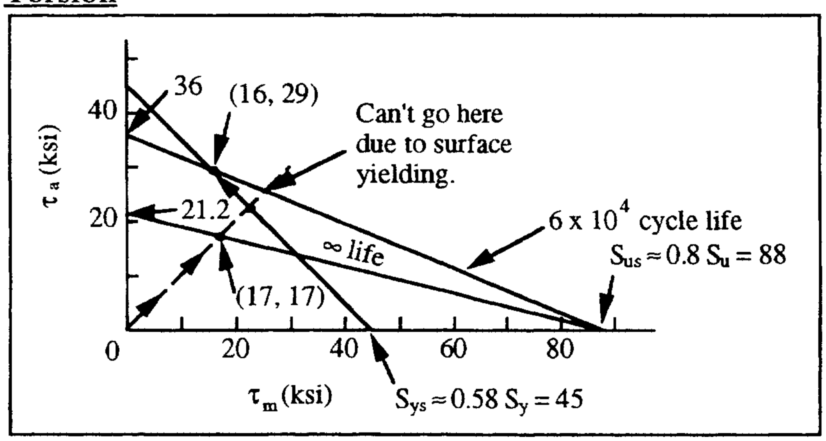
2. Axial



For ∞ life, σ_{max} = 50 ksi ■

For 6 × 10^4 cycles, σ_{max} = 66 ksi ■

3. Torsion



For ∞ life, τ_{max} = 34 ksi ■

For 6 × 10^4 cycles, τ_{max} = 58 ksi* ■

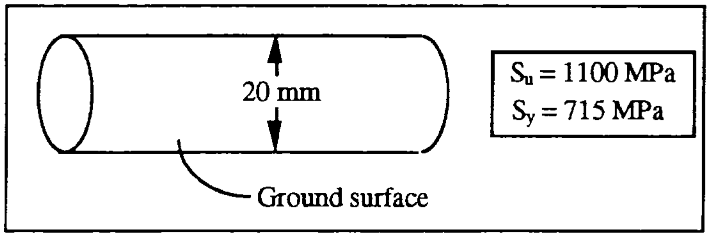
*Only if the yielding indicated is acceptable, but if so, τ_a = 29, and the load stress can be 0-52 ksi.

SOLUTION (8.24)

Known: A steel bar having known S_u and S_y has average ground surfaces.

Find: Determine the fatigue strength corresponding to (1) 10^6 or more cycles and (2) 6 $\times 10^4$ cycles for the case of zero-to-maximum (rather than completely reversed) load fluctuations for bending, axial, and torsional loading.

Schematic and Given Data:



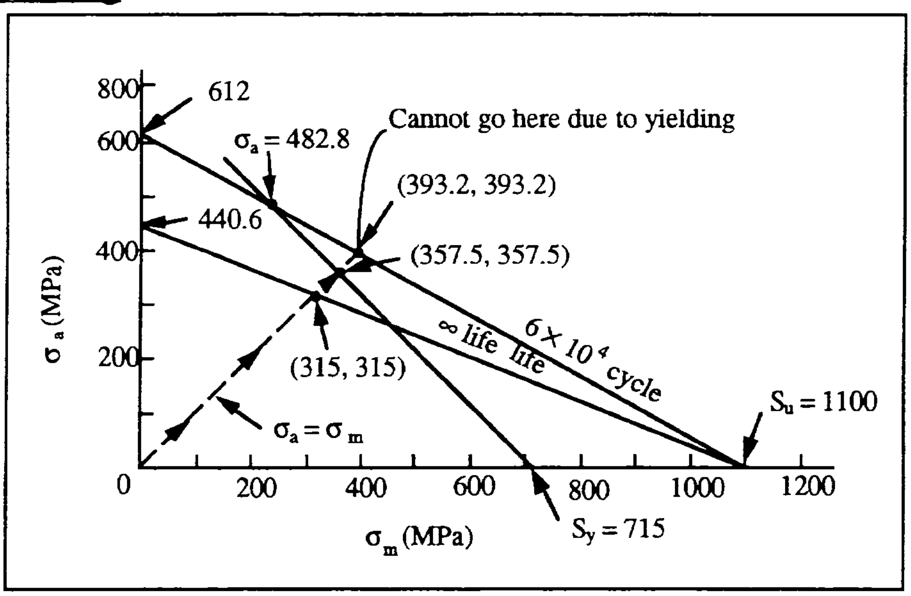
Assumptions:

1. Actual fatigue data is not available for this material.
2. The estimated S-N curves constructed using Table 8.1 are adequate.
3. Fig. 8.13 can be used to estimate surface factor, C_S.

Analysis:

<u>Fine ground surface:</u>

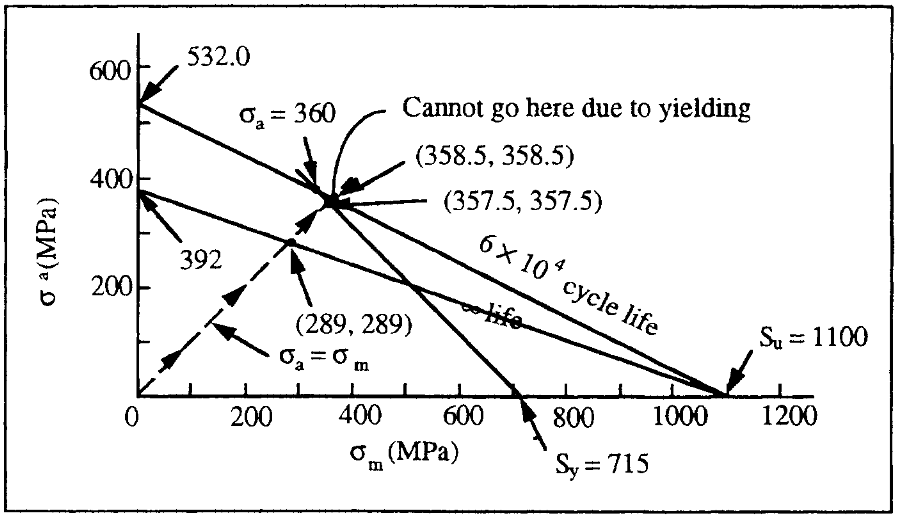
1. <u>Bending</u>



For ∞ life, σ_{max} = 630 MPa ■

For 6 × 10^4 cycles, σ_{max} = 715 MPa if no yielding is permitted; otherwise, σ_{max} = 966 MPa ■

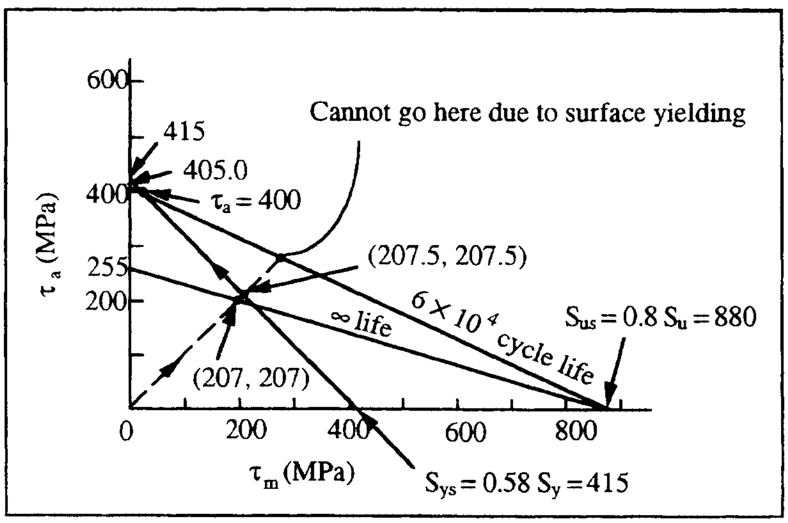
2. Axial



For ∞ life, σ_{max} = 578 MPa ■

For 6 × 10^4 cycles, σ_{max} = 715 MPa if no yielding is permitted; otherwise, σ_{max} = 720 MPa ■

3. Torsion

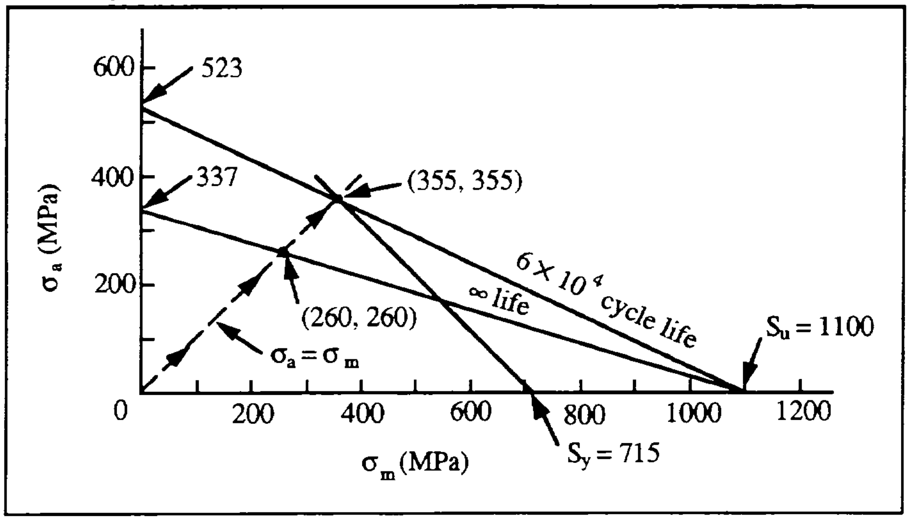


For ∞ life, τ_{max} = 414 MPa ■

For 6 × 10^4 cycle life, τ_{max} = 215 MPa if no yielding is permitted; otherwise, τ_{max} = 800 MPa ■

Average machined surface:

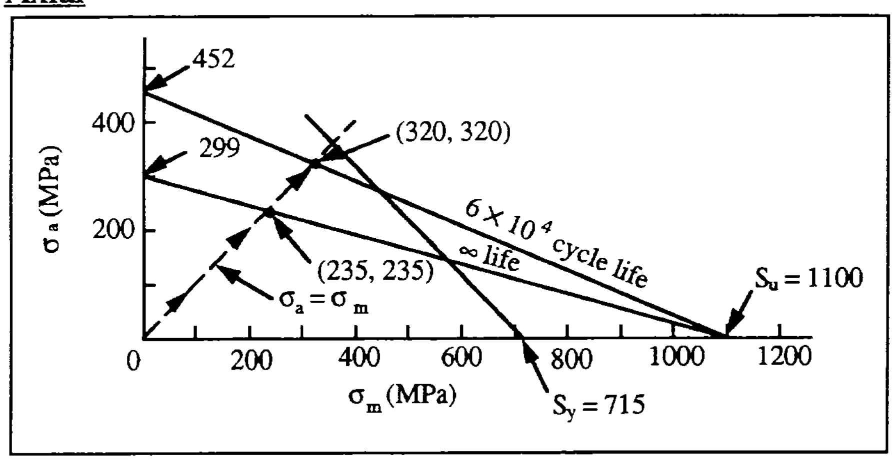
1. Bending



For ∞ life, σ_{max} = 520 MPa ■

For 6×10^4 cycles, σ_{max} = 710 MPa ■

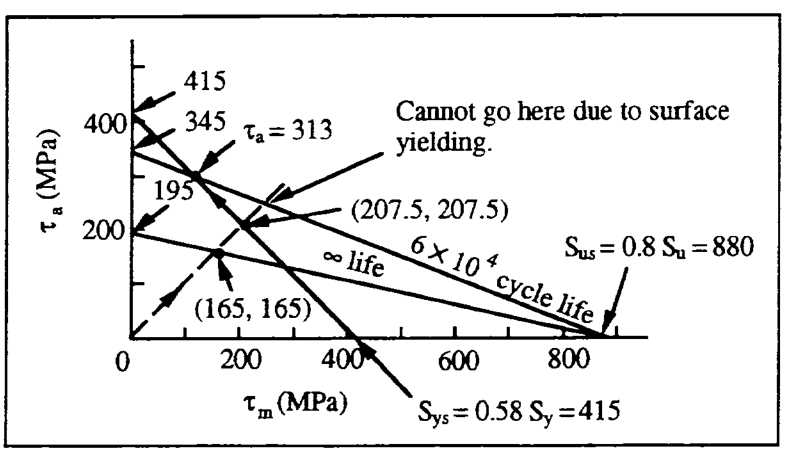
2. Axial



For ∞ life, σ_{max} = 470 MPa ■

For 6×10^4 cycles, σ_{max} = 640 MPa ■

3. Torsion



For ∞ life, $\tau_{max} = 330$ MPa ■

For 6×10^4 cycle life, $\tau_{max} = 415$ MPa if no yielding is permitted; otherwise, $\tau_{max} = 626$ MPa ■

SOLUTION (8.25)

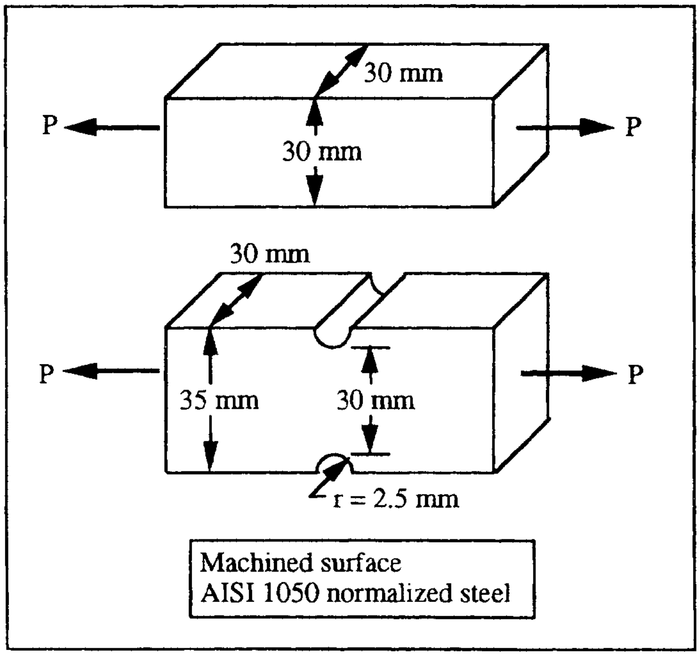
Known: An unnotched bar and a notched bar of known material have the same minimum cross section.

Find: For each bar, estimate

(a) the value of static tensile load P causing fracture

(b) the value of alternating axial load ± P that would be just on the verge of producing eventual fatigue fracture (after perhaps 1-5 million cycles).

Schematic and Given Data:



Assumption: The bar is manufactured as specified with regard to the critical fillet geometry and the bar surface finish.

Analysis:

1. For a static fracture of a ductile material, the notch has little effect. Hence, for both bars,

$$P \approx A \cdot S_u$$

where $S_u = 748.1$ MPa (Appendix C-4a)

$P = (30 \text{ mm})^2 (748.1 \text{ MPa}) = 673 \times 10^3$ N

$P = 670$ kN ■

2. $S_n = S_n' C_L C_G C_S$

where $S_n' = 0.5S_u = 0.5(748.1)$ MPa

$C_L = 1$ (Table 8.1)

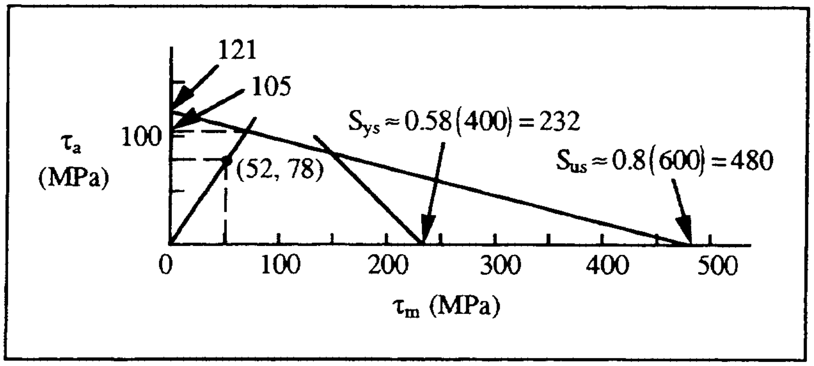
$C_G = 0.8$ (Table 8.1)

7. At critical fillet,

$$\tau_m = 1.63\left(\frac{80 - 16}{2}\right) = 52 \text{ MPa}$$

$$\tau_a = 1.63\left(\frac{80 + 16}{2}\right) = 78 \text{ MPa}$$

8. Thus, for torsional stresses,



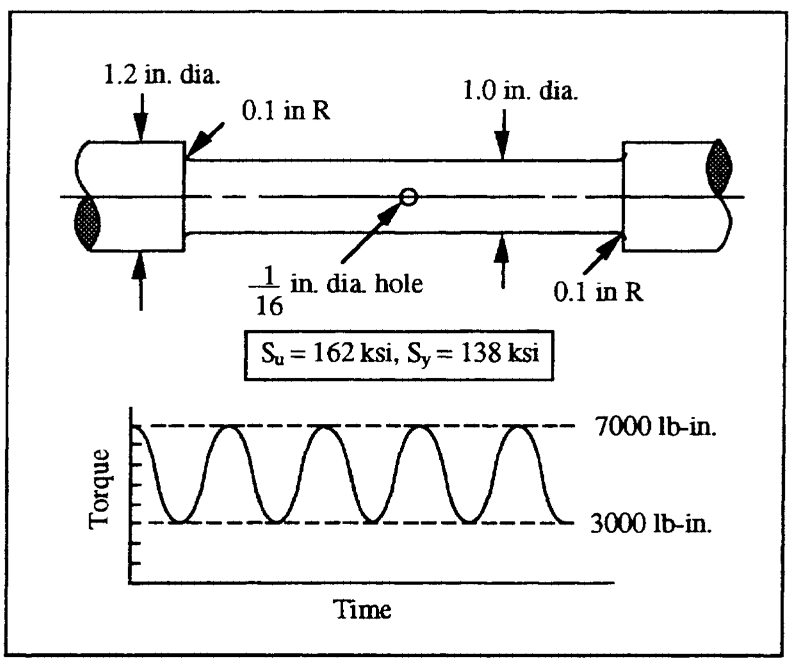
SF = 105/78 = 1.3 ■

SOLUTION (8.39)

Known: A round shaft made of steel having known S_u and S_y is subjected to a torque fluctuation. All critical surfaces are ground.

Find: Estimate the safety factor for infinite fatigue life with respect to an overload that
(a) increases both mean and alternating torque by the same factor,
(b) an overload that increases only the alternating torque.

Schematic and Given Data:



Assumption: The shaft is manufactured as specified with regard to critical radii, hole geometry, and surface finish.

Analysis:

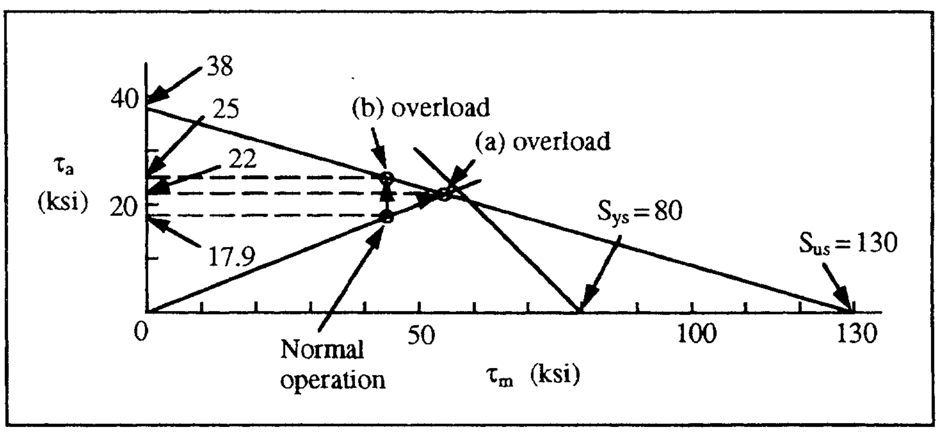
1. $S_{us} = 0.8(162) = 130$ ksi
   $S_{ys} = 0.58(138) = 80$ ksi
2. At the hole,
   from Fig. 4.37, $K_t = 1.75$
   from Fig. 8.24, $q = 0.88$
   $K_f = 1 + (K_t - 1)q$ [Eq. (8.2)]
   $K_f = 1 + (0.75)(0.88) = 1.66$
   (At fillet, $K_t = 1.33$; hence, not as critical as hole)
3. Using the equation in Fig. 4.37,

$$\tau_m = \frac{T_m}{(\pi D^3/16) - (dD^2/6)} K_f$$

$$\tau_m = \frac{5000}{\pi(1/16) - (1/16)(1/6)}(1.66) = 44{,}600 \text{ psi}$$

$$\tau_a = \frac{2000}{\pi/16 - 1/96}(1.66) = 17{,}900 \text{ psi}$$

4. $S_n = S_n' C_L C_G C_s$ [Eq. (8.1)]
   $S_n' = 0.5S_u$ (Fig. 8.5)
   $C_L = 0.58$ (Table 8.1)
   $C_G = 0.9$ (Table 8.1)
   $C_s = 0.89$ (Fig. 8.13)
   $S_n = 0.5(162)(0.58)(0.9)(0.89) = 38$ ksi ■
5.



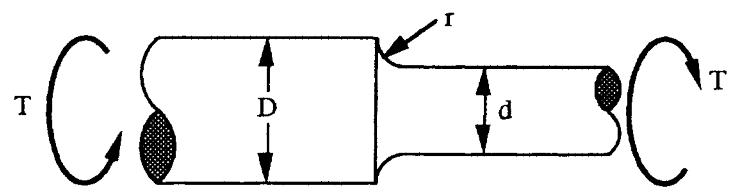
6. For an overload that increases both the mean and the alternating torque by the same factor,
   SF = 22/17.9 = 1.2 ■
   For an overload that increases only the alternating torque,
   SF = 25/17.9 = 1.4 ■

SOLUTION (8.40)

Known: A stepped shaft made of steel having known value of S_u is finished by grinding the surface. In service, it is loaded with a fluctuating zero-to-maximum torque.

Find: Estimate the magnitude of maximum torque which would provide a safety factor of 1.3 with respect to a 75,000 cycle fatigue life.

Schematic and Given Data:



D = 10 mm
d = 8 mm
r = 0.8 mm
S_u = 1200 MPa

Assumption: The shaft is manufactured as specified with regard to the critical fillet geometry and shaft surface finish.

Analysis:

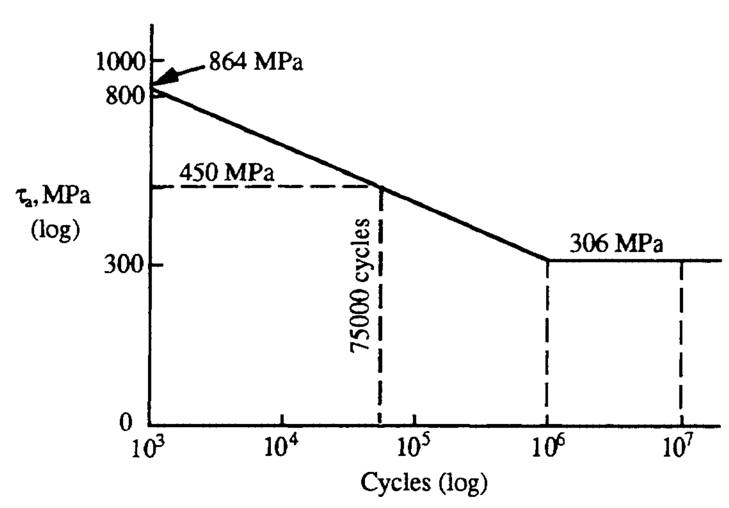
1. $S_n = S_n' C_L C_G C_s$ [Eq. (8.1)]

   $S_n' = 0.5S_u$ (Fig. 8.5)
   $C_L = 0.58$ (Table 8.1)
   $C_G = 1$ (Table 8.1)
   $C_s = 0.88$ (Fig. 8.13)

   $S_n = 0.5(1200)(0.58)(1)(0.88) = 306$ MPa ■

2. For 10^3 cycle strength, from Table 8.1, $S = 0.9\ S_{us}$ where $S_{us} = 0.8S_u$
   Therefore, $S = 0.9(0.8)(1200) = 864$ MPa

3.

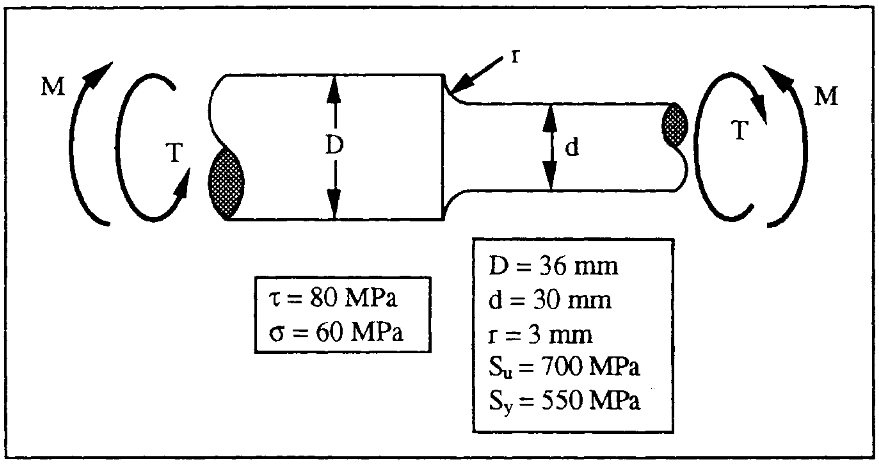


SOLUTION (8.43)

Known: A steel shaft used in a spur gear reducer is subjected to a constant torque together with lateral forces that tend always to bend it downward in the center. The stresses are known, but these values do not take into account stress concentration caused by a shoulder with known dimensions. All surfaces are machined and the strength values and hardness of the steel are known.

Find: Estimate the safety factor with respect to infinite life.

Schematic and Given Data:



Assumption: The shaft is manufactured as specified with regard to the critical fillet and shaft surface finish.

Analysis:

1. We use the Fig. 8.16 relationship for "general biaxial loads":
   - Bending provides an alternating stress:

     $\sigma_a = \sigma_{ea} = 60\ K_f$ MPa

     From Fig. 4.35(a), $K_t = 1.63$

     From Fig. 8.24, $q = 0.84$

     From Eq. 8.2, $K_f = 1 + (0.63)(0.84) = 1.53$

     $\sigma_{ea} = 60(1.53) = 91.8$ MPa
   - Torsion provides a mean stress:

     $\tau_m = \sigma_{em} = 80\ K_f$ MPa

     From Fig. 4.35(c), $K_t = 1.33$

     From Fig. 8.24, $q = 0.86$

     From Eq. 8.2, $K_f = 1 + (0.33)(0.86) = 1.28$

     $\sigma_{em} = 80(1.28) = 102.4$ MPa.

2. $S_n = S_n' C_L C_G C_s$ [Eq. (8.1)]

   $S_n' = 0.5 S_u$ (Fig. 8.5)

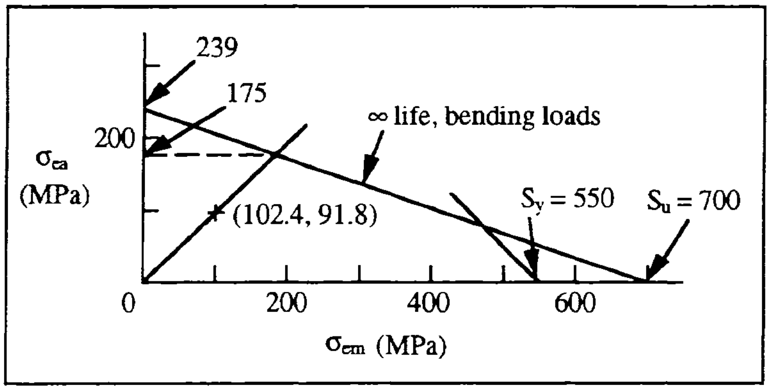
   $C_L = 1$ (Table 8.1)

   $C_G = 0.9$ (Table 8.1)

   $C_s = 0.76$ (Fig. 8.13)

   $S_n = 0.5(700)(1)(0.9)(0.76) = 239$ MPa

3.



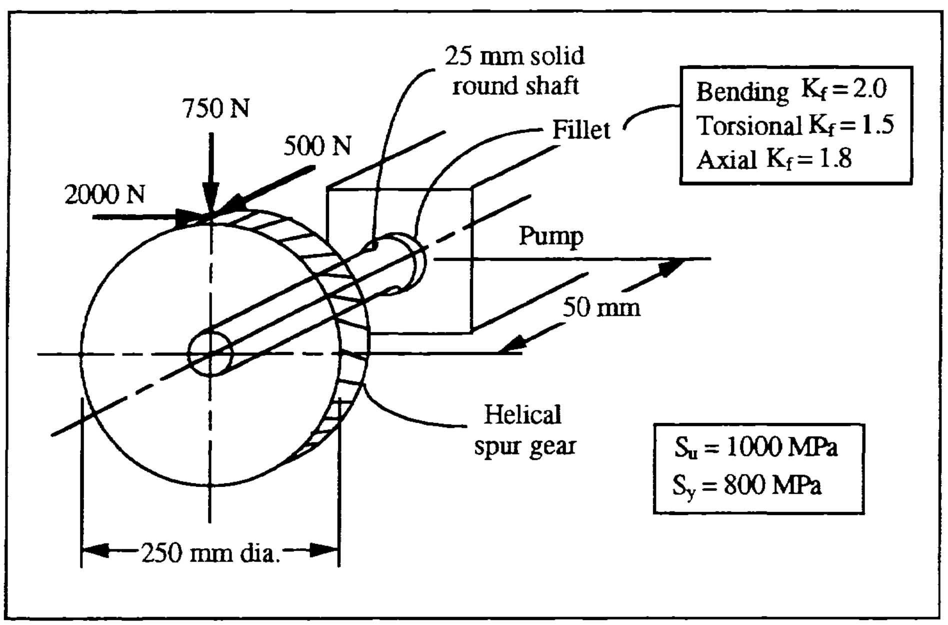
4. SF = 175/91.8 = 1.9 ■

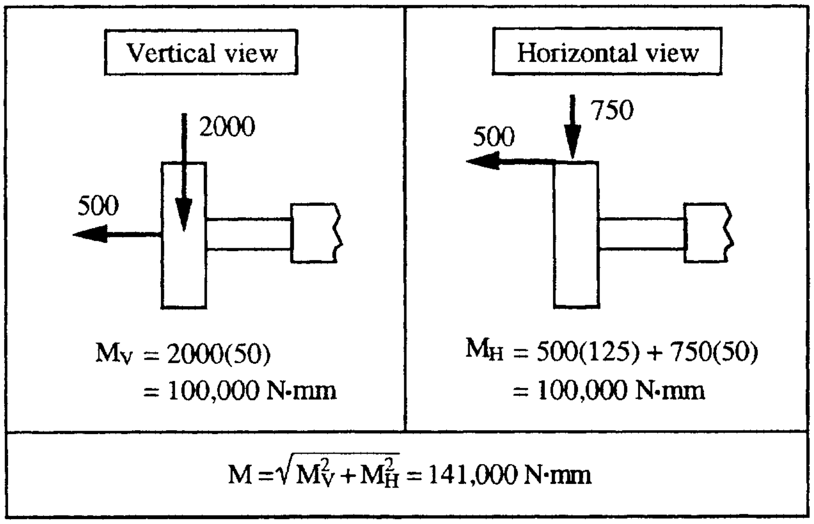
SOLUTION (8.44)

Known: A pump is gear-driven at uniform load and speed. The shaft is supported by bearings mounted in the pump housing. The shaft is made of steel having known values of S_u and S_y. The tangential, axial, and radial components of force applied to the gear are known. The surface of the shaft fillet has been shot-peened, which is estimated to be equivalent to a laboratory mirror-polished surface. Fatigue stress concentration factors for the fillet have been determined.

Find: Estimate the safety factor with respect to eventual fatigue failure at the fillet.

Schematic and Given Data:





Assumption: The shaft is manufactured as specified with regard to shaft geometry and surface finish.

Analysis:

1. $M_V = (2000)(50) = 100{,}000$ N·mm, $M_H = 500(125) + 750(50) = 100{,}000$ N·mm; $M = \sqrt{M_V^2 + M_H^2} = 141{,}000$ N·mm
2. We use the Fig. 8.16 relationship for "general biaxial loads":
   Alternating stress:

   $$\sigma_a = \frac{32M}{\pi d^3}K_f = \frac{32(141{,}000)}{\pi(25)^3}(2) = 183.8 \text{ MPa}$$

   $\sigma_{ea} = 183.8$ MPa
   Mean stresses:

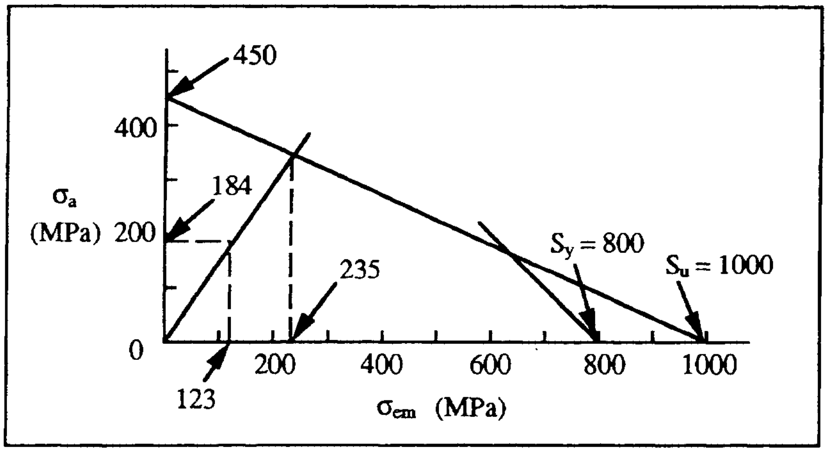
   $$\tau = \frac{16M}{\pi d^3}K_f = \frac{16(2000)(125)}{\pi(25)^3}(1.5) = 122.2 \text{ MPa}$$

   $$\sigma = \frac{P}{A}K_f = \frac{500(4)}{\pi(25)^2}(1.8) = 1.83 \text{ MPa}$$

   $$\sigma_{em} = \frac{\sigma}{2} + \sqrt{\tau^2 + \left(\frac{\sigma}{2}\right)^2} = .92 + \sqrt{122.2^2 + 0.92^2}$$

   $\sigma_{em} = 123.1$ MPa
3. $S_n = S_n' C_L C_G C_s$ [Eq. (8.1)]
   $S_n' = 0.5S_u$ (Fig. 8.5)
   $C_L = 1$ (Table 8.1)
   $C_G = 0.9$ (Table 8.1)
   $C_s = 1$ (Fig. 8.13)
   $S_n = 0.5(1000)(1)(0.9)(1) = 450$ MPa

4.



5. SF = 235/123 = 1.9 ■

SOLUTION (8.45)

Known: A countershaft has helical gear (B), bevel gear (D), and two supporting bearings (A and C). Loads acting on the bevel gear are known. Forces on the helical gears can be determined. Shaft dimensions are known. All shoulder fillets have a radius of 5 mm. Only bearing A takes thrust. The shaft is made of hardened steel having known values of S_u and S_y. All important surfaces are finished by grinding.

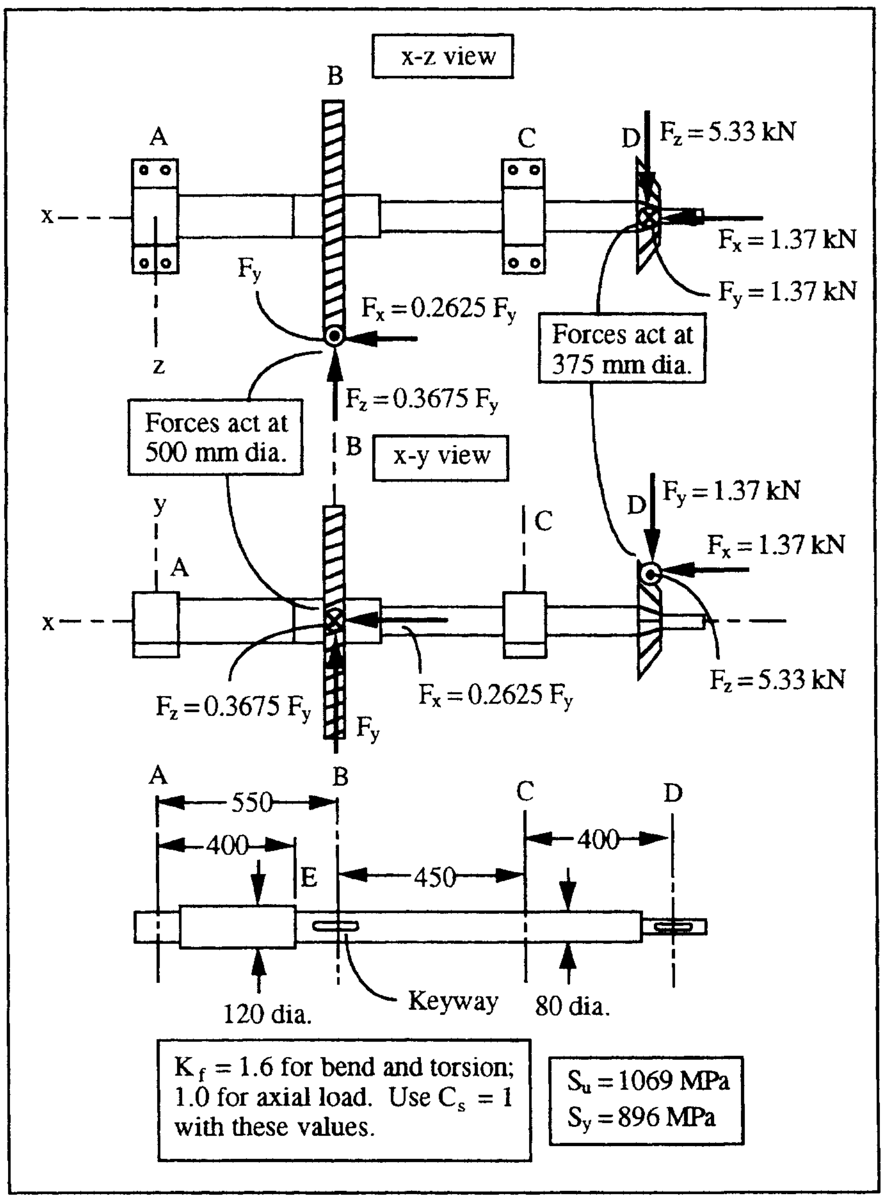
Find:

(a) Draw load, shear force, and bending moment diagrams for the shaft in the xy- and xz- planes. Also draw diagrams showing the intensity of the axial force and torque along the length of the shaft.

(b) At points B, C, and E of the shaft, calculate the equivalent stresses in preparation for making a fatigue safety factor determination. (Note: Refer to Fig. 8.16.)

(c) For a reliability of 99% (and assuming $\sigma = 0.08\ S_n$), estimate the safety factor of the shaft at points B, C, and E.

Schematic and Given Data:



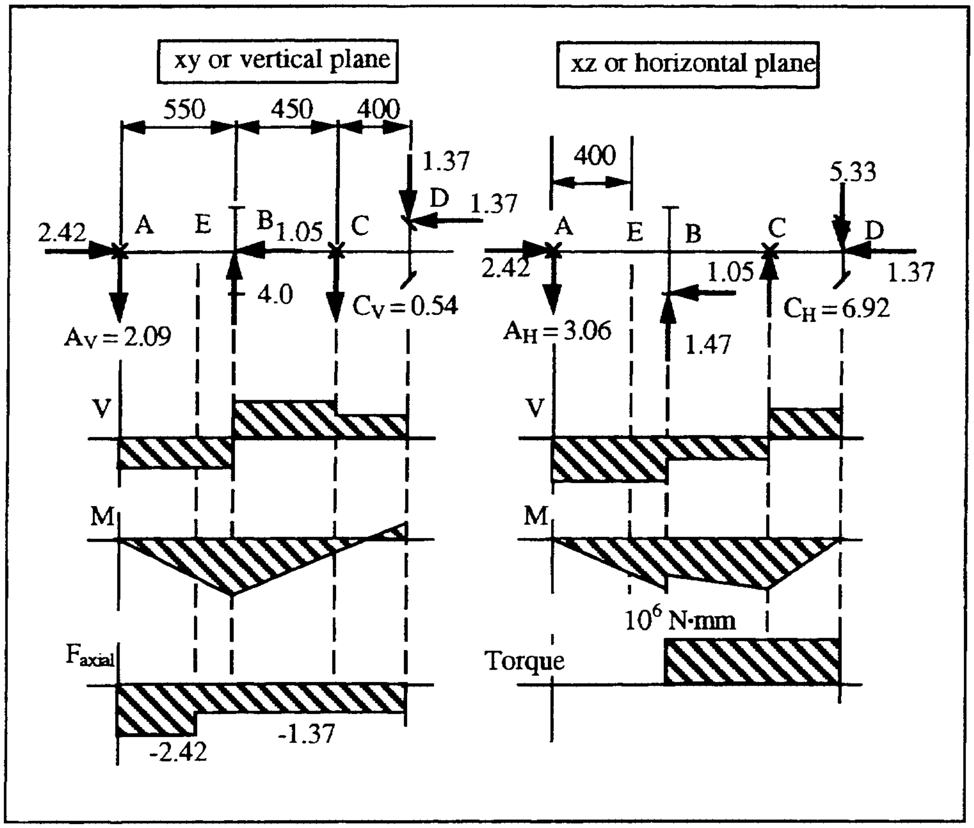
Assumption: The shaft is manufactured as specified with regard to the critical shaft geometry and surface finish.

Analysis:

1. Load determination

(a) Helical gear forces:
For $\sum M_x = 0$, the torque at the two gears must be equal. Therefore, F_y (250 mm) = 5.33(187.5 mm). Hence, $F_y = 4.00$ kN.
From the given data, $F_x = .2625F_y = 1.05$ kN; $F_z = .3675\ F_y = 1.47$ kN.

(b) Determine shaft loads in the xy and xz planes



Vertical forces:

$$\sum M_A = 0: \ C_v = \frac{4(550) + 1.37(187.5) - 1.37(1400)}{1000}$$
$$= 0.54 \text{ kN downward}$$

$$\sum F = 0: \quad A_v = 4 - 0.54 - 1.37 = 2.09 \text{ kN downward}$$

Horizontal forces:

$$\sum M_A = 0: \ C_H = \frac{1.05(250) - 1.47(550) + 5.33(1400)}{1000}$$
$$= 6.92 \text{ kN upward}$$

$$\sum F = 0: \quad A_H = 1.47 + 6.92 - 5.33$$
$$= 3.06 \text{ kN downward}$$

2. Stress determination

(a) At E, the loading is:
Compression of 1.37 kN, $K_t = 2.2$, $q = .94$,
$K_f = 2.13$. Axial stress (mean or constant) =

$$\frac{4PK_f}{\pi d^2} = \frac{4(-1.37)(2.13)}{\pi(80)^2} = -0.581 \text{ MPa}$$

The tension stress is zero.

$$M = \sqrt{(2.09 \times 400)^2 + (3.06 \times 400)^2}$$
$$= 1482 \text{ kN·mm}$$

$K_t = 1.9$, $q = .94$. Therefore, $K_f = 1.85$

$$\text{Bending stress (alternating)} = \frac{32M}{\pi d^3} K_f$$

$$= \frac{32(1482 \times 10^3)}{\pi(80)^3}(1.85) = 54.5 \text{ MPa}$$

From Eq. (a) and Eq. (b) in the figure caption of Fig. 8.16, $\sigma_{em} = 0$;
$\sigma_{ea} = 54.5$ MPa

(b) At B, the loading is:
Axial, $P = -1.37$ kN, $K_f = 1.0$, $\sigma = -0.27$ MPa
Torsion = (4.0)(250) = 1000 kN·mm
Bending : $M = \sqrt{(2.09 \times 550)^2 + (3.06 \times 550)^2} = 2038$ kN·mm
$K_f = 1.6$ for bending and torsion

$$\text{Bending stress (alternating)} = \frac{32M}{\pi d^3} K_f$$

$$= \frac{32(2038 \times 10^3)}{\pi(80)^3}(1.6) = 64.9 \text{ MPa}$$

$$\text{Torsional stress (mean)} = \frac{16T}{\pi d^3} K_f = \frac{16(10)^6}{\pi(80)^3}(1.6) = 15.9 \text{ MPa}$$

$$\sigma_{em} = \frac{-0.27}{2} + \sqrt{(15.9)^2 + \left(\frac{-0.27}{2}\right)^2} = 15.76 \text{ MPa}; \qquad \sigma_{ea} = 64.9 \text{ MPa}$$

(c) At C, the loading is:
Bending:

$$M = \sqrt{(5.33 \times 400)^2 + [1.37 \times (400 - 187.5)]^2} = 2152 \text{ kN·mm}$$

$$\text{Bending stress (alternating)} = \frac{32(2152) \times 10^3}{\pi(80)^3} = 42.8 \text{ MPa}$$

$\sigma_{ea} = 42.8$ MPa

Torsional stress - same as (b) except no stress concentration factor; axial same as (b).

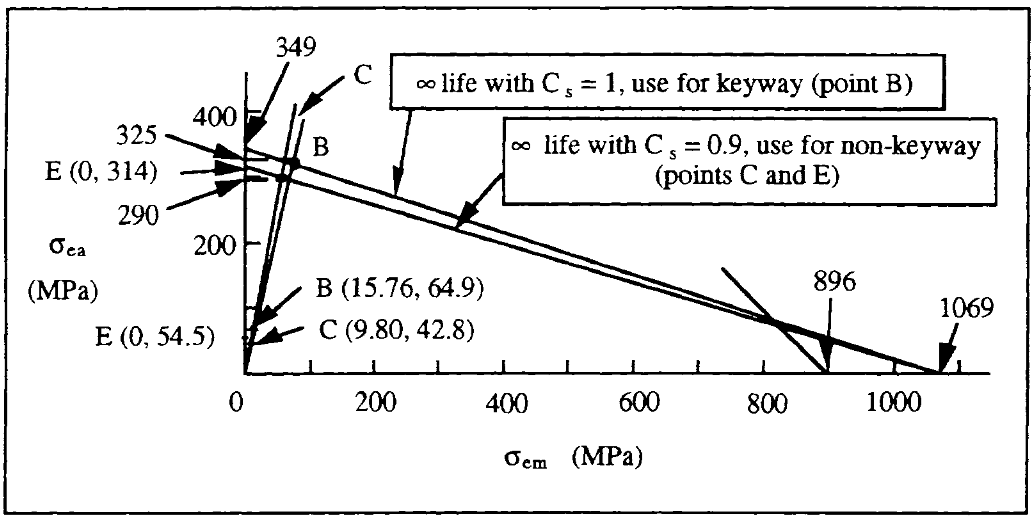
$$\sigma_{em} = \frac{-0.27}{2} + \sqrt{\left(\frac{15.9}{1.6}\right)^2 + \left(\frac{.27}{2}\right)^2} = 9.80 \text{ MPa}$$

3. Strength and safety factor determination
S_u = 155 ksi = 1069 MPa; S_y = 130 ksi = 896 MPa
For working with equivalent bending stress, S_n is

$$S_n = S_n' C_L C_G C_s = \left(\frac{1069}{2}\right)(1)(0.8)^*(0.9)$$
$$= 385 \text{ MPa for } C_s = 0.9$$
*(See note b, Table 8.1)

$$S_n = S_n' C_L C_G C_s = \left(\frac{1069}{2}\right)(1)(0.8)(1.0)$$
$$= 428 \text{ MPa for } C_s = 1.0$$
But for 99% reliability, reduce this by 2.3 standard deviations, which amounts to multiplying by a factor of (1 - 2.3 × .08) = .816
Thus, for 99% reliability,
S_n = 385(.816) = 314 MPa (for C_s = .9)
S_n = 428(.816) = 349 MPa (for C_s = 1.0)

4.



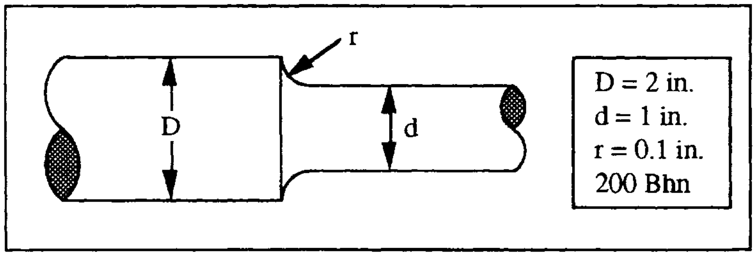
5. Safety factors: (B) SF = 325/64.9 = 5.0
(C) SF = 290/42.8 = 6.8
(E) SF = 314/54.5 = 5.8

SOLUTION (8.46)

Known: A stepped shaft having known dimensions was machined from AISI steel of known hardness. The loading is one of completely reversed torsion. During a typical 30 seconds of operation under overload conditions the nominal (Tc/J) stress in the 1-in.-dia. section was measured.

Find: Estimate the life of the shaft when operating continuously under these conditions.

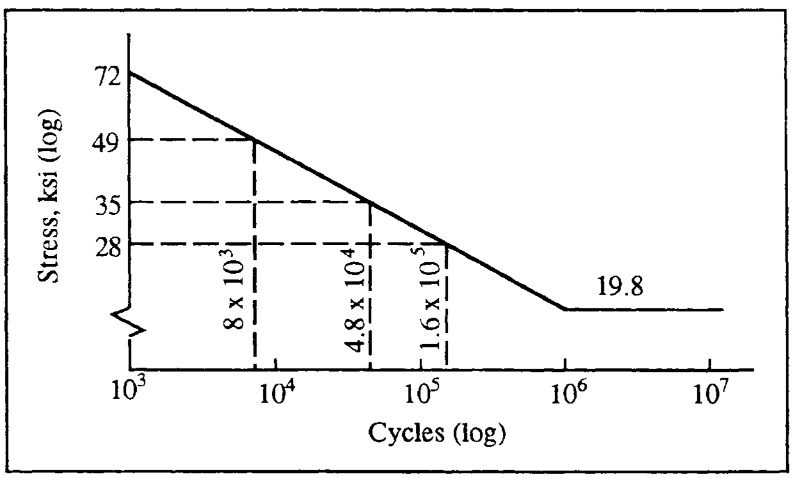
Schematic and Given Data:



Assumption: The shaft is manufactured as specified with regard to the critical fillet geometry and surface finish.

Analysis:

1. At the fillet,
   from Fig. 4.35(c), $K_t = 1.46$
   from Fig. 8.24, $q = 0.86$
   Thus, using Eq. (8.2), $K_f = 1 + (0.46)(0.86) = 1.40$
2. $S_n = S_n' C_L C_G C_s$ [Eq. (8.1)]
   $S_n' = 0.25$ Bhn (Fig. 8.5)
   $C_L = 0.58$ (Table 8.1)
   $C_G = 0.9$ (Table 8.1)
   $C_s = 0.76$ (Fig. 8.13)
   $S_n = 0.25(200)(0.58)(0.9)(0.76) = 19.8$ ksi
3. From Table 8.1,
   10^3 cycle strength $= 0.9S_{us} = 0.9(0.8)S_u$
   $= 0.9(0.8)(0.5)\text{Bhn} = 0.9(0.8)(0.5)(200) = 72$ ksi
4.



5. The 30 second test involves these stresses (in the fillet) above the endurance limit (see graph):
   1 cycles at $\tau_a = 35(1.4) = 49$ ksi
   ($N = 8 \times 10^3$ cycles)
   2 cycles at $\tau_a = 25(1.4) = 35$ ksi
   ($N = 4.8 \times 10^4$ cycles)

4 cycles at $\tau_a = 20(1.4) = 28$ ksi

($N = 1.5 \times 10^5$ cycles)

$$\text{Life used in 30 seconds} = \frac{1}{8 \times 10^3} + \frac{2}{4.8 \times 10^4} + \frac{4}{1.6 \times 10^5} = 1.916 \times 10^{-4}$$

$$\text{Estimated life} = \frac{1}{1.916 \times 10^{-4}} = 5217 \text{ periods of 30 seconds}$$

Estimated life ≈ 43 hours ■
