# Contents

# 6 Gears

## 6.1 Introduction

*Gears* are toothed elements that transmit rotary motion from one shaft to another. Gears are generally rugged and durable and their power transmission efficiency is as high as 98 percent. Gears are usually more costly than chains and belts. The American Gear Manufacturers Association (AGMA) has established standard tolerances for various degrees of gear manufacturing precision. *Spurs gears* are the simplest and most common type of gears. They are used to transfer motion between parallel shafts, and they have teeth that are parallel to the shaft axes.

## 6.2 Geometry and Nomenclature

The basic requirement of gear-tooth geometry is the condition of angular velocity ratios that are exactly constant, i.e., the angular velocity ratio between a 30-tooth and a 90-tooth gear must be precisely 3 in every position. The action of a pair of gear teeth satisfying this criterion is called conjugate gear-tooth action.

Law of conjugate gear-tooth action:
*The common normal to the surfaces at the point of contact of two gears in rotation must always intersect the line of centers at the same point* P*, called the pitch point.*

The law of conjugate gear-tooth action can be satisfied by various tooth shapes, but the one of current importance is the involute of the circle. An *involute* (of the circle) is the curve generated by any point on a taut thread as it unwinds from a circle, called the *base circle* [Fig. 6.1(a)]. The involute can also be defined as the locus of a point on a taught string that is unwrapped from a cylinder. The circle that represents the cylinder is the base circle. Figure 6.1(b) represents an involute generated from a base circle of radius $r_b$ starting at the point $A$. The radius of curvature of the involute at any point $I$ is given by

$$\rho = \sqrt{r^2 - r_b^2}, \tag{6.1}$$

where $r = OI$. The *involute pressure angle* at $I$ is defined as the angle between the normal to the involute $IB$ and the normal to $OI$, $\phi = \angle IOB$.

In any pair of meshing gears, the smaller of the two is called the pinion and the larger one the gear. The term "gear" is used in a general sense to indicate either of the members and also in a specific sense to indicate the larger of the two. The angular velocity ratio between a pinion and a gear is (Fig. 6.2):

$$i = \omega_p/\omega_g = -d_g/d_p, \tag{6.2}$$

where $\omega$ is the angular velocity and $d$ is the *pitch diameter*, and the minus sign indicates that the two gears rotate in opposite directions. The *pitch circles* are the two circles, one for each gear, that remain tangent throughout the engagement cycle. The point of tangency is the pitch point. The diameter of the pitch circle is the pitch diameter. If the angular speed is expressed in rpm then the symbol $n$ is preferred instead of $\omega$. The diameter (without a qualifying adjective) of a gear always refers to its pitch diameter. If other diameters (base, root, outside, etc.) are intended, they are always specified. Similarly, $d$, without subscripts, refers to pitch diameter. The pitch diameters of a pinion and gear are distinguished by subscripts $p$ and $g$ ($d_p$ and $d_g$, are their symbols, see Fig. 6.2). The *center distance* is

$$c = (d_p + d_g)/2 = r_p + r_g, \tag{6.3}$$

where $r = d/2$ is the *pitch circle radius*.

In Fig. 6.3 line $tt$ is the common tangent to the pitch circles at the pitch point and $AB$ is the common normal to the surfaces at $C$, the point of contact of two gears. The angle of $AB$ with the line $tt$ is called the *pressure angle*, $\phi$. The most common pressure angle used, with both English and SI units, is $20^0$. In the United States $25^0$ is also standard, and $14.5^0$ was formerly an alternative standard value. The pressure angle affects the force that tends to separate the two meshing gears.

The involute profiles are augmented outward beyond the pitch circle by a distance called the *addendum*, $a$, [Fig. 6.4(a)]. The outer circle is usually termed the *addendum circle*, $r_a = r + a$. Similarly, the tooth profiles are extended inward from the pitch circle, a distance called the *dedendum*, $b$. The involute portion can extend inward only to the base circle. A fillet at the base of the tooth merges the profile into the dedendum circle. The fillet decreases the bending stress concentration. The *clearance* is the amount by which the dedendum in a given gear exceeds the addendum of its meshing gear.

The *circular pitch* is designated as $p$, and measured in inches (English units) or millimeters (SI units). If $N$ is the number of teeth in the gear (or pinion), then

$$p = \pi d/N, \;\; p = \pi d_p/N_p, \;\; p = \pi d_g/N_g. \tag{6.4}$$

More commonly used indices of gear-tooth size are *diametral pitch*, $P_d$ (used only with English units), and *module*, $m$ (used only with SI). Diametral pitch is defined as the number of teeth per inch of pitch diameter [see Fig. 6.4(b)]:

$$P_d = N/d, \;\; P_d = N_p/d_p, \;\; P_d = N_g/d_g. \tag{6.5}$$

Module $m$, which is essentially the complementary of $P_d$, is defined as the pitch diameter in millimeters divided by the number of teeth (number of millimeters of pitch diameter per tooth):

$$m = d/N, \;\; m = d_p/N_p, \;\; m = d_g/N_g. \tag{6.6}$$

One can easily verify that

$$p\, P_d = \pi \;\; (p \text{ in inches; } P_d \text{ in teeth per inch})$$

$$p/m = \pi \;\; (p \text{ in millimeters; } m \text{ in millimeters per tooth})$$

$$m = 25.4/P_d.$$

With English units the word "pitch", without a qualifying adjective, denotes diametral pitch (a "12-pitch gear" refers to a gear with $P_d = 12$ teeth per inch of pitch diameter). With SI units "pitch" means circular pitch (a "gear of pitch = 3.14 mm" refers to a gear having a circular pitch of $p = 3.14$ mm).

Standard diametral pitches, $P_d$ (English units), in common use are

1 to 2 by increments of 0.25

2 to 4 by increments of 0.5

4 to 10 by increments of 1

10 to 20 by increments of 2

20 to 40 by increments of 4.

With SI units, commonly used standard values of module $m$ are

0.2 to 1.0 by increments of 0.1

1.0 to 4.0 by increments of 0.25

4.0 to 5.0 by increments of 0.5.

Addendum, minimum dedendum, whole depth, and clearance for gears with English units in common use are [5]:

| | $14\frac{1}{2}^\circ$ Full-depth Involute or composite | $20^\circ$ Full-depth involute | $20^\circ$ Stub involute |
|---|---|---|---|
| addendum $a$ | $1/P_d$ | $1/P_d$ | $0.8/P_d$ |
| minimum dedendum $b$ | $1.157/P_d$ | $1.157/P_d$ | $1/P_d$ |
| whole depth | $2.157/P_d$ | $2.157/P_d$ | $1.8/P_d$ |
| clearance | $0.157/P_d$ | $0.157/P_d$ | $0.2/P_d$ |

For SI units the standard values for full-depth involute teeth with a pressure angle of $20^0$ are addendum $a = m$ and minimum dedendum $b = 1.25\,m$.

## 6.3 Interference and Contact Ratio

The contact of segments of tooth profiles which are not conjugate is called *interference*. The involute tooth form is only defined outside the base circle. In some cases, the dedendum will extend below the base circle, then the portion of tooth below the base circle will not be an involute and will interfere with the tip of the tooth on the meshing gear, which is an involute. Interference will occur, preventing rotation of the meshing gears, if either of the addendum circles extends beyond tangent points $A$ and $B$ (Fig. 6.5) which are called interference points. In Fig. 6.5 both addendum circles extend beyond the interference points.

The maximum possible addendum circle radius, of pinion or gear, without interference is

$$r_{a(max)} = \sqrt{r_b^2 + c^2 \sin^2 \phi}, \tag{6.7}$$

where $r_b = r \cos \phi$ is the base circle radius of pinion or gear. The base circle diameter is

$$d_b = d \cos \phi. \tag{6.8}$$

The average number of teeth in contact as the gears rotate together is the contact ratio $CR$, which is calculated from the following equation (for external gears):

$$CR = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - c \sin \phi}{p_b}, \tag{6.9}$$

where $r_{ap}$, $r_{ag}$ are addendum radii of the pinion and gear, and $r_{bp}$, $r_{bg}$ are base circle radii of the pinion and gear. The base pitch, $p_b$, is computed with

$$p_b = \pi d_b / N = p \cos \phi. \tag{6.10}$$

The base pitch is like the circular pitch except that it represents an arc of the base circle rather than an arc of the pitch circle.
For internal gears the contact ratio is

$$CR = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} - \sqrt{r_{ag}^2 - r_{bg}^2} + c \sin \phi}{p_b}, \tag{6.11}$$

The greater the contact ratio, the smoother and quieter the operation of the gears. If the contact ratio is 2 then two pairs of teeth are in contact at all the times. The acceptable values for contact ratio are $CR > 1.2$.

Gears are commonly specified according to AGMA Class Number, a code that denotes important quality characteristics. Quality numbers denote tooth-elements tolerances. The higher the number, the tighter the tolerance. Gears are heat treated by case hardening, nitriding, precipitation hardening, or through hardening. In general, harder gears are stronger and last longer than soft ones.

## 6.4 Ordinary Gear Trains

A gear train is any collection of two or more meshing gears. Figure 6.6(a) shows a simple gear train with three gears in series. The train ratio is computed with the relation

$$i_{13} = \frac{\omega_1}{\omega_3} = \frac{\omega_1}{\omega_2}\frac{\omega_2}{\omega_3} = \left(-\frac{N_2}{N_1}\right)\left(-\frac{N_3}{N_2}\right) = \frac{N_3}{N_1}. \tag{6.12}$$

Only the sign of the overall ratio is affected by the intermediate gear 2 which is called an *idler*.

Figure 6.6(b) shows a compound gear train, without idler gears, with the train ratio

$$i_{14} = \frac{\omega_1}{\omega_2}\frac{\omega_{2'}}{\omega_3}\frac{\omega_{3'}}{\omega_4} = \left(-\frac{N_2}{N_1}\right)\left(-\frac{N_3}{N_{2'}}\right)\left(-\frac{N_4}{N_{3'}}\right) = -\frac{N_2 N_3 N_4}{N_1 N_{2'} N_{3'}}. \tag{6.13}$$

## 6.5 Epicyclic Gear Trains

When at least one of the gear axes rotates relative to the frame in addition to the gear's own rotation about its own axes, the train is called a *planetary gear train* or *epicyclic gear train*. The term "epicyclic" comes from the fact that points on gears with moving axes of rotation describe epicyclic paths. When a generating circle (planet gear) rolls on the outside of another circle, called a directing circle (sun gear), each point on the generating circle describes an epicycloid, as shown in Fig. 6.7.

Generally, the more planet gears there are the greater the torque capacity of the system. For better load balancing, new designs have two sun gears and up to 12 planetary assemblies in one casing.

In the case of simple and compound gears it is not difficult to visualize the motion of the gears and the determination of the speed ratio is relatively easy. In the case of epicyclic gear trains it is often difficult to visualize the motion of the gears. A systematic procedure, using the contour method, is presented below. The contour method is applied to determine the distribution of velocities for epicyclic gear trains.

The velocity equations for a simple closed kinematic chain are [1, 24]:

$$\sum_{(i)} \boldsymbol{\omega}_{i,i-1} = \mathbf{0} \ \text{ and } \ \sum_{(i)} \mathbf{r}_{A_i} \times \boldsymbol{\omega}_{i,i-1} = \mathbf{0},$$

where $\boldsymbol{\omega}_{i,i-1}$ is the relative angular velocity of the rigid body $(i)$ with respect to the rigid body $(i-1)$ and $\mathbf{r}_{A_i}$ is the position vector of the joint between the rigid body $(i)$ and the rigid body $(i-1)$ with respect to a "fixed" reference frame.

The epicyclic (planetary) gear train shown in Fig. 6.8 consists of a central gear 2 (sun gear) and another gear 3 (planet gear) in mesh with 2 at $B$. Gear 3 is carried by the arm 1 hinged at $A$. The ring gear 4 meshes with the planet gear 3 and pivots at $A$, so it can be easily tapped as an output member. The sun gear and the ring gear are concentric. The sun gear, the ring gear, and the arm can be accessed to tap the angular velocity and torque either as an input or an output. There are four moving bodies 1, 2, 3, and 4, ($n = 4$) connected by:

• four full joints ($c_5 = 4$): one hinge between the arm 1 and the planet gear 3 at $C$, one hinge between the frame 0 and the shaft of the sun gear 2 at $A$, one hinge between the frame 0 and the ring gear 4 at $A$, and one hinge between the frame 0 and the arm 1 at $A$.

• two half joints ($c_4 = 2$): one between the sun gear 2 and the planet gear 3, and one between the planet gear 3 and the ring gear 4.
The system has two degrees of freedom: $M = 3\,n - 2\,c_5 - c_4 = 3{\cdot}4 - 2{\cdot}4 - 2 = 2$. The sun gear has $N_2$-tooth external gear, the planet gear has $N_3$-tooth external gear, and the ring gear has $N_4$-tooth internal gear.

If the arm and the sun gear rotate with input angular speeds $\omega_1$ and $\omega_2$, find the absolute output angular velocity of the ring gear.

The velocity analysis is carried out using the contour method. The system shown in Fig. 6.8 has a total of five elements ($p = 5$): the frame 0 and four moving links 1, 2, 3, and 4. There are six joints ($l = 6$): four full joints and two half joints. The number of independent contours is given by

$$n_c = l - p + 1 = 6 - 5 + 1 = 2.$$

This gear system has two independent contours. The graph of the kinematic chain is represented in Fig. 6.9.

*First contour*

The first contour is formed by the elements 0, 1, 3, 2, and 0 (clockwise path). For the velocity analysis, the following vectorial equations can be written

$$\begin{aligned} &\boldsymbol{\omega}_{10} + \boldsymbol{\omega}_{31} + \boldsymbol{\omega}_{23} + \boldsymbol{\omega}_{02} = \mathbf{0}, \\ &\mathbf{r}_{AC} \times \boldsymbol{\omega}_{31} + \mathbf{r}_{AB} \times \boldsymbol{\omega}_{23} = \mathbf{0}, \end{aligned} \tag{6.14}$$

where the input angular velocities are

$$\boldsymbol{\omega}_{10} = \omega_1\,\mathbf{\imath} \text{ and } \boldsymbol{\omega}_{02} = -\omega_2\,\mathbf{\imath},$$

and the unknown angular velocities are

$$\boldsymbol{\omega}_{31} = \omega_{31}\,\mathbf{\imath} \text{ and } \boldsymbol{\omega}_{23} = \omega_{23}\,\mathbf{\imath}.$$

The sign of the relative angular velocities is selected positive and then the numerical computation gives the true orientation of the vectors.

The vectors $\mathbf{r}_{AB}$, $\mathbf{r}_{AC}$, and $\mathbf{r}_{AD}$ are defined as

$$\mathbf{r}_{AB} = x_B\,\mathbf{\imath} + y_B\,\mathbf{\jmath},\quad \mathbf{r}_{AC} = x_C\,\mathbf{\imath} + y_C\,\mathbf{\jmath},\quad \mathbf{r}_{AD} = x_D\,\mathbf{\imath} + y_D\,\mathbf{\jmath}, \tag{6.15}$$

where

$$\begin{aligned} y_B &= r_2 = m\,N_2/2, \\ y_C &= r_2 + r_3 = m\,(N_2 + N_3)/2, \\ y_D &= r_2 + 2\,r_3 = m\,N_2/2 + m\,N_3. \end{aligned}$$

The module of the gears is $m$. Equation (6.14) becomes

$$\omega_1\,\mathbf{\imath} + \omega_{31}\,\mathbf{\imath} + \omega_{23}\,\mathbf{\imath} - \omega_2\,\mathbf{\imath} = \mathbf{0},$$
$$\begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_C & y_C & 0 \\ \omega_{31} & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_B & y_B & 0 \\ \omega_{23} & 0 & 0 \end{vmatrix} = \mathbf{0}. \tag{6.16}$$

Equation (6.16) can be projected on a "fixed" reference frame $xOyz$

$$\begin{aligned} \omega_1 + \omega_{31} + \omega_{23} - \omega_2 &= 0, \\ y_C\,\omega_{31} + y_B\,\omega_{23} &= 0. \end{aligned} \tag{6.17}$$

Equation (6.17) represents a system of two equations with two unknowns, $\omega_{31}$ and $\omega_{23}$. Solving the algebraic equations, the following values are obtained:

$$\omega_{31} = N_2\,(\omega_1 - \omega_2)/N_3,$$
$$\omega_{23} = -\omega_1 + \omega_2 - N_2\,(\omega_1 - \omega_2)/N_3.$$

*Second contour*

The second closed contour contains the elements 0, 1, 3, 4, and 0 (Fig. 6.9). The contour velocity equations can be written as (counterclockwise path)

$$\begin{aligned} &\boldsymbol{\omega}_{10} + \boldsymbol{\omega}_{31} + \boldsymbol{\omega}_{43} + \boldsymbol{\omega}_{04} = \mathbf{0}, \\ &\mathbf{r}_{AC} \times \boldsymbol{\omega}_{31} + \mathbf{r}_{AD} \times \boldsymbol{\omega}_{43} = \mathbf{0}, \end{aligned} \tag{6.18}$$

where the known angular velocities are $\boldsymbol{\omega}_{10}$, $\boldsymbol{\omega}_{31}$, and the unknown angular velocities are

$$\boldsymbol{\omega}_{43} = \omega_{43}\,\mathbf{\imath} \text{ and } \boldsymbol{\omega}_{04} = \omega_{04}\,\mathbf{\imath}.$$

Equation (6.18) can be written as

$$\omega_1\,\mathbf{\imath} + \omega_{31}\,\mathbf{\imath} + \omega_{43}\,\mathbf{\imath} + \omega_{04}\,\mathbf{\imath} = \mathbf{0},$$
$$\begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_C & y_C & 0 \\ \omega_{31} & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_D & y_D & 0 \\ \omega_{43} & 0 & 0 \end{vmatrix} = \mathbf{0}. \tag{6.19}$$

From Eq. (6.19), the absolute angular velocity of the ring gear is

$$\omega_{40} = -\omega_{04} = \frac{2N_2\omega_1 + 2N_3\omega_1 - N_2\omega_2}{N_2 + 2N_3}.$$

## 6.6 Differential

Figure 6.10(a) is a schematic drawing of the ordinary bevel-gear automotive differential. The drive shaft pinion 1 and the ring gear 2 are normally hypoid gears. The ring gear 2 acts as the planet carrier for the planet gear 3, and its speed can be calculated as for a simple gear train when the speed of the drive shaft is given. Sun gears 4 and 5 are connected, respectively, to each rear wheel.

When the car is traveling in a straight line, the two sun gears rotate in the same direction with exactly the same speed. Thus for straight-line motion of the car, there is no relative motion between the planet gear 3 and ring 2. The planet gear 3, in effect, serves only as a key to transmit motion from the planet carrier to both wheels.

When the vehicle is making a turn, the wheel on the inside of the turn makes fewer revolutions than the wheel with a larger turning radius. Unless this difference in speed is accommodated in some manner, one or both of the tires would have to slide in order to make the turn. The differential permits the two wheels to rotate at different velocities while at the same time delivering power to both. During a turn, the planet gear 3 rotates about its own axis, thus permitting gears 4 and 5 to revolve at different velocities. The purpose of a differential is to differentiate between the speeds of the two wheels. In the usual passenger-car differential, the torque is divided equally whether the car is traveling in a straight line or on a curve. Sometimes the road conditions are such that the tractive effort developed by the two wheels is unequal. In this case the total tractive effort available will be only twice that at the wheel having the least traction, because the differential divides the torque equally. If one wheel should happen to be resting on snow or ice, the total effort available is very small and only a small torque will be required to cause the wheel to spin. Thus, the car sits with one wheel spinning and the other at rest with no tractive effort. And, if the car is in motion and encounters slippery surfaces, then all traction as well as control is lost.

It is possible to overcome the disadvantages of the simple bevel-gear differential by adding a coupling unit which is sensitive to wheel speeds. The object of such a unit is to cause most of the torque to be directed to the slow-moving wheel. Such a combination is then called a limited-slip differential.

**Angular velocities diagram**

The velocity analysis is carried out using the contour equation method and the graphical angular velocities diagram.

There are five moving elements (1, 2, 3, 4, and 5 ) connected by
• five full joints ($c_5 = 5$): one between the frame 0 and the drive shaft pinion gear 1, one between the frame 0 and the ring gear 2, one between the planet carrier arm 2 and the planet gear 3, one between the frame 0 and the sun gear 4, and one between the frame 0 and the sun gear 5.
• three half joints ($c_4 = 3$): one between the drive shaft pinion gear 1 and the ring gear 2, one between the planet gear 3 and the sun gear 4, and one between the planet gear 3 and the sun gear 5.

The system possesses two degrees of freedom:

$$M = 3\,n - 2\,c_5 - c_4 = 3 \cdot 5 - 2 \cdot 5 - 3 = 2.$$

The input data are the absolute angular velocities of the two wheels, $\omega_{40}$ and $\omega_{50}$.

The system shown in Fig. 6.10(a) has six elements (0, 1, 2, 3, 4, and 5) and eight joints ($c_4 + c_5$). The number of independent contours is given by

$$n_c = 8 - p + 1 = 8 - 6 + 1 = 3.$$

This gear system has three independent contours. The graph of the kinematic chain and the independent contours are represented in Fig. 6.10(b).

The first closed contour contains the elements 0, 4, 3, 5, and 0 (clockwise path). For the velocity analysis, the following vectorial equations can be written

$$\boldsymbol{\omega}_{40} + \boldsymbol{\omega}_{34} + \boldsymbol{\omega}_{53} + \boldsymbol{\omega}_{05} = \mathbf{0},$$

or

$$\boldsymbol{\omega}_{40} + \boldsymbol{\omega}_{34} = \boldsymbol{\omega}_{50} + \boldsymbol{\omega}_{35}. \tag{6.20}$$

The unknown angular velocities are $\boldsymbol{\omega}_{34}$ and $\boldsymbol{\omega}_{35}$. The relative angular velocity of the planet gear 3 with respect to the sun gear 4 is parallel to the $Ia$ line and the relative angular velocity of the planet gear 3 with respect to the sun gear 5 is parallel to $Ib$. Equation (6.20) can be solved graphically as shown in Fig. 6.10(c). The vectors $OA$ and $OB$ represent the velocities $\boldsymbol{\omega}_{50}$ and $\boldsymbol{\omega}_{40}$. At $A$ and $B$ two parallels at $Ib$ and $Ia$ are drawn. The intersection between the two lines is the point $C$. The vector $BC$ represents the relative angular velocity of the planet gear 3 with respect to the sun gear 4, and the vector $AC$ represents the relative angular velocity of the planet gear 3 with respect to the sun gear 5.

The absolute angular velocity of planet gear 3 is

$$\boldsymbol{\omega}_{30} = \boldsymbol{\omega}_{40} + \boldsymbol{\omega}_{34}.$$

The vector $OC$ represents the absolute angular velocity of planet gear.

The second closed contour contains the elements 0, 4, 3, 2, and 0 (counterclockwise path). For the velocity analysis, the following vectorial equations can be written

$$\boldsymbol{\omega}_{40} + \boldsymbol{\omega}_{34} + \boldsymbol{\omega}_{23} + \boldsymbol{\omega}_{02} = \mathbf{0}. \quad (6.21)$$

Using the velocities diagram [Fig. 6.10(c)] the vector $DC$ represents the relative angular velocity of the planet gear 3 with respect to the ring gear 2, $\boldsymbol{\omega}_{23}$, and the $OD$ represents the absolute angular velocity of the ring gear 2, $\boldsymbol{\omega}_{20}$.

Figure 6.10(c) gives

$$\omega_{20} = |OD| = \frac{1}{2}(\omega_{40} + \omega_{50}),$$
$$\omega_{32} = |DC| = \frac{1}{2}(\omega_{50} - \omega_{40}) \tan \alpha. \quad (6.22)$$

When the car is traveling in a straight line, the two sun gears rotate in the same direction with exactly the same speed, $\omega_{50} = \omega_{40}$ , and there is no relative motion between the planet gear and the ring gear, $\omega_{32} = 0$. When the wheels are jacked up $\omega_{50} = -\omega_{40}$, the absolute angular velocity of the ring gear 2 is zero.

## 6.7 Gear Force Analysis

The force between meshing teeth (neglecting the sliding friction) can be resolved at the pitch point (P in Fig. 6.11) into two components:

- tangential component $F_t$, which accounts for the power transmitted;
- radial component $F_r$, which does no work but tends to push the gears apart.

The relationship between these components is

$$F_r = F_t \tan \phi, \quad (6.23)$$

where $\phi$ is the pressure angle.

The pitch line velocity in feet per minute is equal to

$$V = \pi\, d\, n/12 \;\; (\text{ft/min}), \quad (6.24)$$

where $d$ is the pitch diameter in inches of the gear rotating $n$ rpm.
In SI units,

$$V = \pi \, d \, n/60\,000 \ \ \text{(m/s)}, \tag{6.25}$$

where $d$ is the pitch diameter in millimeters of the gear rotating $n$ rpm.
The transmitted power in horsepower is

$$H = F_t \, V/33\,000 \ \ \text{(hp)}, \tag{6.26}$$

where $F_t$ is in pounds and $V$ is in feet per minute.
In SI units the transmitted power in watts is

$$H = F_t \, V \ \ \text{(W)}, \tag{6.27}$$

where $F_t$ is in newtons and $V$ is in meters per second.
The transmitted torque can be expressed as

$$M_t = 63\,000 \, H/n \quad \text{(lb·in)}, \tag{6.28}$$

where $H$ is in horsepower and $n$ is in rpm.
In SI units,

$$M_t = 9549 \, H/n \quad \text{(N·m)} \ , \tag{6.29}$$

where the power $H$ is in kW and $n$ in rpm.
For the gear force analysis the following assumptions will be made:

- all the gears mesh along their pitch circles
- friction losses are negligible
- all the tooth loads are transferred at the pitch point
- centripetal forces will not be considered.

## 6.8 Strength of Gear Teeth

Hall et al. present an analysis of the strength of gear teeth [5]. The flank of the driver tooth makes contact with the tip of the driven tooth at the beginning of action between a pair of gear teeth. The total load $F$ is carried by one tooth, and is normal to the tooth profile (see Fig. 6.12). The bending stress at the base of the tooth is produced by the tangential load component $F_t$, which is perpendicular to the centerline of the tooth. The friction and the radial component $F_r$ are neglected. The parabola shown in Fig. 6.12 outlines a beam of uniform strength. The weakest section of the gear tooth is at section $A - A$ where the parabola is tangent to the tooth outline.

The bending stress $\sigma$ is

$$\sigma = \frac{M\,c}{I} = \frac{M\,(t/2)}{B\,t^3/12} = \frac{6M}{Bt^2} = \frac{6F_t h}{Bt^2},$$

and

$$F_t = \sigma B(t^2/6h) = \sigma B\,(t^2/6h\,p)\,p, \tag{6.30}$$

where $M = F_t\,h$ is the bending moment, $h$ is the distance between the section $A - A$ and the point where the load is applied, and $t$ is the tooth thickness. In the previous equations $B$ is the face width and is limited to a maximum of 4 times the circular pitch, i.e. $B = k\,p$, where $k \leq 4$.
The form factor

$$\gamma = \frac{t^2}{6\,h\,p}, \tag{6.31}$$

is a dimensionless quantity tabulated in Table 6.1.
Substituting $\gamma$ in the Eq. (6.30) gives

$$F_t = \sigma Bp\gamma, \tag{6.32}$$

or

$$F_t = \sigma p^2 k\gamma = \sigma\pi^2 k\gamma/P_d^2, \tag{6.33}$$

which is the Lewis equation.

In the design problem the diameters are either known or unknown.

• If the diameters are unknown the stress is

$$\sigma = \frac{2M_t P_d^3}{k\pi^2\gamma N}, \tag{6.34}$$

where $M_t$ is the torque on the weaker gear, $k = 4$ (upper limit), and $N$ is the number of teeth on the weaker gear. The minimum number of teeth, $N$, is usually limited to 15. The stress $\sigma$ should be less than or equal to allowable stress.

• If the diameters are known then the allowable value for the ratio $P_d^2/\gamma$ which controls the design is

$$P_d^2/\gamma = \sigma k\pi^2/F_t, \tag{6.35}$$

where $\sigma$ is the allowable stress, $k = 4$ (upper limit), $F_t = 2M_t/d$ is the transmitted force, and $M_t$ is the torque on the weaker gear.

If the diameters are known, design for the largest number of teeth; if the diameters are unknown, design for the smallest pitch diameters possible. The most economical design is given by the largest diametral pitch.

From Eq. (6.33) the force that can be transmitted to a gear tooth is a function of the product $\sigma_0\gamma$. For two gears in contact, the weaker gear will have the smaller $\sigma_0\gamma$ value. For gears made of the same material, the smaller gear will be the weaker and control design.

**Allowable tooth stresses**

The allowable stress for gear tooth design is

$$\begin{aligned} \text{Allowable } \sigma &= \sigma_0 \left(\frac{600}{600+V}\right) \text{ for } V < 2000 \text{ ft/min} \\ &= \sigma_0 \left(\frac{1200}{1200+V}\right) \text{ for } 2000 < V < 4000 \text{ ft/min} \\ &= \sigma_0 \left(\frac{78}{78+\sqrt{V}}\right) \text{ for } V > 4000 \text{ ft/min}, \end{aligned} \tag{6.36}$$

where $\sigma_0$ is the endurance strength in psi, and $V$ is the pitch line velocity in ft/min. The endurance strength is:

$\sigma_0 = 8000$ psi for cast iron,

$\sigma_0 = 12\,000$ psi for bronze, and

$\sigma_0 = [10\,000, ..., 50\,000]$ psi for carbon steels.

In general, $\sigma_0 \approx (1/3)$ ultimate strength of the material.

**Dynamic tooth loads**

The dynamic forces on the teeth are produced by velocity changes due to inaccuracies of the tooth profiles, and misalignments in mounting, spacing, tooth deflection, and so forth.. The dynamic load $F_d$ proposed by Buckingham is

$$F_d = \frac{0.05\,V(B\,C + F_t)}{0.05\,V + \sqrt{B\,C + F_t}} + F_t, \tag{6.37}$$

where $F_d$ is the dynamic load (lb), $F_t = M_t/r$, and $C$ is a constant that depends on the tooth material, form, and the accuracy of the tooth cutting process (tooth error, $e$). The constant $C$ is tabulated in Table 6.2. Figure 6.13(a) shows the relation of permissible errors in tooth profiles function of pitch line velocity, $V$, and Fig. 6.13(b) represents the connection between the errors $e$ and diametral pitch, $P_d$. The dynamic force $F_d$ must be less than the allowable endurance load, $F_0 = \sigma_0 \, B \, \gamma \, p$.

**Wear tooth loads**

The wear load $F_w$ is

$$F_w = d_p \, B \, K \, Q, \tag{6.38}$$

where $d_p$ is the pitch diameter of smaller gear (pinion), $K$ is the stress factor for fatigue, $Q = 2 \, N_g/(N_p + N_g)$, $N_g$ is the number of teeth on gear, and $N_p$ is the number of teeth on pinion.

The stress factor for fatigue has the following expression:

$$K = \frac{\sigma_{es}^2 (\sin \phi)(1/E_p + 1/E_g)}{1.4}, \tag{6.39}$$

where $\sigma_{es}$ is the surface endurance limit of a gear pair (psi), $E_p$ is the modulus of elasticity of the pinion material (psi), $E_g$ is the modulus of elasticity of the gear material (psi), and $\phi$ is the pressure angle. Values for the modulus of elasticity are [7]:

| material | $E$ (psi) | $E$ (GPa) |
|---|---|---|
| steel | $30 \times 10^6$ | 207 |
| cast iron | $19 \times 10^6$ | 131 |
| aluminum bronze | $17.5 \times 10^6$ | 121 |
| tin bronze | $16 \times 10^6$ | 110 |

An estimated value for surface endurance is

$$\sigma_{es} = (400)(\text{BHN}) - 10\,000 \, \text{psi}, \tag{6.40}$$

where BHN may be approximated by the average Brinell Hardness Number of the gear and pinion. The wear load $F_w$ is an allowable load and must be greater than the dynamic load $F_d$. Table 6.3 presents several tentative values of $K$ for various materials and tooth forms.

## 6.9 Examples

**Example 6.1.** Two involute spur gears of module 5, with 19 and 28 teeth operate at a pressure angle of 20°. Determine whether there will be interference when standard full-depth teeth are used. Find the contact ratio.

Solution.

A standard full-depth tooth has the addendum of $a = m = 5$ mm.
The gears will mesh at their pitch circles, and the pitch circle radii of pinion and gear are
$r_p = m\,N_p/2 = 5\,(19)/2 = 47.5\,\text{mm}$, and
$r_g = m\,N_g/2 = 5\,(28)/2 = 70\,\text{mm}$.
The theoretical center distance is
$c = (d_p + d_g)/2 = r_p + r_g = 47.5 + 70 = 117.5$ mm.
The base circle radii of pinion and gear are
$r_{bp} = r_p \cos\phi = 47.5\,\cos 20° = 44.635$ mm, and
$r_{bg} = r_g \cos\phi = 70\,\cos 20° = 65.778$ mm.
The addendum circle radii of pinion and gear are
$r_{ap} = r_p + a = m(N_p + 2)/2 = 52.5$ mm, and
$r_{ag} = r_g + a = m(N_g + 2)/2 = 75$ mm.
The maximum possible addendum circle radii of pinion and gear, without interference, are
$r_{a(max)p} = \sqrt{r_{bp}^2 + c^2 \sin^2\phi} = 60.061 \text{ mm} > r_{ap} = 52.5$ mm, and
$r_{a(max)g} = \sqrt{r_{bg}^2 + c^2 \sin^2\phi} = 77.083 \text{ mm} > r_{ag} = 75$ mm.
Clearly, the use of standard teeth would not cause interference.
The contact ratio is

$$CR = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - c\sin\phi}{\pi m \cos\phi} = 1.590,$$

which should be a suitable value ($CR > 1.2$).

**Example 6.2.** A planetary gear train is shown in Fig. 6.14(a). The system consists of an input sun gear 1 and a planet gear 2 in mesh with 1 at $B$. Gear $2'$ is fixed on the shaft of gear 2. The system of gears 2 and $2'$ is carried by the arm 3. The gear $2'$ meshes with the fixed frame 0 at $E$.

There are three moving gears (1, 2, and 3) connected by:
• three full joints ($c_5 = 3$): one at $A$, between the frame 0 and the sun gear 1; one at $C$, between the arm 3 and the planet gear system 2; and another

at $D$, between the frame 0 and the arm 3.
• two half joints ($c_4 = 2$): one at $B$, between the sun gear 1 and the planet gear 2, and another at $E$, between the planet gear $2'$ and the frame 0. The system has one degree of freedom. The sun gear has a radius of the pitch circle equal to $r_1$, the planet gear 2 has a radius of the pitch circle equal to $r_2$, the arm 3 has a length equal to $r_3$, and the planet gear $2'$ has a radius of the pitch circle equal to $r_4$.

The sun gear rotates with the input angular velocity $\omega_1$. Find the speed ratio $i_{13}$ between the sun gear 1 and the arm 3.

Solution.

The system shown in Fig. 6.14(a) has four elements (0, 1, 2, 3) and five joints. The number of independent loops is given by

$$n_c = l - p + 1 = 5 - 4 + 1 = 2.$$

This gear system has two independent contours. The diagram representing the kinematic chain and the independent contours is shown in Fig. 6.14(b).

The position vectors $\mathbf{r}_{AB}$, $\mathbf{r}_{AC}$, $\mathbf{r}_{AD}$, and $\mathbf{r}_{AE}$ are defined as follows:

$$\begin{aligned}
\mathbf{r}_{AB} &= x_B\,\mathbf{\imath} + y_B\,\mathbf{\jmath} = x_B\,\mathbf{\imath} + r_1\,\mathbf{\jmath},\\
\mathbf{r}_{AC} &= x_C\,\mathbf{\imath} + y_C\,\mathbf{\jmath} = x_C\,\mathbf{\imath} + (r_1 + r_2)\,\mathbf{\jmath},\\
\mathbf{r}_{AD} &= x_D\,\mathbf{\imath} + y_D\,\mathbf{\jmath} = x_D\,\mathbf{\imath} + (r_1 + r_2 - r_3)\,\mathbf{\jmath},\\
\mathbf{r}_{AE} &= x_E\,\mathbf{\imath} + y_E\,\mathbf{\jmath} = x_E\,\mathbf{\imath} + (r_1 + r_2 - r_4)\,\mathbf{\jmath}.
\end{aligned}$$

*First contour*

The first closed contour contains the elements 0, 1, 2, and 0 (following the clockwise path). For the velocity analysis, the following vectorial equations can be written:

$$\begin{aligned}
\boldsymbol{\omega}_{10} + \boldsymbol{\omega}_{21} + \boldsymbol{\omega}_{02} &= \mathbf{0},\\
\mathbf{r}_{AB} \times \boldsymbol{\omega}_{21} + \mathbf{r}_{AE} \times \boldsymbol{\omega}_{02} &= \mathbf{0},
\end{aligned} \tag{6.41}$$

where the input angular velocity is

$$\boldsymbol{\omega}_{10} = \omega_{10}\,\mathbf{\imath},$$

and the unknown angular velocities are

$$\begin{aligned}
\boldsymbol{\omega}_{21} &= \omega_{21}\,\mathbf{\imath},\\
\boldsymbol{\omega}_{02} &= \omega_{02}\,\mathbf{\imath}.
\end{aligned}$$

Equation (6.41) becomes

$$\omega_1\,\imath + \omega_{21}\,\imath + \omega_{02}\,\imath = \mathbf{0},$$
$$\begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_B & y_B & 0 \\ \omega_{21} & 0 & 0 \end{vmatrix} + \begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_E & y_E & 0 \\ \omega_{02} & 0 & 0 \end{vmatrix} = \mathbf{0}. \tag{6.42}$$

Equation (6.42) projected onto a "fixed" reference frame $xOyz$ is

$$\begin{aligned} \omega_1 + \omega_{21} + \omega_{02} &= 0, \\ y_B\,\omega_{21} + y_E\,\omega_{02} &= 0. \end{aligned} \tag{6.43}$$

Equation (6.43) represents a system of two equations with two unknowns $\omega_{21}$ and $\omega_{02}$. Solving the algebraic equations, the following value is obtained for the absolute angular velocity of planet gear 2:

$$\omega_{20} = -\,\omega_{02} = -\,\frac{r_1\,\omega_1}{r_2 - r_4}. \tag{6.44}$$

*Second contour*

The second closed contour contains the elements 0, 3, 2, and 0 (counterclockwise path). For the velocity analysis, the following vectorial equations can be written:

$$\begin{aligned} \boldsymbol{\omega}_{30} + \boldsymbol{\omega}_{23} + \boldsymbol{\omega}_{02} &= \mathbf{0}, \\ \mathbf{r}_{AD} \times \boldsymbol{\omega}_{30} + \mathbf{r}_{AC} \times \boldsymbol{\omega}_{23} + \mathbf{r}_{AE} \times \boldsymbol{\omega}_{02} &= \mathbf{0}, \end{aligned} \tag{6.45}$$

The unknown angular velocities are

$$\begin{aligned} \boldsymbol{\omega}_{30} &= \omega_{21}\,\imath, \\ \boldsymbol{\omega}_{23} &= \omega_{23}\,\imath. \end{aligned}$$

Solving Eq. (6.45), the following value is obtained for the absolute angular velocity of the arm 3:

$$\omega_{30} = \frac{r_1\,r_4\,\omega_1}{r_3\,(-r_2 + r_4)}. \tag{6.46}$$

The speed ratio is

$$i_{13} = \frac{\omega_{10}}{\omega_{30}} = \frac{\omega_1}{\omega_{30}} = \frac{r_3\,(-r_2 + r_4)}{r_1\,r_4}. \tag{6.47}$$

**Example 6.3.** Figure 6.15 shows a planetary gear train. The schematic representation of the planetary gear train is depicted in Fig. 6.16(a). The system consists of an input sun gear 1 and a planet gear 2 in mesh with 1 at $B$. Gear 2 is carried by the arm $S$ fixed on the shaft of gear 3, as shown. Gear 3 meshes with the output gear 4 at $F$. The fixed ring gear 4 meshes with the planet gear 2 at $D$.

There are four moving gears (1, 2, 3, and 4) connected by:

• four full joints ($c_5 = 4$): one at $A$, between the frame 0 and the sun gear 1; one at $C$, between the arm $S$ and the planet gear 2; one at $E$, between the frame 0 and the gear 3, and another at $G$, between the frame 0 and the gear 3.

• three half joints ($c_4 = 3$): one at $B$, between the sun gear 1 and the planet gear 2; one at $D$, between the planet gear 2 and the ring gear; and another at $F$, between the gear 3 and the output gear 4. The module of the gears is $m = 5$ mm. The system has one degree of freedom. The sun gear has $N_1 = 19$ external gear teeth, the planet gear has $N_2 = 28$ external gear teeth, and the fixed ring gear has $N_5 = 75$ internal gear teeth. Gear 3 has $N_3 = 18$ external gear teeth, and the output gear has $N_4 = 36$ external gear teeth. The sun gear rotates with an input angular speed $n_1 = 2970$ rpm ($\omega_1 = \omega_{10} = \pi n_1/30 = 311.018$ rad/s). Find the absolute output angular velocity of gear 4, the velocities of the pitch points $B$ and $F$, and the velocity of joint $C$.

Solution.

The velocity analysis is carried out using the contour equation method. The system shown in Fig. 6.16(a) has five elements (0, 1, 2, 3, 4) and seven joints. The number of independent loops is given by

$$n_c = l - p + 1 = 7 - 5 + 1 = 3.$$

This gear system has three independent contours. The diagram representing the kinematic chain and the independent contours is shown in Fig. 6.16(b).

The position vectors $\mathbf{r}_{AB}$, $\mathbf{r}_{AC}$, $\mathbf{r}_{AD}$, $\mathbf{r}_{AF}$, and $\mathbf{r}_{AG}$ are defined as follows:

$$\mathbf{r}_{AB} = x_B\,\mathbf{\imath} + y_B\,\mathbf{\jmath} = x_B\,\mathbf{\imath} + r_1\,\mathbf{\jmath} = x_B\,\mathbf{\imath} + \frac{mN_1}{2}\,\mathbf{\jmath},$$
$$\mathbf{r}_{AC} = x_C\,\mathbf{\imath} + y_C\,\mathbf{\jmath} = x_C\,\mathbf{\imath} + (r_1 + r_2)\,\mathbf{\jmath} = x_C\,\mathbf{\imath} + \frac{m(N_1 + N_2)}{2}\,\mathbf{\jmath}.$$

$$\mathbf{r}_{AD} = x_D\,\mathbf{\imath} + y_D\,\mathbf{\jmath} = x_D\,\mathbf{\imath}(r_1 + 2r_2)\,\mathbf{\jmath} = x_D\,\mathbf{\imath} + \frac{m(N_1 + 2N_2)}{2}\,\mathbf{\jmath},$$
$$\mathbf{r}_{AF} = x_F\,\mathbf{\imath} + y_F\,\mathbf{\jmath} = x_F\,\mathbf{\imath} + r_3\,\mathbf{\jmath} = x_F\,\mathbf{\imath} + \frac{mN_3}{2}\,\mathbf{\jmath},$$
$$\mathbf{r}_{AG} = x_G\,\mathbf{\imath} + y_G\,\mathbf{\jmath} = x_G\,\mathbf{\imath} + (r_3 + r_4)\,\mathbf{\jmath} = x_G\,\mathbf{\imath} + \frac{m(N_3 + N_4)}{2}\,\mathbf{\jmath}.$$

*First contour*

The first closed contour contains the elements 0, 1, 2, and 0 (following the clockwise path). For the velocity analysis, the following vectorial equations can be written:

$$\sum_{(i)} \boldsymbol{\omega}_{i,i-1} = \mathbf{0} \implies \boldsymbol{\omega}_{10} + \boldsymbol{\omega}_{21} + \boldsymbol{\omega}_{02} = \mathbf{0},$$
$$\sum_{(i)} \mathbf{r}_{A_i} \times \boldsymbol{\omega}_{i,i-1} = \mathbf{0} \implies \mathbf{r}_{AB} \times \boldsymbol{\omega}_{21} + \mathbf{r}_{AD} \times \boldsymbol{\omega}_{02} = \mathbf{0}, \qquad (6.48)$$

where the input angular velocity is

$$\boldsymbol{\omega}_{10} = \omega_{10}\,\mathbf{\imath},$$

and the unknown angular velocities are

$$\boldsymbol{\omega}_{21} = \omega_{21}\,\mathbf{\imath},\ \ \boldsymbol{\omega}_{02} = \omega_{02}\,\mathbf{\imath}.$$

The sign of the relative angular velocities is selected to be positive, and then the numerical results give the real orientation of the vectors.

Equation (6.48) becomes

$$\omega_1\,\mathbf{\imath} + \omega_{21}\,\mathbf{\imath} + \omega_{02}\,\mathbf{\imath} = \mathbf{0},$$
$$\begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_B & y_B & 0 \\ \omega_{21} & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_D & y_D & 0 \\ \omega_{02} & 0 & 0 \end{vmatrix} = \mathbf{0}. \qquad (6.49)$$

Equation (6.49) projected on a “fixed” reference frame $xOyz$ is

$$\begin{aligned} \omega_1 + \omega_{21} + \omega_{02} &= 0, \\ y_B\,\omega_{21} + y_D\,\omega_{02} &= 0. \end{aligned} \qquad (6.50)$$

Equation (6.50) represents a system of two equations with two unknowns, $\omega_{21}$ and $\omega_{02}$. Solving the algebraic equations, the following value is obtained for the absolute angular velocity of the planet gear 2:

$$\omega_{20} = -\,\omega_{02} = -\,\frac{N_1\,\omega_1}{2\,N_2} = -\,\frac{19\,(311.018)}{2\,(28)} = -105.524\,\text{rad/s}. \qquad (6.51)$$

*Second contour*

The second closed contour contains the elements 0, 3, 2, and 0 (following the counterclockwise path). For the velocity analysis, the following vectorial equations can be written:

$$\boldsymbol{\omega}_{30} + \boldsymbol{\omega}_{23} + \boldsymbol{\omega}_{02} = \mathbf{0},$$
$$\mathbf{r}_{AE} \times \boldsymbol{\omega}_{30} + \mathbf{r}_{AC} \times \boldsymbol{\omega}_{23} + \mathbf{r}_{AD} \times \boldsymbol{\omega}_{02} = \mathbf{0}. \tag{6.52}$$

The unknown angular velocities are

$$\boldsymbol{\omega}_{30} = \omega_{21}\,\mathbf{1},\;\; \boldsymbol{\omega}_{23} = \omega_{23}\,\mathbf{1}.$$

Solving Eq. (6.52) the following value is obtained for the absolute angular velocity of gear 3 and arm $S$

$$\omega_{30} = \frac{N_1\,\omega_1}{2\,(N_1 + N_2)} = \frac{19\,(311.018)}{2\,(19+28)} = 62.865\,\text{rad/s}.$$

The absolute angular velocity of the output gear 4 is

$$\begin{aligned}\omega_{40} &= -\frac{\omega_{30}\,N_3}{N_4} = -\frac{N_1\,N_3\,\omega_1}{2\,(N_1+N_2)\,N_4} = -\frac{19\,(18)\,(311.018)}{2\,(19+28)\,(36)}\\ &= -31.432\,\text{rad/s}.\end{aligned}$$

*Linear velocities of pitch points*

The velocity of the pitch point $B$ is

$$v_B = \omega_{10}\,r_1 = 311.018\,(0.005)\,(19)/2 = 14.773\,\text{m/s},$$

and the velocity of the pitch point $F$ is

$$v_F = \omega_{40}\,r_4 = 31.432\,(0.005)\,(36)/2 = 2.828\,\text{m/s}.$$

The velocity of the joint $C$ is

$$v_C = \omega_{30}\,(r_1 + r_2) = 62.865\,(0.005)\,(19+28)/2 = 7.386\,\text{m/s}.$$

*Gear geometrical dimensions*

For standard external gear teeth the addendum is $a = m$.

Gear 1

pitch circle diameter $d_1 = mN_1 = 95.0$ mm;

addendum circle diameter $d_{a1} = m(N_1 + 2) = 105.0$ mm;
dedendum circle diameter $d_{d1} = m(N_1 - 2.5) = 82.5$ mm.

Gear 2

pitch circle diameter $d_2 = mN_2 = 140.0$ mm;
addendum circle diameter $d_{a2} = m(N_2 + 2) = 150.0$ mm;
dedendum circle diameter $d_{d2} = m(N_2 - 2.5) = 127.5$ mm.

Gear 3

pitch circle diameter $d_3 = mN_3 = 90.0$ mm;
addendum circle diameter $d_{a3} = m(N_3 + 2) = 100.0$ mm;
dedendum circle diameter $d_{d3} = m(N_3 - 2.5) = 77.5$ mm.

Gear 4

pitch circle diameter $d_4 = mN_4 = 180.0$ mm;
addendum circle diameter $d_{a4} = m(N_4 + 2) = 190.0$ mm;
dedendum circle diameter $d_{d4} = m(N_4 - 2.5) = 167.5$ mm.

Gear 5 (internal gear)

pitch circle diameter $d_5 = mN_5 = 375.0$ mm;
addendum circle diameter $d_{a5} = m(N_5 - 2) = 365.0$ mm;
dedendum circle diameter $d_{d5} = m(N_5 + 2.5) = 387.5$ mm.

*Number of planet gears*

The number of necessary planet gears $k$ is given by the assembly condition

$$(N_1 + N_5)/k = INTEGER,$$

and for the planetary gear train $k = 2$ planet gears. The vicinity condition between the sun gear and the planet gear,

$$m\,(N_1 + N_2)\,\sin(\pi/k) > d_{a2}$$

is thus verified.

**Example 6.4.** Figure 6.17 shows a two stage gear reducer with identical pairs of gears. An electric motor with the power $H = 2$ kW and $n_1 = 900$ rpm is coupled to the shaft $a$. On this shaft there is rigidly connected the input driver gear 1, with the number of teeth, $N_1 = N_p = 17$. The speed reducer uses a countershaft $b$ with two rigidly connected gears, 2 and $2'$, having $N_2 = N_g = 51$ teeth and $N_{2'} = N_p = 17$ teeth. The output gear 3 has $N_3 = N_g = 51$ teeth and is rigidly fixed to the shaft $c$ coupled to the driven machine. The input shaft $a$ and output shaft $c$ are collinear. The countershaft $b$ turns freely in bearings $A$ and $B$. The gears mesh along the pitch diameter and the shafts are parallel. The diametral pitch for each stage is $P_d = 5$, and the pressure angle is $\phi = 20^0$. The distance between the bearings is $s = 100$ mm, and the distance $l = 25$ mm (Fig. 6.17). The gear reducer is a part of an industrial machine intended for continuous one-shift (8 hours per day). Select identical extra-light series (L00) ball bearings for $A$ and $B$.

Solution

*Geometry*

The pitch diameters of pinions 1 and $2'$ are $d_1 = d_{2'} = d_p = N_p/P_d = 17/5 = 3.4$ in. The pitch diameters of gears 2 and 3 are $d_2 = d_3 = d_g = N_g/P_d = 51/5 = 10.2$ in. The circular pitch is $p = \pi/P_d = 3.14/5 = 0.63$ in.

*Angular speeds*

The following relation exists for the first stage:

$$\frac{n_1}{n_2} = \frac{N_2}{N_1} \quad \Rightarrow \quad n_2 = n_1 \frac{N_1}{N_2} = 900 \frac{17}{51} = 300 \ \text{rpm},$$

and for the second stage:

$$\frac{n_2}{n_3} = \frac{N_3}{N_{2'}} \quad \Rightarrow \quad n_3 = n_2 \frac{N_{2'}}{N_3} = 300 \frac{17}{51} = 100 \ \text{rpm}.$$

The angular speed of the coutershaft $b$ is $n_b = n_2 = 300$ rpm, and the angular speed of the driven shaft $c$ is $n_c = n_3 = 100$ rpm.

*Torque carried by each of the shafts*

The relation between the power $H_a$ of the motor and the torque $M_a$ in shaft $a$ is

$$H_a = \frac{M_a \, n_a}{9549},$$

and the torque $M_a$ in shaft $a$ is

$$M_a = \frac{9549 \, H_a}{n_a} = \frac{9549 \, (2 \text{ kW})}{900 \text{ rpm}} = 21.22 \text{ N} \cdot \text{m}.$$

The torque in shaft $b$ is

$$M_b = \frac{9549 \, H_a}{n_b} = M_a \frac{N_2}{N_1} = 21.22 \, \frac{51}{17} = 63.66 \text{ N} \cdot \text{m},$$

and the torque in shaft $c$ is

$$M_c = \frac{9549 \, H_a}{n_c} = M_b \frac{N_3}{N_{2'}} = 63.66 \, \frac{51}{17} = 190.98 \text{ N} \cdot \text{m}.$$

*Bearing reactions*

All the gear radial and tangential force is transferred at the pitch point $P$. The tangential force on the motor pinion is

$$F_t = \frac{M_a}{r_p} = \frac{21.22}{0.0431} = 492.34 \text{ N},$$

where $r_p = d_p/2 = 1.7$ in $= 0.0431$ m. The radial force on the motor pinion is

$$F_r = F_t \tan\phi = 492.34 \tan 20^0 = 179.2 \text{ N}.$$

The force on the motor pinion 1 at $P$ (Fig. 6.17) is

$$\mathbf{F}_{21} = F_{r21}\mathbf{\jmath} + F_{t21}\mathbf{k} = 179.2\mathbf{\jmath} - 492.34\mathbf{k} \text{ N}.$$

The force on the countershaft gear 2 at $P$ is

$$\mathbf{F}_{12} = -\mathbf{F}_{21} = F_{r12}\mathbf{\jmath} + F_{t12}\mathbf{k} = -179.2\mathbf{\jmath} + 492.34\mathbf{k} \text{ N}.$$

The forces on the countershaft pinion $2'$ at $R$ are three times as large, i.e.,

$$F_{t'} = \frac{M_b}{r_p} = \frac{63.66}{0.0431} = 1477 \text{ N},$$

$$F_{r'} = F_{t'} \tan\phi = 1477 \tan 20^0 = 537.6 \text{ N},$$

and

$$\mathbf{F}_{32'} = F_{r32'}\mathbf{\jmath} + F_{t32'}\mathbf{k} = -537.6\mathbf{\jmath} - 1477\mathbf{k} \text{ N}.$$

The unknown forces applied to bearings $A$ and $B$ can be written as

$$\mathbf{F}_A = F_{Ay}\mathbf{\jmath} + F_{Az}\mathbf{k},$$

$$\mathbf{F}_B = F_{By}\mathbf{\jmath} + F_{Bz}\mathbf{k}.$$

The sketch of the countershaft as a free body in equilibrium is shown in Fig. 6.18. To determine these forces two vectorial equations are used. Sum of moments of all forces that act on the countershaft with respect to $A$ are zero:

$$\sum \mathbf{M}_A = \mathbf{r}_{AP} \times \mathbf{F}_{12} + \mathbf{r}_{AR} \times \mathbf{F}_{32'} + \mathbf{r}_{AB} \times \mathbf{F}_B =$$

$$\begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ -l & r_2 & 0 \\ 0 & F_{r12} & F_{t12} \end{vmatrix} + \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ s+l & r_{2'} & 0 \\ 0 & F_{r32'} & F_{t32'} \end{vmatrix} +$$

$$\begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ s & 0 & 0 \\ 0 & F_{By} & F_{Bz} \end{vmatrix} = \mathbf{0},$$

or

$$\sum \mathbf{M}_A \cdot \mathbf{j} = lF_{t12} - (s+l)F_{t32'} - sF_{Bz} = 0,$$
$$\sum \mathbf{M}_A \cdot \mathbf{k} = -lF_{r12} + (s+l)F_{r32'} + sF_{By} = 0.$$

From the above equations $F_{By} = 627.2$ N, and $F_{Bz} = 1969.33$ N. The radial force at $B$ is

$$F_B = \sqrt{{F_{By}}^2 + {F_{Bz}}^2} = 2066.8 \text{ N}.$$

Sum of all forces that act on the countershaft are zero

$$\sum \mathbf{F} = \mathbf{F}_{12} + \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_{32'} = \mathbf{0},$$

or

$$-F_{r12} + F_{Ay} + F_{By} - F_{r32'} = 0,$$
$$F_{t12} + F_{Az} + F_{Bz} - F_{t32'} = 0. \tag{6.53}$$

From Eq. (6.53) $F_{Ay} = 89.6$ N, and $F_{Az} = -984.67$ N. The radial force at $A$ is

$$F_A = \sqrt{{F_{Ay}}^2 + {F_{Az}}^2} = 988.73 \text{ N}.$$

*Ball bearing selection*

Since the radial force at $B$ is greater than the radial force at $A$, $F_B > F_A$, the bearing selection will be based on bearing $B$.
The equivalent radial force for radial ball bearings is $F_e = F_B$ =2066.8 N.
The ball bearings operate 8 hours per day, 5 days per week.
From Table 3.3 choose $K_a = 1.1$ for gearing.
From Table 3.4 choose (conservatively) 30 000 hour life.
The life in revolutions is

$L = 300$ rpm $\times$ 30 000 h $\times$ 60 min/h = 540 $\times 10^6$ rev.

For standard 90 percent reliability ($K_r$=1, Fig. 3.9), and for $L_R = 90 \times 10^6$ rev (for use with Table. 3.2), the rated capacity is

$$\begin{aligned} C_{req} &= K_a F_e \left(\frac{L}{K_r L_R}\right)^{0.3} \\ &= (1.1)\,(2066.8) \left[\frac{540 \times 10^6}{(1)\,90 \times 10^6}\right]^{0.3} = 3891.67\,\text{N} \\ &\approx 3.9\,\text{kN}. \end{aligned}$$

From Table 3.2 with 3.9 kN for L00 series $\Longrightarrow C$=4.2 kN and $d$=35 mm bore. From Table 3.1 with 35 mm bore and L00 series the bearing number is L07. The shaft size requirement may necessitate use of a larger bore bearing.

**Example 6.5.** Figure 6.19(a) shows a gearset. Gear 1 is the driving or input gear, it rotates with the angular speed $\omega_{10}$, $(\omega_{10} > 0)$, and transmits an unknown motor torque $M_{mot}$. The output (driven) gear 2 is attached to a shaft that drives a machine. The external torque exerted by the machine on gear 2 is opposite to the absolute angular velocity of the output gear, $\boldsymbol{\omega}_{20}$, and is given by

$$\mathbf{M}_{ext} = -|\mathbf{M}_{ext}|\frac{\boldsymbol{\omega}_{20}}{|\boldsymbol{\omega}_{20}|}.$$

The radii of the pitch circles of the two gears in contact are $r_1$ and $r_2$, and the pressure angle is $\phi$. Find the motor torque (equilibrium moment) $M_{mot}$ and the bearing reactions in terms of $r_1$, $r_2$, $\omega_{10}$, and $|\mathbf{M}_{ext}|$.

Use the following numerical application: $r_1 = 1$ m, $r_2 = 0.5$ m, $\phi = 20°$, $\omega_{10} = \frac{\pi}{3}$ rad/s, and $|\mathbf{M}_{ext}| = 400$ N·m.

Solution.

The angular speed ratio between the gears is

$$\frac{\omega_{10}}{\omega_{20}} = -\frac{r_1}{r_2}.$$

Thus, the angular speed $\omega_{20}$ of the output gear is

$$\omega_{20} = -\frac{r_2\,\omega_{10}}{r_1} = -\frac{1(\pi/3)}{0.5} = -\frac{2\,\pi}{3}\ \text{rad/s}.$$

The angular velocity vector of the output gear is

$$\boldsymbol{\omega}_{20} = \omega_{20}\,\mathbf{ı} = -\frac{2\,\pi}{3}\ \mathbf{ı}\ \ \text{rad/s}.$$

The free-body diagrams of the gears are shown in Fig. 6.19(b). The external torque exerted by the machine on the gear 2 is

$$\mathbf{M}_{ext} = M_{ext}\,\mathbf{ı} = -|\mathbf{M}_{ext}|\frac{\boldsymbol{\omega}_{20}}{|\boldsymbol{\omega}_{20}|} = -400\frac{-(2\,\pi/3)\ \mathbf{ı}}{2\,\pi/3} = 400\ \mathbf{ı}\ \ \text{N}\cdot\text{m}.$$

Gear 2

The moment equation for gear 2 with respect to its center $C$ gives

$$\sum \mathbf{M}_C^{(2)} = \mathbf{r}_{CB} \times \mathbf{F}_{12} + \mathbf{M}_{ext} = \mathbf{0}, \tag{6.54}$$

where $\mathbf{F}_{12} = \mathbf{F}_{r12}\mathbf{j} + \mathbf{F}_{t12}\mathbf{k}$ is the reaction force of gear 1 on gear 2, and $\mathbf{r}_{CB} = -r_2\mathbf{j}$. Equation (6.54) becomes

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -r_2 & 0 \\ 0 & F_{r12} & F_{t12} \end{vmatrix} + M_{ext}\mathbf{i} = \mathbf{0}. \tag{6.55}$$

From Eq. (6.55) the tangential force is

$$F_{t12} = \frac{M_{ext}}{r_2} = \frac{400}{0.5} = 800 \text{ N}.$$

The radial reaction force $F_{r12}$ is

$$F_{r12} = F_{t12} \tan\phi = \frac{M_{ext}}{r_2} \tan\phi = 800 \tan 20^\circ = 291.176 \text{ N}.$$

The force equation for gear 2 gives

$$\sum \mathbf{F}^{(2)} = \mathbf{F}_{12} + \mathbf{F}_{02} = \mathbf{0},$$

and the reaction force of the ground on gear 2 is

$$\mathbf{F}_{02} = -\mathbf{F}_{12} = -291.176\mathbf{j} - 800\mathbf{k} \text{ N}.$$

Gear 1 (driver)

The moment equation for gear 1 with respect its center $A$ gives

$$\sum \mathbf{M}_A^{(1)} = \mathbf{r}_{AB} \times \mathbf{F}_{21} + \mathbf{M}_{mot} = \mathbf{0}, \tag{6.56}$$

where $\mathbf{F}_{21} = -\mathbf{F}_{12}$, and $\mathbf{r}_{AB} = r_1\mathbf{j}$. Equation (6.56) becomes

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & r_1 & 0 \\ 0 & -F_{r12} & -F_{t12} \end{vmatrix} + M_{mot}\mathbf{i} = \mathbf{0},$$

and the motor torque $M_{mot}$ is

$$M_{mot} = F_{t12}\, r_1 = \frac{r_1}{r_2} M_{ext} = \frac{1}{0.5} 400 = 800 \text{ N} \cdot \text{m}.$$

The force equation for gear 1 is

$$\sum \mathbf{F}^{(1)} = \mathbf{F}_{21} + \mathbf{F}_{01} = \mathbf{0},$$

and the reaction force of the ground on the gear 1 is

$$\mathbf{F}_{01} = -\mathbf{F}_{21} = \mathbf{F}_{12} = 291.176\mathbf{\jmath} + 800\mathbf{k} \text{ N}.$$

**Example 6.6.** A planetary gear train is shown in Fig. 6.20(a). The planet gear 2 rotates around the sun gear 1. The arm 3 is connected to the planet gear at the point $C$ (pin joint) and to the ground 0 at the point $D$ (pin joint). The sun gear is connected to the ground at the point $A$ (pin joint). A motor drives the sun gear with the angular speed $\omega_{10} = 2\,\pi/3$ rad/s ($n_1 = 20$ rpm). A second motor is connected to the arm and has the angular speed $\omega_{30} = -\pi/3$ rad/s ($n_3 = -10$ rpm). The radii of the pitch circles of the sun gear 1 and planet gear 2 are $r_1 = 1$ m and $r_2 = 0.5$ m, respectively.

An external moment $\mathbf{M}_{ext} = -|\mathbf{M}_{ext}|\dfrac{\boldsymbol{\omega}_{20}}{|\boldsymbol{\omega}_{20}|}$ acts on the planet gear 2, where $|\mathbf{M}_{ext}| = 400$ N·m. The pressure angle of the gears is $\phi = 20°$. Find the equilibrium moments (motor moments) $M_{1mot}$ and $M_{3mot}$ that act on gear 1 and arm 3, and the reaction forces.

Solution.

First, the angular velocity $\boldsymbol{\omega}_{20}$ of the planet gear will be calculated.
The gear train has one contour: $0 - A - 1 - B - 2 - C - 3 - D - 0$.
For this contour the relations between the relative angular velocities of the links are

$$\begin{aligned} \boldsymbol{\omega}_{10} + \boldsymbol{\omega}_{21} + \boldsymbol{\omega}_{32} + \boldsymbol{\omega}_{03} &= \mathbf{0}, \\ \mathbf{r}_{AB} \times \boldsymbol{\omega}_{21} + \ \mathbf{r}_{AC} \times \boldsymbol{\omega}_{32} &= \mathbf{0}, \end{aligned} \tag{6.57}$$

where $\boldsymbol{\omega}_{10} = \omega_{10}\,\mathbf{\imath}$, $\boldsymbol{\omega}_{03} = -\boldsymbol{\omega}_{30} = \omega_{30}\,\mathbf{\imath}$, $\mathbf{r}_{AB}$, and $\mathbf{r}_{AC}$ are known.
Equation (6.57) can be solved simultaneously with respect to the two unknowns, $\boldsymbol{\omega}_{21}$ and $\boldsymbol{\omega}_{32}$. The solutions are

$$\boldsymbol{\omega}_{21} = -3\,\pi\,\mathbf{\imath} \text{ rad/s} \quad \text{and} \quad \boldsymbol{\omega}_{32} = 2\,\pi\,\mathbf{\imath} \text{ rad/s}.$$

The angular speed $\boldsymbol{\omega}_{20}$ of the planet gear 2 is

$$\boldsymbol{\omega}_{20} = \boldsymbol{\omega}_{10} + \boldsymbol{\omega}_{21} = -\frac{7\,\pi}{3}\mathbf{\imath} \text{ rad/s} \ \ (n_2 = -70\,\text{rpm}).$$

Figure 6.20(b) shows the free-body diagrams of gear 2, gear 1, and arm 3. The external torque $\mathbf{M}_{ext}$ on the driven gear 2 is

$$\mathbf{M}_{ext} = M_{ext}\,\boldsymbol{\imath} = -|\mathbf{M}_{ext}|\,\frac{\boldsymbol{\omega}_{20}}{|\boldsymbol{\omega}_{20}|} = -400\,\frac{-7\,\pi/3\,\boldsymbol{\imath}}{7\,\pi/3} = 400\,\boldsymbol{\imath}\;\;\text{N}\cdot\text{m},$$

and the force analysis starts with the driven planet gear.

Gear 2

The sum of the moments with respect to the center $C$ for the planet gear 2 gives

$$\sum \mathbf{M}_C^{(2)} = \mathbf{r}_{CB} \times \mathbf{F}_{12} + \mathbf{M}_{ext} = \mathbf{0}, \quad \text{or} \quad \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & -r_2 & 0 \\ 0 & F_{r12} & F_{t12} \end{vmatrix} + M_{ext}\boldsymbol{\imath} = \mathbf{0}. \quad (6.58)$$

where $\mathbf{F}_{12} = F_{r12}\,\boldsymbol{\jmath} + F_{t12}\,\mathbf{k}$ and $\mathbf{r}_{CB} = -r_2\,\boldsymbol{\jmath}$. From Eq. (6.58) it results

$$F_{t12} = \frac{M_{ext}}{r_2} = \frac{400}{0.5} = 800\ \text{N}.$$

The radial reaction force $F_{r12}$ is

$$F_{r12} = F_{t12}\tan\phi = \frac{M_{ext}}{r_2}\tan\phi = 800\ \tan 20^\circ = \ 291.176\ \text{N}.$$

The sum of the forces for the planet gear 2 is

$$\sum \mathbf{F}^{(2)} = \mathbf{F}_{12} + \mathbf{F}_{32} = \mathbf{0},$$

and the reaction force of arm 3 on gear 2 is

$$\mathbf{F}_{32} = -\mathbf{F}_{12} = F_{32y}\,\boldsymbol{\jmath} + F_{32z}\,\mathbf{k} = -291.176\,\boldsymbol{\jmath} - 800\,\mathbf{k}\;\;\text{N}.$$

Gear 1

The sum of the moments for gear 1 with respect to its center $A$ is

$$\sum \mathbf{M}_A^{(1)} = \mathbf{r}_{AB} \times \mathbf{F}_{21} + \mathbf{M}_{1mot} = \mathbf{0},$$

where $\mathbf{F}_{21} = -\mathbf{F}_{12}$ and $\mathbf{r}_{AB} = r_1\,\boldsymbol{\jmath}$. The motor torque (equilibrium moment) $M_{1mot}$ is

$$M_{1mot} = -F_{t21}\,r_1 = F_{t12}\,r_1 = \frac{M_{ext}}{r_2}\,r_1 = 800\ \text{N}\cdot\text{m}.$$

The sum of the forces for gear 1 is

$$\sum \mathbf{F}^{(1)} = \mathbf{F}_{21} + \mathbf{F}_{01} = \mathbf{0},$$

and the reaction force of the ground on gear 1 is

$$\mathbf{F}_{01} = -\mathbf{F}_{21} = 291.176\,\mathbf{\jmath} + 800\,\mathbf{k} \ \ \text{N}.$$

Arm 3

The sum of the moments for arm 3 with respect to the point $D$ is

$$\sum \mathbf{M}_D^{(3)} = \mathbf{r}_{DC} \times \mathbf{F}_{23} + \mathbf{M}_{3mot} = \mathbf{0},$$

where $\mathbf{F}_{23} = -\mathbf{F}_{32}$ and $\mathbf{r}_{DC} = (r_1 + r_2)\mathbf{\jmath}$. The motor torque (equilibrium moment) $M_3$ can be computed as

$$M_{3mot} = -F_{23z}\,(r_1 + r_2) = -F_{t12}\,(r_1 + r_2) = -\frac{M_{ext}}{r_2}\,(r_1 + r_2) = -1200 \text{ N} \cdot \text{m}.$$

For the arm 3 the sum of forces equation is

$$\sum \mathbf{F}^{(3)} = \mathbf{F}_{23} + \mathbf{F}_{03} = \mathbf{0},$$

and the reaction force of the ground on arm 3 is

$$\mathbf{F}_{03} = -\mathbf{F}_{23} = -291.176\,\mathbf{\jmath} - 800\,\mathbf{k} \ \ \text{N}.$$

**Example 6.7.** A planetary gear train with one degree of freedom is shown in Fig. 6.21(a). The sun gear 1 is connected to the ground with a pin joint at point $A$. The arm 3 is connected with pin joints to the planet gear 2 at point $C$ and to the ground at point $D$. The planet gear 2 is also in contact to gear 4 (as internal gear) which is fixed to the ground ($4 = 0$). The angular speed of the motor that drives the sun gear is $\omega_{10} = 2\,\pi/3$ rad/s ($n_1 = 20$ rpm). The radii of the pitch circles of the sun gear 1 and planet gear 2 are $r_1 = 1$ m and $r_2 = 0.5$ m. An external moment $\mathbf{M}_{ext} = -|\mathbf{M}_{ext}|\dfrac{\boldsymbol{\omega}_{30}}{|\boldsymbol{\omega}_{30}|}$ acts on the driven arm 3, where $|\mathbf{M}_{ext}| = 400$ N·m. The pressure angle of the gears is $\phi = 20°$. Find the equilibrium moment (motor moment) $M_{mot}$ that acts on the sun gear and the reaction forces.

Solution.

Contour $0 - A - 1 - B - 2 - E - 0$

For the relative angular velocities of the gears the following relations can be written:

$$\boldsymbol{\omega}_{10} + \boldsymbol{\omega}_{21} + \boldsymbol{\omega}_{02} = \mathbf{0},$$
$$\mathbf{r}_{AB} \times \boldsymbol{\omega}_{21} + \mathbf{r}_{AE} \times \boldsymbol{\omega}_{02} = \mathbf{0},$$

where $\boldsymbol{\omega}_{10} = 2\,\pi/3\,\mathbf{\imath}$ rad/s. The solutions of the system are $\boldsymbol{\omega}_{21}$ and $\boldsymbol{\omega}_{02}$:

$$\boldsymbol{\omega}_{21} = -4\,\pi/3\,\mathbf{\imath} \text{ rad/s}, \;\; \boldsymbol{\omega}_{02} = 2\,\pi/3\,\mathbf{\imath} \text{ rad/s}.$$

The angular speed $\boldsymbol{\omega}_{20}$ of the planet gear 2 can be computed as

$$\boldsymbol{\omega}_{20} = -\boldsymbol{\omega}_{02} = -2\,\pi/3\,\mathbf{\imath} \text{ rad/s}.$$

Contour $0 - E - 2 - C - 3 - D - 0$

For this contour the relative angular velocity equations are

$$\boldsymbol{\omega}_{20} + \boldsymbol{\omega}_{32} + \boldsymbol{\omega}_{03} = \mathbf{0},$$
$$\mathbf{r}_{AE} \times \boldsymbol{\omega}_{20} + \mathbf{r}_{AC} \times \boldsymbol{\omega}_{32} = \mathbf{0}.$$

The relative angular speed $\boldsymbol{\omega}_{32}$ is

$$\boldsymbol{\omega}_{32} = \frac{26.6\,\pi}{30}\,\mathbf{\imath} \;\; \text{rad/s}.$$

The angular speed $\boldsymbol{\omega}_{30}$ of the arm 3 is

$$\boldsymbol{\omega}_{30} = -\boldsymbol{\omega}_{03} = \boldsymbol{\omega}_{20} + \boldsymbol{\omega}_{32} = \frac{6.6\,\pi}{30}\,\mathbf{\imath} \;\; \text{rad/s}.$$

Figure 6.21(b) shows the free body diagrams of arm 3, gear 2, and gear 1. The external torque $\mathbf{M}_{ext}$ on the driven arm 3 is

$$\mathbf{M}_{ext} = M_{ext}\,\mathbf{\imath} = -|\mathbf{M}_{ext}|\,\frac{\boldsymbol{\omega}_{30}}{|\boldsymbol{\omega}_{30}|} = -400\,\mathbf{\imath} \;\; \text{N} \cdot \text{m},$$

and the force analysis starts with the driven planet gear.

Arm 3 (driven)

The moment equation with respect to point $D$ for arm 3 is

$$\sum \mathbf{M}_D^{(3)} = \mathbf{r}_{DC} \times \mathbf{F}_{23} + \mathbf{M}_{ext} = \mathbf{0}, \tag{6.59}$$

where $\mathbf{F}_{23} = \mathbf{F}_{23y}\,\boldsymbol{\jmath} + \mathbf{F}_{23z}\,\mathbf{k}$. Equation (6.59) gives

$$\begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ x_C - x_D & r_1 + r_2 & 0 \\ 0 & F_{32y} & F_{32z} \end{vmatrix} + M_{ext}\boldsymbol{\imath} = \mathbf{0}. \tag{6.60}$$

From Eq. (6.60) it results

$$F_{23z} = -\frac{M_{ext}}{r_1 + r_2} = -\frac{-400}{1 + 0.5} = 266.666 \text{ N}.$$

Gear 2

The sum of the moments for gear 2 with respect to its center $C$ is

$$\sum \mathbf{M}_C^{(2)} = \mathbf{r}_{CE} \times \mathbf{F}_{02} + \mathbf{r}_{CB} \times \mathbf{F}_{12} = \mathbf{0},$$

or

$$\begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & r_2 & 0 \\ 0 & F_{r02} & F_{t02} \end{vmatrix} + \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & -r_2 & 0 \\ 0 & F_{r12} & F_{t12} \end{vmatrix} = \mathbf{0},$$

or

$$r_2\,F_{t02} - r_2\,F_{t12} = 0 \implies F_{t02} = F_{t12}.$$

The radial component $F_r$ tends to push the gears apart:

$$F_{r12} = F_{t12}\,\tan\phi \quad \text{and} \quad F_{r02} = -F_{t12}\,\tan\phi.$$

For planet gear 2 the force equation is

$$\sum \mathbf{F}^{(2)} = \mathbf{F}_{12} + \mathbf{F}_{02} + \mathbf{F}_{32} = \mathbf{0}, \tag{6.61}$$

where $\mathbf{F}_{12} = F_{r12}\,\boldsymbol{\jmath} + F_{t12}\,\mathbf{k}$, $\mathbf{F}_{02} = -F_{r12}\,\boldsymbol{\jmath} + F_{t12}\,\mathbf{k}$, and $\mathbf{F}_{32} = -\mathbf{F}_{23}$. Equation (6.61) gives

$$F_{r12} - F_{r12} - F_{23y} = 0 \implies F_{23y} = 0,$$
$$F_{t12} + F_{t12} - F_{23z} = 0 \implies F_{t12} = \frac{F_{23z}}{2} = \frac{266.666}{2} = 133.333 \text{ N}.$$

The radial reaction forces $F_{r12}$ and $F_{r02}$ are

$$F_{r12} = -F_{r02} = F_{t12}\tan\phi = 133.333\tan 20^\circ = 48.529 \text{ N}.$$

For the arm 3 the force equation is

$$\sum \mathbf{F}^{(3)} = \mathbf{F}_{23} + \mathbf{F}_{03} = \mathbf{0},$$

and the reaction force $\mathbf{F}_{03}$ is

$$\mathbf{F}_{03} = -\mathbf{F}_{23} = -266.666\,\mathbf{k} \quad \text{N}.$$

Gear 1 (driver)

For gear 1 the sum of the moments with respect to its center $A$ is

$$\sum \mathbf{M}_A^{(1)} = \mathbf{r}_{AB} \times \mathbf{F}_{21} + \mathbf{M}_{mot} = \mathbf{0},$$

where $\mathbf{F}_{21} = -\mathbf{F}_{12}$. The motor torque $M_{mot}$ is

$$M_{mot} = -F_{t21}\, r_1 = F_{t12}\, r_1 = 133.333 \quad \text{N} \cdot \text{m}.$$

The sum of the forces for the gear 1 is

$$\sum \mathbf{F}^{(1)} = \mathbf{F}_{21} + \mathbf{F}_{01} = \mathbf{0},$$

and the reaction force of the ground on gear 1 is

$$\mathbf{F}_{01} = -\mathbf{F}_{21} = \mathbf{F}_{12} = 48.529\,\mathbf{\jmath} + 133.333\,\mathbf{k} \quad \text{N}.$$

The *Mathematica*™ program for this example is given in Program 6.1.

**Example 6.8.** A driver pinion made of steel with the endurance strength $\sigma_0 = 15\,000$ psi rotates at $n_p = 1200$ rpm. The pinion is surface hardened to BHN 250. The gear is made of cast iron with the endurance strength $\sigma_0 = 8000$ psi and rotates at $n_g = 300$ rpm. The teeth have standard 20° stub involute profiles. The maximum power to be transmitted is 33 hp. Determine the proper diametral pitch, numbers of teeth, and face width for the gears from the standpoint of strength, dynamic load, and wear.

Solution

The diameters of the gears are unknown. In order to determine the smallest diameter gears that can be used, the minimum number of teeth for the pinion will be selected, $N_p = 16$. Then the number of teeth for the gear is

$$N_g = -N_p\, i = N_p \frac{n_p}{n_g} = 16\,\frac{1200}{300} = 64,$$

where $i = -n_p/n_g = 4$ is the speed ratio. Next it will be determined which is weaker, the gear or the pinion. The load carrying capacity of the tooth is a function of the $\sigma_0\gamma$ product.

From Table 6.1 for a 20° stub involute gear with 16 teeth, the form factor is $\gamma_p = 0.115$. For the pinion, the load carrying capacity is

$$F_p = \sigma_{0p}\gamma_p = 15\,000\,(0.115) = 1725.$$

From Table 6.1 for a 20° stub involute gear with 64 teeth, the calculated form factor is $\gamma_g = 0.155$. For the gear, the load carrying capacity is

$$F_g = \sigma_{0g}\gamma_g = 8000\,(0.155) = 1240.53.$$

Since $F_g < F_p$ the gear is weaker and the gear will be analyzed.

The moment transmitted by the gear is

$$M_t = \frac{63\,000\;H}{n_g} = \frac{63\,000\,(33)}{300} = 6930 \;\; \text{lb}\cdot\text{in.}$$

The diameters are unknown and the induced stress is

$$\sigma = \frac{2\,M_t\,P_d^3}{k\,\pi^2\,\gamma_g\,N_g} = \frac{2(6930)P_d^3}{4\pi^2(0.155)(64)} = 35.375\,P_d^3, \tag{6.62}$$

where a maximum value of $k = 4$ was considered.

To determine an approximate $P_d$, assume the allowable stress

$$\sigma \approx \frac{\sigma_0}{2} = \frac{8000}{2} = 4000 \;\; \text{psi.}$$

Equation (6.62) yields

$$4000 = 35.375\,P_d^3 \quad \Longrightarrow \quad P_d \approx 4.835.$$

Try a diametral pitch $P_d = 5$ teeth per inch. Then the pitch diameter is

$$d_g = \frac{N_g}{P_d} = \frac{64}{5} = 12.8 \text{ in.}$$

The pitch line velocity is

$$V = d_g\,\pi\,n_g/12 = 12.8\,\pi(300)/12 = 1005.31 \text{ ft/min.}$$

The allowable stress for $V < 2000$ ft/min is given by Eq. (6.36):

$$\sigma = 8000\left(\frac{600}{600 + 1005.31}\right) = 2990.08\,\text{psi}.$$

Using Eq. (6.62) the induced stress is

$$\sigma = \frac{2\,M_t\,P_d^3}{k\,\pi^2\,\gamma_g\,N_g} = \frac{2(6930)(5^3)}{4\pi^2(0.155)(64)} = 4421.96 \text{ psi}.$$

The gear is weak because the induced stress is larger than the allowable stress.

Try a stronger tooth and select $P_d = 4$ teeth per inch. The pitch diameter is

$$d_g = \frac{N_g}{P_d} = \frac{64}{4} = 16 \text{ in}.$$

The pitch line velocity is

$$V = d_g\,\pi n_g\,/12 = 16\,\pi\,(300)/12 = 1256.64 \text{ ft/min}.$$

Because the pitch line velocity is less than 2000 ft/min, the allowable stress is

$$\sigma = 8000\left(\frac{600}{600 + 1256.64}\right) = 2585.32\,\text{psi}.$$

Using Eq. (6.62) the induced stress is

$$\sigma = \frac{2(6930)(4^3)}{4\pi^2(0.155)(64)} = 2264.04 \text{ psi}.$$

The gear is strong because the induced stress is smaller than the allowable stress.

The parameter $k$ can be reduced from the maximum value of $k = 4$. Equation (6.62) with the allowable stress, $\sigma = 2585.32$ psi, gives

$$2585.32 = \frac{2(6930)(4^3)}{k\,\pi^2(0.155)(64)} \implies k = 3.502.$$

The face width is

$$B = k\,p = k\,(\pi/P_d) = 3.502\,(\pi/4) = 2.751 \text{ in.},$$

where the circular pitch is

$$p = \pi/P_d = \pi/4 = 0.785 \ \text{ in.}$$

Then
$P_d = 4$, $B = 2\frac{3}{4}$ in., $r_g = d_g/2 = 16/2 = 8$ in., and
$r_p = -r_g/i = 8/4 = 2$ in.
The center distance is
$c = r_p + r_g = 2 + 8 = 10$ in.
The addendum of the gears is
$a = 0.8/P_d = 0.8/4 = 0.2$ in.,
while the minimum dedendum for 20° full-depth involute gears is
$b = 1/P_d = 1/4 = 0.25$ in.
The radii of the base circle for the pinion and the gear are
$r_{bp} = r_p \cos\phi = 2 \cos 20° = 1.879$ in., and
$r_{bg} = r_g \cos\phi = 8 \cos 20° = 7.517$ in., respectively.
The maximum possible addendum circle radius of pinion or gear without interference is

$$r_{a(max)} = \sqrt{r_b^2 + c^2 \sin^2\phi}.$$

Hence, for the pinion and for the gear
$r_{ap(max)} = \sqrt{1.879^2 + 10^2 \sin^2 20°} = 3.902$ in.,
$r_{ag(max)} = \sqrt{7.517^2 + 10^2 \sin^2 20°} = 8.259$ in.
The addendum radii of the meshing pinion and gear are
$r_{ap} = r_p + a = 2 + 0.2 = 2.2$ in., and
$r_{ag} = r_g + a = 8 + 0.2 = 8.2$ in.
Since $r_{a(max)} > r_a$, there is no interference.
The contact ratio is calculated from the equation

$$CR = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - c\sin\phi}{p_b},$$

where the base pitch is $p_b = \pi\, d_b/N = p \cos\phi = 0.785 \cos 20° = 0.738$ in.
The contact ratio is $CR = 1.353$, which is a suitable value ($> 1.2$).

Next, the tentative design will be checked from the standpoint of dynamic load and wear effects.

The allowable endurance load is

$$F_0 = \sigma_0\, B\, \gamma_g\, p = 8000(2.75)(0.155)(\pi/4) = 2679.36 \ \text{ lb.}$$

Equation (6.38) gives the allowable wear load

$$F_w = d_p\, B\, K\, Q = 4(2.75)(170.11)(1.6) = 2993.94 \ \text{ lb},$$

where

$$Q = \frac{2\, N_g}{N_p + N_g} = \frac{2\, 64}{16 + 64} = 1.6,$$

the surface endurance limit of a gear pair is

$$\sigma_{es} = (400)(\text{BHN}) - 10\,000 = (400)(250) - 10\,000 = 9000 \ \text{ psi},$$

and the stress factor for fatigue is

$$\begin{aligned} K &= \frac{\sigma_{es}^2 (\sin\phi)(1/E_p + 1/E_g)}{1.4} = \frac{9000^2 (\sin 20^\circ)\left(\dfrac{1}{30 \times 10^6} + \dfrac{1}{19 \times 10^6}\right)}{1.4} \\ &= 170.11 \text{ lb}. \end{aligned}$$

For $V = 1256.64$ ft/min, from Fig. 6.13(a), the permissible error is 0.00225 in. From Fig. 6.13(b), for carefully cut gears with $P_d = 4$, the tooth error is $e = 0.0012$ in. From Table 6.2 the deformation factor for dynamic load check is $C = 1416$.

The dynamic load $F_d$ proposed by Buckingham is

$$\begin{aligned} F_d &= \frac{0.05\, V (B\, C + F_t)}{0.05\, V + \sqrt{B\, C + F_t}} + F_t \\ &= \frac{0.05\,(1256.64)[(2.75)\,(1416) + 866.25]}{0.05\,(1256.64) + \sqrt{(2.75)\,(1416) + 866.25}} + 866.25 \\ &= 3135.7 \text{ lb}. \end{aligned}$$

where $F_t = M_t/r_g = 6930/8 = 866.25$ lb.

The design is unsatisfactory because the dynamic force $F_d$ must be less than the allowable endurance load $F_0$ and less than the wear load $F_w$.

From Fig. 6.13(b) select a precision gear with an error of action $e = 0.00051$ for $P_d = 4$. From Table 6.2 the deformation factor for dynamic load check is $C = 601.8$. Recalculating the dynamic load $F_d$ for $C = 601.8$ gives

$$\begin{aligned} F_d &= \frac{0.05\,(1256.64)[(2.75)\,(601.8) + 866.25]}{0.05\,(1256.64) + \sqrt{(2.75)\,(601.8) + 866.25}} + 866.25 \\ &= 2267.89 \text{ lb}. \end{aligned}$$

The design is satisfactory because the dynamic force $F_d$ is less than the allowable endurance load $F_0$ and less than the wear load $F_w$

$$F_d < F_0 \quad \text{and} \quad F_d < F_w.$$

The *Mathematica*™ program for this example is given in Program 6.2.

## 6.10 Problems

6.1 A planetary gear train is shown in Fig. 6.22. Gear 1 has $N_1 = 36$ external gear teeth, gear 2 has $N_2 = 40$ external gear teeth, and gear $2'$ has $N_{2'} = 21$ external gear teeth. Gears 2 and $2'$ are fixed on the same shaft $CC'$. Gear 3 has $N_3 = 30$ external gear teeth, and gear $3'$ has $N_{3'} = 24$ external gear teeth . Gears 3 and $3'$ are fixed on the same shaft $EE'$. The planet gear 4 has $N_4 = 18$ external gear teeth, and the planet gear 5 has $N_5 = 14$ external gear teeth. Gear 1 rotates with a constant input angular speed $n_1 = 320$ rpm. The module of the gears is $m = 30$ mm. The pressure angle of the gears is 20°. a) Determine whether there will be interference when standard full-depth teeth are used and find the contact ratios of the meshing gears. b) Find the angular velocity of the output planet arm 6, $\boldsymbol{\omega}_6$. c) Find the equilibrium moment on gear 1 if an external moment $\mathbf{M}_{ext} = -|\mathbf{M}_{ext}|\dfrac{\boldsymbol{\omega}_6}{|\boldsymbol{\omega}_6|}$ acts on the arm 6, where $|\mathbf{M}_{ext}| = 600$ N·m.

6.2 The planetary gear train considered in Fig. 6.23 has gears with the same module $m = 24$ mm. The sun gear 1 has $N_1 = 22$ external gear teeth, the planet gear 2 has $N_2 = 18$ external gear teeth, gear 3 has $N_3 = 20$ external gear teeth, and gear 4 has $N_4 = 54$ external gear teeth. Gears 3 and $3'$ are fixed on the same shaft. The sun gear 1 rotates with an input angular speed $n_1 = 290$ rpm, and arm 5 rotates with $n_5 = 110$ rpm. The pressure angle of the gears is 20°. a) Find the number of DOF for the planetary gear train. b) Determine whether there will be interference when standard full-depth teeth are used and find the contact ratios of the meshing gears. c) Find the angular velocity of the output gear 4, $\boldsymbol{\omega}_4$. d) Find the equilibrium moments on gear 1 and arm 5 if an external moment $\mathbf{M}_{ext} = -|\mathbf{M}_{ext}|\dfrac{\boldsymbol{\omega}_4}{|\boldsymbol{\omega}_4|}$ acts on gear 4, where $|\mathbf{M}_{ext}| = 800$ N·m.

6.3 A planetary gear train is depicted in Fig. 6.24. Gear 1 has $N_1 = 15$ external gear teeth, gear 2 has $N_2 = 27$ external gear teeth, gear $2'$ has $N_{2'} = 18$ external gear teeth, gear 3 has $N_3 = 24$ external gear teeth, and gear $3'$ has $N_{3'} = 16$ external gear teeth. The planet gears 2 and $2'$ are fixed on the same link and the planet gears 3 and $3'$ are fixed on the same shaft. Gear 1 rotates with the input angular speed $n_1 = 440$ rpm, and arm 5 rotates at $n_5 = 80$ rpm. The module of the gears is $m = 24$

and the pressure angle of the gears is 20°. a) Determine whether there will be interference when standard full-depth teeth are used and find the contact ratios of the meshing gears. b) Find the angular velocity of the output gear 4, $\boldsymbol{\omega}_4$. c) Find the equilibrium moments on gear 1 and arm 5 if an external moment $\mathbf{M}_{ext} = -|\mathbf{M}_{ext}|\dfrac{\boldsymbol{\omega}_4}{|\boldsymbol{\omega}_4|}$ acts on gear 4, where $|\mathbf{M}_{ext}| = 1000$ N·m.

6.4 A planetary gear train is shown in Fig. 6.25. Gear 1 has $N_1 = 11$ external gear teeth, the planet gear 2 has $N_2 = 22$ external gear teeth, gear $2'$ has $N_{2'} = 17$ external gear teeth, gear 3 has $N_3 = 51$ internal gear teeth, the sun gear $3'$ has $N_{3'} = 12$ external gear teeth, and the planet gear 4 has $N_4 = 32$ external gear teeth. Gears 2 and $2'$ are fixed on the same shaft and gears 3 and $3'$ are fixed on the same shaft. The sun gear 1 rotates with an input angular speed of $n_1 = 550$ rpm. The module of the gears is $m = 26$ mm and the pressure angle of the gears is 20°. a) Determine whether there will be interference when standard full-depth teeth are used and find the contact ratios of the meshing gears. b) Find the angular velocity of the arm 5, $\boldsymbol{\omega}_5$. c) Find the joint forces if an electric motor with the power $H = 4$ kW is coupled to gear 1.

6.5 A planetary gear train is shown in Fig. 6.26. The ring gear 1 has $N_1 = 60$ internal gear teeth, the planet gear 2 has $N_2 = 25$ external gear teeth, and the planet gear $2'$ has $N_{2'} = 15$ external gear teeth. Gears 2 and $2'$ are fixed on the same shaft. The planet gear 3 has $N_3 = 20$ teeth and gears 3 and $3'$ are fixed on the same shaft. The ring gear 4 has $N_4 = 90$ internal gear teeth. Gear 1 rotates with the input angular speed $n_1 = 100$ rpm, and arm 5 rotates at $n_5 = -150$ rpm ($n_1$ is opposite to $n_5$). The module of the gears is $m = 28$ and the pressure angle of the gears is 20°. a) Determine whether there will be interference when standard full-depth teeth are used and find the contact ratios of the meshing gears. b) Find the angular velocity of the output ring gear 4, $\boldsymbol{\omega}_4$. c) Find the joint forces and the equilibrium moments on gear 1 and arm 5 if an external moment $\mathbf{M}_{ext} = -|\mathbf{M}_{ext}|\dfrac{\boldsymbol{\omega}_4}{|\boldsymbol{\omega}_4|}$ acts on 4, where $|\mathbf{M}_{ext}| = 1000$ N·m.

6.6 A planetary gear train is shown in Fig. 6.27. The sun gear 1 has $N_1 = 11$ teeth, the planet gear 2 has $N_2 = 19$ teeth, gear 3 has $N_3 = 40$

internal gear teeth, gear 4 has $N_4 = 29$ external gear teeth, and gear 5 has $N_5 = 24$ external gear teeth. Gear 1 rotates with a constant input angular speed $n_1 = 200$ rpm. The module of the gears is $m = 22$ and the pressure angle of the gears is 20°. a) Determine whether there will be interference when standard full-depth teeth are used and find the contact ratios of the meshing gears. b) Find the angular velocity of the output ring gear 6, $\boldsymbol{\omega}_6$. c) An external moment $\mathbf{M}_{ext} = -|\mathbf{M}_{ext}|\dfrac{\boldsymbol{\omega}_6}{|\boldsymbol{\omega}_6|}$ acts on gear 6, where $|\mathbf{M}_{ext}| = 900$ N·m. Find the joint forces and the equilibrium moment on gear 1.

6.7 A planetary gear train is shown in Fig. 6.28. The sun ring gear 1 has $N_1 = 28$ teeth, the planet gear 2 has $N_2 = 21$ teeth, and the planet gear $2'$ has $N_{2'} = 16$ teeth. Gears 2 and $2'$ are fixed on the same shaft. Gear 1 rotates with the input angular speed $n_1 = 370$ rpm. The module of the gears is $m = 20$ and the pressure angle of the gears is 20°. a) Determine whether there will be interference when standard full-depth teeth are used and find the contact ratios of the meshing gears. b) Find the angular velocity of the arm gear 4, $\boldsymbol{\omega}_4$. c) An external moment $\mathbf{M}_{ext} = -|\mathbf{M}_{ext}|\dfrac{\boldsymbol{\omega}_4}{|\boldsymbol{\omega}_4|}$ acts on the arm gear 4, where $|\mathbf{M}_{ext}| = 800$ N·m. Find the joint forces and the equilibrium moment on gear 1.

6.8 A planetary gear train is shown in Fig. 6.29. The ring gear 1 has $N_1 = 75$ internal gear teeth, the planet gear 2 has $N_2 = 35$ teeth, the planet gear $2'$ has $N_{2'} = 20$ teeth, gear 3 has $N_3 = 11$ teeth, gear 4 has $N_4 = 13$ external gear teeth, and the ring gear 5 has $N_5 = 50$ internal gear teeth. Gears 2 and $2'$ are fixed on the same shaft. The module of the gears is $m = 42$ and the pressure angle of the gears is 20°. a) Determine whether there will be interference when standard full-depth teeth are used and find the contact ratios of the meshing gears. b) Find the angular velocity of the ring gear 5, $\boldsymbol{\omega}_5$. c) An external moment $\mathbf{M}_{ext} = -|\mathbf{M}_{ext}|\dfrac{\boldsymbol{\omega}_5}{|\boldsymbol{\omega}_5|}$ acts on the output ring gear 5, where $|\mathbf{M}_{ext}| = 500$ N·m. Find the joint forces and the equilibrium moment.

6.9 A planetary gear train is shown in Fig. 6.30. An electric motor with the power $H = 2$ kW and 319 rpm is coupled to sun gear 1. Gear 1 has

$N_1 = 12$ teeth, the planet gear 2 has $N_2 = 17$ teeth, gear 3 has $N_3 = 20$ teeth, gear 4 has $N_4 = 11$ teeth, gear $4'$ has $N_{4'} = 17$ external gear teeth, and the ring gear 5 has $N_5 = 51$ internal gear teeth. Gears 4 and $4'$ are fixed on the same shaft. The module of the gears is $m = 33$ mm and the pressure angle of the gears is 20°. a) Find the angular velocity of the output ring gear 5. b) Find the joint forces.

6.10 The planetary gear train considered is shown in Fig. 6.31. The sun gear 1 has $N_1 = 22$ teeth, the planet gear 2 has $N_2 = 20$ teeth, the planet gear $2'$ has $N_{2'} = 35$ teeth, the sun gear 4 has $N_4 = 15$ teeth, and the planet gear 5 has $N_5 = 16$ teeth. Gears 2 and $2'$ are fixed on the same shaft. The ring gear 3 rotates with the input angular speed $n_3 = 200$ rpm and the ring gear 6 rotates at the input angular speed $n_6 = 150$ rpm. The module of the gears is $m = 24$ mm. Find the absolute angular velocity of gear 1.

6.11 The planetary train with two planet gears is shown in Fig. 6.32. The sun gear 1 has an angular speed of 600 rpm and is driven by a motor with the moment 20 N·m. The planet gears are 2 and $2'$', each having $N_2 = N_{2'} = 20$ teeth. The ring gear 4 has $N_4 = 70$ teeth. A brake holds the ring gear 4 fixed. The module of the gears is $m = 2$ mm and the pressure angle of the gears is 20°. The arm 3 drives a machine. Determine: a) the circular pitch of the gears; b) the angular speed of the arm; c) the pitch line velocity of each gear; d) the joint forces; e) the output moment; f) the moment to be applied to the ring to keep it fixed.

6.12 A driver spur pinion of cast steel with the endurance strength $\sigma_0 =$ 20 000 psi rotates at $n = 1500$ rpm and transmits 35 hp. The driven gear is made of cast iron with the endurance strength $\sigma_0 = 8000$ psi. The transmission ratio is 3.5 to 1 (external gearing). Both gears have 14.5° pressure angles, and are full-depth involute gear teeth. Design for strength and determine the smallest diameter gears and the face width.

6.13 A cast steel spur pinion ($\sigma_0 = 15\,000$ psi) rotating at 900 rpm is to drive a bronze spur gear ($\sigma_0 = 12\,000$ psi) at 300 rpm. The power to be transmitted is 10 hp. The teeth have standard 20° stub involute profiles. Determine the smallest diameter gear that can be used and the necessary face width.

6.14 Two spur gears are to be designed with a minimum size. The following requirements are given: speed of the pinion 600 rpm, power to be transmitted 15 hp, velocity ratio 3 to 1 external gearing, endurance strength for pinion $\sigma_0 = 30\,000$ psi, endurance strength for gear $\sigma_0 = 20\,000$ psi, tooth profile 20° stub involute. For strength design determine the necessary face width and diametral pitch.

6.15 A driver made of mild steel pinion with the endurance strength $\sigma_0 = 15\,000$ psi rotates at $n_1 = 1750$ rpm and transmits 6 hp. The transmission ratio is $i = -3.5$. The gear is made of bronze and has the endurance strength $\sigma_0 = 12\,000$ psi. The gears have 20° pressure angles and full-depth involute gear teeth. Design a gear with the smallest diameter that can be used. No less than 15 teeth are to be used on either gear.

6.16 A pinion made of cast iron with the endurance strength $\sigma_0 = 20\,000$ psi rotates at $n_p = 900$ rpm and transmits 30 hp. The transmission ratio is $i = -7/3$. The gear is made of cast iron and has the endurance strength $\sigma_0 = 8000$ psi. The gears have 20° pressure angles and full-depth involute gear teeth. The diameter of the pinion is $d_p = 4$ in. Design for the greatest number of teeth.

## 6.11 Working Model Simulation for Gear Trains

**Working Model Simulation for Example 6.5**
The gearset shown in Fig. 6.19(a) will be simulated using Working Model software. Gear 1 ($r_1 = r_g = 1$ m) rotates with the angular speed $n_1 = 10$ rpm. The external torque exerted on the pinion 2 ($r_2 = r_p = 0.5$ m) is $M_{ext} = 400$ N·m.

*Step 1: Opening Working Model.*
1. Click on the Working Model program icon to start the program.
2. Create a new Working Model document by selecting "New" from the "File" menu.
3. Set up the workspace.

In the "View" menu: select "Workspace", check Coordinates and X,Y Axes from the Navigation box, and check all the objects from the Toolbars box except Simple; turn off Grid Snap and turn on Object Snap; select "Numbers and Units" and change the Unit System to SI (degrees).

*Step 2. Creating the gears.*
This step creates the two gears for the system.
1. Create the gear 1.

Click on the "Circle" tool in the toolbar to sketch out a disk. Click on the disk and modify its radius at the bottom of the screen to $r_1 = 1$ m.
2. Create the pinion 2.

Click on the "Circle" tool in the toolbar to sketch out a disk. Click on the disk and modify its radius at the bottom of the screen to $r_2 = 0.5$ m.
3. Change the properties of the gears.

Press the Shift key and click on the main gear and the pinion, respectively. Select "Properties" in the "Window" menu and change the material to Steel, the coefficients of static and kinetic friction to 0.0 (no friction), the coefficient of restitution to 1.0 (perfect elastic), and the charge to 0.0 (no charge), as shown in Fig. 6.33.

Remark. In order to make the objects clearly visible the commands "Zoom in" and "Zoom out" can be used by clicking on the icons at the top of the screen.

*Step 3. Connecting the gears to the ground.*

This step connects a motor to gear 1 and the pinion 2 to the ground using a pin joint.

1. Select the gear 1 and modify its center coordinates at the bottom of the screen to $x = 0$ and $y = 0$ (the center of axis).

2. Click on the "Motor" tool, place the cursor over the "snap point" on the center of the main gear and then click again. This connects the motor to the ground and the gear.

3. Select "Numbers and units" in the "View" menu and change the "Rot. Velocity" to Revs/min. Select the "Properties" box in the "Window" menu and change the "value" to $n_1 = 10$ rpm.

The screen should look like that shown in Fig. 6.34.

4. Select the pinion and modify its center coordinates at the bottom of the screen to $x = 0$ and $y = 1.5$ m.

5. Click on the "Pin joint" tool and then click again on the center of the pinion. This connects the pinion to the ground with the pin joint.

The screen should look like that shown in Fig. 6.35.

*Step 4. Connecting the gears.*

This step connects the gear and the pinion using the "Gear" tool.

1. Click on the "Gear" tool from the toolbox and then click on the centers of the gear and the pinion, respectively. This connects the two gears with a rigid rod.

By default, each pair of gears has a rigid rod constraint between the two mass centers. The rod maintains a constant distance between the two objects.

The screen should look like that shown in Fig. 6.36.

*Step 5. Running the simulation.*

1. Click on "Run" in the toolbar to start the simulation.

2. Click on "Reset" in the toolbar. The simulation resets to the initial frame 0.

3. Select the pinion, then go to "Measure" menu and "Velocity" submenu. Apply the "Rotational graph" command to measure the rotational velocity of the pinion. Click on the arrow in the right upper corner of the measurement window to change it from graphic to numerical. Select the gear and apply the same command to measure the rotational velocity of the gear.

*Step 6. Adding an external torque.*

1. Click on the "Torque" tool from the toolbox and then click on the pinion (anywhere). This will apply an external torque to the pinion.

2. Select the torque and modify its value to $M_{ext} = 400$ N·m in the "Properties" menu. Apply the command "Torque" from the "Measure" menu to measure the torque applied.

3. Select the motor and apply the command "Torque Transmitted" from the "Measure" menu to measure the torque of the motor.

The screen should look like that shown in Fig. 6.37.

*Results.*

The angular speed of the gear $n_1 = 10$ rpm and the external torque $M_{ext} = 400$ N·m are given. It results in the angular speed of the pinion $n_2 = -20$ rpm and the motor torque $M_{mot} = 800$ N·m.

**Working Model Simulation for Example 6.6**

The planetary gear train with two degrees of freedom shown in Fig. 6.20(a) will be simulated using Working Model software. The sun gear 1 ($r_1 = 1$ m) has the angular speed $n_1 = 20$ rpm. The planet gear 2 has the pitch radius $r_2 = 0.5$ m. The arm has the angular speed $n_3 = -10$ rpm. The external torque exerted on the gear 2 is $M_{ext} = 400$ N·m.

*Step 1. Creating the gears and the arm.*

1. Create the sun gear 1.

Click on the "Circle" tool in the toolbar and sketch out a disk. Click on the disk and modify its radius at the bottom of the screen as $r_1 = 1$ m.

2. Create the planet gear 2.

Click on the "Circle" tool in the toolbar and sketch out a disk. Click on the disk and modify its radius at the bottom of the screen as $r_2 = 0.5$ m.

3. Create arm 3.

Click on the "Rectangle" tool in the toolbar and sketch out a rectangle. Click on the rectangle and modify its the dimensions as $h = 1.5$ m and $w = 0.1$ m.

The rod created with a set of gears cannot have torques applied to it or have an anchor or a motor placed on it. That is why the rectangle 3 will be used instead of the rod to model the arm connected to the planet gear 2 and the ground.

4. Select the arm 3 and modify the coordinates of its center as $x = 0$ and $y = 0.75$ m at the bottom of the screen.

5. Select the planet gear 2 and modify the coordinates of its center as $x = 0$ and $y = 1.5$ m at the bottom of the screen.

*Step 2. Connecting the planet gear and the arm.*

1. Click on the "Pin joint" tool on the toolbox and connect the planet gear and the rectangle by clicking again on the center of the circle.

2. Click on the "Motor" tool on the toolbox and then click again on the center of the axis. This connects the motor to the ground and the arm. Click on the motor and change the value of the velocity to 10 rpm in the "Properties" window ($n_3 = -10$ rpm).

The screen should look like that shown in Fig. 6.38.

*Step 3. Connecting the sun gear to the ground.*

1. Click on the "Motor" tool on the toolbox and then click again on the center of the sun gear 1.

2. Select the motor and open the "Properties" window. Change the value of the velocity to 20 rpm ($n_1 = 20$ rpm), as shown in Fig. 6.39.

3. In the "Properties" window, select the base point of the motor and change its coordinates to $x = 0$ and $y = 0$. This moves the motor along with the sun gear to the center of axis (the motor is still connected to the ground).

*Step 4. Connecting the gears.*

1. Click on the sun gear 1 and select "Move to front" from the "Object" menu. Do the same command for the planet gear 2.

2. Click on the "Gear" tool from the toolbox. With the gear selected, click on the center of the sun gear 1 and then again on the center of the planet gear 2. The two circles are now connected with a gear.

3. Click on the rectangle 3 and select the command "Bring to front" from the "Object" menu.

The screen should look like that shown in Fig. 6.40.

*Step 5. Running the simulation.*

1. Select all the bodies and choose the command "Do not collide" from the "Object" menu.

2. Click on "Run" in the toolbar to start the simulation.

3. Click on “Reset” in the toolbar. The simulation resets to the initial frame 0.

4. Click on the planet gear 2 and select the “Measure” menu and the “Velocity” and “Rotational graph” submenus to measure the rotational velocity $n_2$. Click on the arrow in the right upper corner of the measurement window to change it from graphic to numerical. Apply the same command to visualize the rotational velocity $n_1$ of the sun gear 1.

*Step 6. Adding an external torque.*

1. Click on the “Torque” tool from the toolbox and then click on gear 2 (anywhere on the disk). This will apply an external torque to gear 2.

2. Select the torque and modify its value to $M_{ext} = 400$ N·m in the “Properties” menu.

3. Select the submenu “Torque” from the “Measure” menu to measure the torque applied.

The screen should look like that shown in Fig. 6.41.

*Results.*

The angular speed of gear 1, $n_1 = 20$ rpm, the angular speed of arm 3, $n_3 = -10$ rpm, and the external torque, $M_{ext} = 400$ N·m, are given. It results in the angular speed of gear 2, $n_2 = -70$ rpm, the motor torque on gear 1, $M_{1mot} = 800$ N·m, and the motor torque on arm 3, $M_{3mot} = -1200$ N·m.

**Working Model Simulation for Example 6.7**

The planetary gear train with one degree of freedom shown in Fig. 6.21(a) will be simulated using Working Model software. The sun gear 1 ($r_1 = 1$ m) has the angular speed $n_1 = 20$ rpm. The planet gear 2 ($r_2 = 0.5$ m) is connected to the sun gear 1 and the fixed ring gear 4. The external torque exerted on arm 3 is $M_{ext} = -400$ N·m.

*Step 1. Creating the gears and the arm.*

1. Open a new file and make a drawing as shown in Fig. 6.41, following the steps from the previous example of planetary gears.

2. Select the motor connected to the rectangle 3 and erase it using the “Delete” command from the “Edit” menu.

3. Click on the “Pin joint” tool from the toolbox and then click again on the end of the rectangle 3. This connects arm 3 to the ground with a pin joint.

4. Click on the “Circle” tool from the toolbox and draw a disk with the radius $r_4 = 2$ m. Modify the coordinates of its center as $x = y = 0$ at the bottom of the screen. Select the command “Send to back” from the “Object” menu.

5. Click on the “Anchor” tool from the toolbox and then click again on gear 4. This fixes gear 4 to the ground.

The screen should look like that shown in Fig. 6.42.

*Step 2. Connecting gear 2 and gear 4.*

1. Click on gear 2 and select the command “Bring to front” from the “Object” menu. Apply the same command to gear 4.

2. Click on the “Gear” tool, then click on the center of gear 4 and the center of gear 2, respectively. Double-click on the gear and check the box “Internal gear” on the “Properties” window. Choose gear 4 as internal gear.

The screen should look like that shown in Fig. 6.43.

*Step 3. Running the simulation.*

1. Select gear 1 and choose “Bring to front” command. Apply the same command to arm 3.

2. Select all the bodies and choose the command “Do not collide” from the “Object” menu.

3. Click on “Run” in the toolbar to start the simulation.

4. Click on “Reset” in the toolbar. The simulation resets to the initial frame 0.

5. Click on the planet gear 2 and select the “Measure” menu and the “Velocity” and “Rotational graph” submenus to measure the rotational velocity $n_2$. Click on the arrow in the right upper corner of the measurement window to change it from graphical to numerical. Apply the same command to visualize the rotational velocity $n_1$ of the sun gear 1 and the rotational velocity $n_3$ of the arm 3.

*Step 4. Adding an external torque.*

1. Click on the “Torque” tool from the toolbox and then click on arm 3 (anywhere on the rectangle). This will apply an external torque to arm 3.

2. Select the torque and modify its value to $M_{ext} = -400$ N·m in the “Properties” menu.

3. Select the submenu “Torque” from the “Measure” menu to measure the torque applied.

The screen should look like that shown in Fig. 6.44.

*Results.*

The angular speed of gear 1, $n_1 = 20$ rpm, and the external torque, $M_{ext} = -400$ N·m, are given. It results in the angular speed of gear 2, $n_2 = -20$ rpm, the angular speed of arm 3, $n_3 = 6.667$ rpm, and the motor torque on gear 1, $M_{mot} = 133.333$ N·m.

## 6.12 Programs

**Program 6.1**

**Program 6.2**

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Table 6.1. Form factors $\gamma$ - for use in Lewis strength equation

| Number of teeth | $14\frac{1}{2}^\circ$ Full-depth involute or composite | $20^\circ$ Full-depth involute | $20^\circ$ Stub involute |
|---|---|---|---|
| 12 | 0.067 | 0.078 | 0.099 |
| 13 | 0.071 | 0.083 | 0.103 |
| 14 | 0.075 | 0.088 | 0.108 |
| 15 | 0.078 | 0.092 | 0.111 |
| 16 | 0.081 | 0.094 | 0.115 |
| 17 | 0.084 | 0.096 | 0.117 |
| 18 | 0.086 | 0.098 | 0.120 |
| 19 | 0.088 | 0.100 | 0.123 |
| 20 | 0.090 | 0.102 | 0.125 |
| 21 | 0.092 | 0.104 | 0.127 |
| 23 | 0.094 | 0.106 | 0.130 |
| 25 | 0.097 | 0.108 | 0.133 |
| 27 | 0.099 | 0.111 | 0.136 |
| 30 | 0.101 | 0.114 | 0.139 |
| 34 | 0.104 | 0.118 | 0.142 |
| 38 | 0.106 | 0.122 | 0.145 |
| 43 | 0.108 | 0.126 | 0.147 |
| 50 | 0.110 | 0.130 | 0.151 |
| 60 | 0.113 | 0.134 | 0.154 |
| 75 | 0.115 | 0.138 | 0.158 |
| 100 | 0.117 | 0.142 | 0.161 |
| 150 | 0.119 | 0.146 | 0.165 |
| 300 | 0.122 | 0.150 | 0.170 |
| Rack | 0.124 | 0.154 | 0.175 |

*Source*: A. S. Hall, A. R. Holowenko, and H. G. Laughlin, *Theory and Problems of Machine Design*, Schaum's Outline Series, McGraw-Hill, New York, 1961.

Table 6.2. Values of deformation factor $C$ - for dynamic load check

| Materials | | Involute | Tooth error inches | | | |
|---|---|---|---|---|---|---|
| Pinion | Gear | tooth form | 0.0005 | 0.001 | 0.002 | 0.003 |
| cast iron | cast iron | $14\frac{1}{2}^\circ$ | 400 | 800 | 1600 | 2400 |
| steel | cast iron | $14\frac{1}{2}^\circ$ | 550 | 1100 | 2200 | 3300 |
| steel | steel | $14\frac{1}{2}^\circ$ | 800 | 1600 | 3200 | 4800 |
| cast iron | cast iron | 20° full depth | 415 | 830 | 1660 | 2490 |
| steel | cast iron | 20° full depth | 570 | 1140 | 2280 | 3420 |
| steel | steel | 20° full depth | 830 | 1660 | 3320 | 4980 |
| cast iron | cast iron | 20° stub | 430 | 860 | 1720 | 2580 |
| steel | cast iron | 20° stub | 590 | 1180 | 2360 | 3540 |
| steel | steel | 20° stub | 860 | 1720 | 3440 | 5160 |

*Source*: A. S. Hall, A. R. Holowenko, and H. G. Laughlin, *Theory and Problems of Machine Design*, Schaum's Outline Series, McGraw-Hill, New York, 1961.

Table 6.3.

Values for surface endurance limit $\sigma_{es}$ and stress fatigue factor $K$

| Average Brinell Hardness Number of steel pinion and steel gear | | Surface endurance limit $\sigma_{es}$ | Stress fatigue factor $K$: $14\frac{1}{2}^\circ$ | Stress fatigue factor $K$: $20^\circ$ |
|---|---|---|---|---|
| 150 | | 50,000 | 30 | 41 |
| 200 | | 70,000 | 58 | 79 |
| 250 | | 90,000 | 96 | 131 |
| 300 | | 110,000 | 144 | 196 |
| 400 | | 150,000 | 268 | 366 |
| Brinell Hardness Number, BHN | | | | |
| Steel pinion | Gear | | | |
| 150 | C.I. | 50,000 | 44 | 60 |
| 200 | C.I. | 70,000 | 87 | 119 |
| 250 | C.I. | 90,000 | 144 | 196 |
| 150 | Phosphor Bronze | 50,000 | 46 | 62 |
| 200 | Phosphor Bronze | 65,000 | 73 | 100 |
| C.I. Pinion | C.I. Gear | 80,000 | 152 | 208 |
| C.I. Pinion | C.I. Gear | 90,000 | 193 | 284 |

*Source*: A. S. Hall, A. R. Holowenko, and H. G. Laughlin, *Theory and Problems of Machine Design*, Schaum's Outline Series, McGraw-Hill, 1961.

# Figure captions

Figure 6.1. (a) Development of involute curve; (b) pressure angle.

Figure 6.2. Gears of pitch diameter $d$ rotating at angular velocity $\omega$.

Figure 6.3. Pressure angle *phi*.

Figure 6.4. (a) Nomenclature of gear teeth; (b) standard diametral pitches compared with tooth size.

Figure 6.5. Interference of spur gears.

Figure 6.6. Gear train; (a) simple gear trains, and (b) compound gear trains.

Figure 7 Epicycloid curve

Figure 6.8. Epicycloid gear train with two degrees of freedom.

Figure 6.9. Contour diagram of the epicycloid gear train with two two degrees of freedom.

Figure 6.10. (a) Automotive differential planetary gear train; (b) graph attached to the differential mechanism; (c) angular velocities diagram.

Figure 6.11. Gear tooth forces. Driving pinion 1 and driven gear 2 are shown separately.

Figure 6.12. Load carried by the gear tooth.

Figure 6.13. (a) Errors in tooth profiles versus pitch line velocity and (b) errors in tooth profiles versus diametral pitch.

Figure 6.14. (a) Planetary gear train for Example 6.2; (b) contour diagram.

Figure 6.15. Drawing for the planetary gear train for Example 6.3.

Figure 6.16. (a) Schematic representation of the planetary gear train for Example 6.3; (b) contour diagram.

Figure 6.17. Two stage gear reducer for Example 6.4.

Figure 6.18. Free-body diagrams for Example 6.4.

Figure 6.19. (a) Two gears in contact for Example 6.5; (b) free-body diagrams of the gears.

Figure 6.20. (a) Gear train for Example 6.6; (b) free-body diagrams of the gears.

Figure 6.21. (a) Planetary gear train for Example 6.6; (b) free-body diagrams.

Figure 6.22. Planetary gear train for Problem 6.1.

Figure 6.23. Planetary gear train for Problem 6.2.

Figure 6.24. Planetary gear train for Problem 6.3.

Figure 6.25. Planetary gear train for Problem 6.4.

Figure 6.26. Planetary gear train for Problem 6.5.
Figure 6.27. Planetary gear train for Problem 6.6.
Figure 6.28. Planetary gear train for Problem 6.7.
Figure 6.29. Planetary gear train for Problem 6.8.
Figure 6.30. Planetary gear train for Problem 6.9.
Figure 6.31 Planetary gear train for Problem 6.10
Figure 6.32. Planetary gear train for Problem 6.11.
Figure 6.33.
Figure 6.34.
Figure 6.35.
Figure 6.36.
Figure 6.37.
Figure 6.38.
Figure 6.39.
Figure 6.40.
Figure 6.41.
Figure 6.42.
Figure 6.43.
Figure 6.44.