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(* packages that have some differential
equations to solve and define some utility functions *)
Needs["DifferentialEquations`NDSolveProblems`"];
Needs["DifferentialEquations`NDSolveUtilities`"];
Needs["DifferentialEquations`InterpolatingFunctionAnatomy`"];
Needs["GUIKit`"];

ClearAll["Global`*"];
Off[General::spell];
Off[General::spell1];

Print["Kinematics"];

m = 2;
L = 1;
g = 9.81;
k = 2000;

alpha = {0, 0, theta'[t]};
Print["angular acceleration of RB: alpha=", alpha];

xC = (L/2) * Cos[theta[t]];
yC = (L/2) * Sin[theta[t]];
rC = {xC, yC, 0};
Print["position vector of C: rC=", rC];

xA = L * Cos[theta[t]];
yA = L * Sin[theta[t]];
rA = {xA, yA, 0};
Print["position vector of A: rA=", rA];

Print["Forces"];

G = {0, -mg, 0};
Print["gravitational force at C: G=", G];

delta = L Sin[theta[t]] - L Sin[Pi/6];

Fe = {0, -k delta, 0};
Print["elastic force at A: Fe=", Fe];

IC = m * L^2 / 12;
Print["mass moment of inertia wrt C: ICz=", IC];
IO = IC + m * (L/2)^2;
Print["mass moment of inertia wrt O: IOz=ICz+m*(L/2)^2=",
Simplify[IO]];

Print["Summ of the moments in O at t -> t1:"]
eqI = Simplify[IO * alpha - Cross[rC, G]][[3]];
solutionI = Solve[eqI == 0, theta'[t]][[1]];
Print["moment equation: IO alpha = sum M wrt O = rC x G"];
Print["Solution: theta'[t]=", theta'[t] /. Simplify[solutionI]];

(*-----*)
t0 = 0;
soldif = NDSolve[{eqI == 0, theta[0] == N[Pi]/3, theta'[0] == 0}, theta,
{t, t0, Infinity},
Method -> {EventLocator, "Event" -> (theta[t] - Pi/6)}];

t1 = InterpolatingFunctionDomain[First[theta /. soldif]][[1, -1]];
theta1 = (Evaluate[theta[t] /. soldif] /. t -> t1)[[1]];
omega1 = (Evaluate[D[theta[t] /. soldif, t] /. t -> t1])[1];

Print[" "]

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Print["t1 = ", t1, " [s]"]
Print["θ1 = ", thet1, " [rad] = ", thet1 180/Pi, " [deg]"]
Print["ω1 = ", omeg1, " [rad/s]"]

thetaplot =
Plot[Evaluate[theta[t] /. soldif] * 180/Pi, {t, t0, t1}, AxesLabel → {"t[s]", "theta[deg]"}]
omegaplot = Plot[Evaluate[D[theta[t] /. soldif, t]],
{t, t0, t1}, AxesLabel → {"t[s]", "omega[rad/s]"}]
alphaplot = Plot[Evaluate[D[theta[t] /. soldif, {t, 2}]],
{t, t0, t1}, AxesLabel → {"t[s]", "alpha[rad/s^2]"}]

(*-----*)
Print["Summ of the moments in O at t=t1:"]
eqII = Simplify[IO*alpha - Cross[rC, G] - Cross[rA, Fe]][[3]];
solutionII = Solve[eqII == 0, theta'[t]][[1]];
Print["moment equation at t1: IO alpha = sum M wrt O = rC x G + rA x Fe"];
Print["Solution: theta'[t]=", theta'[t] /. Simplify[solutionII]];

soldif1 = NDSolve[{eqII == 0, theta[t1] == thet1, theta'[t1] == omeg1},
theta[t], {t, t1, .43}, MaxSteps → 10000,
Method → {EventLocator, "Event" → {theta'[t], theta[t] - Pi/6},
"EventAction" → {tm = t; thetam = theta[t];
omegam = theta'[t], t2 = t; theta2 = theta[t]; omega2 = theta'[t]}
}];
Print[" "]
Print["maximum compression"]
Print["tm = ", tm, " [s]"]
Print["θm = ", thetam, " [rad] = ", thetam*180/Pi, " deg"]
Print["ωm = ", Chop[omegam], " [rad/s]"]
Print[" "]
Print["end of restitution"]
Print["t2 = ", t2, " [s]"]
Print["θ2 = ", theta2, " [rad] = ", theta2*180/Pi, " deg"]
Print["ω2 = ", omega2, " [rad/s]"]

deltam = (Evaluate[delta /. soldif1] /. t → tm)[[1]];
Fem = (Evaluate[-k Evaluate[delta /. soldif1] /. t → tm][[1]]);
Print["deltam = ", deltam, " [m]"]
Print["Fem = ", Fem, " [N]"]

thetaplot1 =
Plot[Evaluate[theta[t] /. soldif1] * 180/Pi, {t, t1, t2}, AxesLabel → {"t[s]", "theta [deg]"}]
omegaplot1 = Plot[Evaluate[D[theta[t] /. soldif1, t]],
{t, t1, t2}, AxesLabel → {"t[s]", "omega [rad/s]"}]
alphaplot1 = Plot[Evaluate[D[theta[t] /. soldif1, {t, 2}]],
{t, t1, t2}, AxesLabel → {"t[s]", "alpha [rad/s^2]"}]

Plot[Evaluate[delta /. soldif1], {t, t1, t2},
AxesLabel → {"t[s]", "delta[m]"}, PlotRange → {{0, .5}, {0, -.07}}]

Plot[Evaluate[-k Evaluate[delta /. soldif1]], {t, t1, t2},
AxesLabel → {"t[s]", "Fe[N]"}, PlotRange → {{0, .5}, {0, 130}}]

(*-----*)
soldiff = NDSolve[{eqI == 0, theta[t2] == theta2, theta'[t2] == omega2}, theta,
{t, t2, Infinity},
Method → {EventLocator, "Event" → theta'[t]};

t3 = InterpolatingFunctionDomain[First[theta /. soldiff]][[1, -1]];
theta3 = (Evaluate[theta[t] /. soldiff] /. t → t3)[[1]];
omega3 = (Evaluate[D[theta[t] /. soldiff, t] /. t → t3)[[1]];

Print[" "]

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Print["t3 = ", t3, " [s]"]
Print["θ3 = ", theta3, " [rad] = ", theta2*180/Pi, " deg"]
Print["ω3 = ", omega3, " [rad/s]"]

thetaplotf =
  Plot[Evaluate[theta[t] /. soldiff] * 180/Pi, {t, t2, t3}, AxesLabel → {"t[s]", "theta[deg]"}]
omegaplotf = Plot[Evaluate[D[theta[t] /. soldiff, t]],
  {t, t2, t3}, AxesLabel → {"t[s]", "omega[rad/s]"}]
alphaplotf = Plot[Evaluate[D[theta[t] /. soldiff, {t, 2}]],
  {t, t2, t3}, AxesLabel → {"t[s]", "alpha[rad/s^2]"}]

(*-----*)
Print["all the graphics"]
alltheta = Show[thetaplot, thetaplot1, thetaplotf, PlotRange → Automatic]
allomega = Show[omegaplot, omegaplot1, omegaplotf, PlotRange → {{0, 0.8}, All}]

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Kinematics

angular acceleration of RB: $\alpha = \{0, 0, \theta''[t]\}$

position vector of C: $r_C = \left\{ \frac{1}{2} \cos[\theta[t]], \frac{1}{2} \sin[\theta[t]], 0 \right\}$

position vector of A: $r_A = \{\cos[\theta[t]], \sin[\theta[t]], 0\}$

Forces

gravitational force at C: $G = \{0, -19.62, 0\}$

elastic force at A: $F_e = \left\{ 0, -2000 \left(-\frac{1}{2} + \sin[\theta[t]] \right), 0 \right\}$

mass moment of inertia wrt C: $I_{Cz} = \frac{1}{6}$

mass moment of inertia wrt O: $I_{Oz} = I_{Cz} + m \left(\frac{L}{2} \right)^2 = \frac{2}{3}$

Summ of the moments in O at t -> t1:

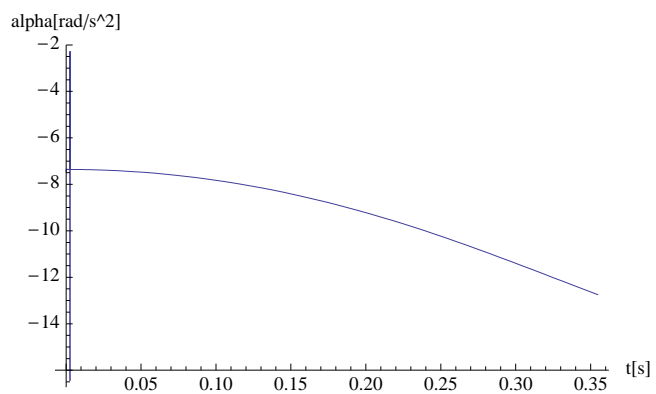
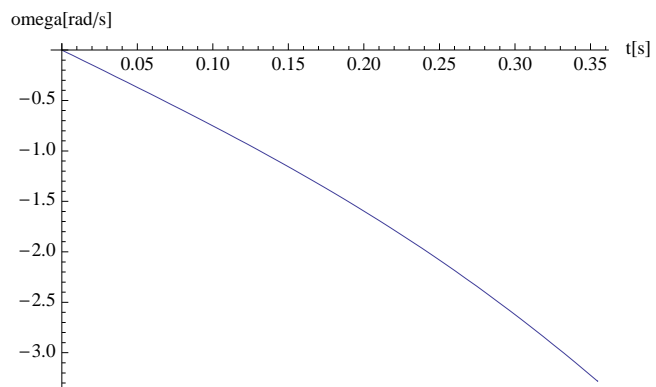
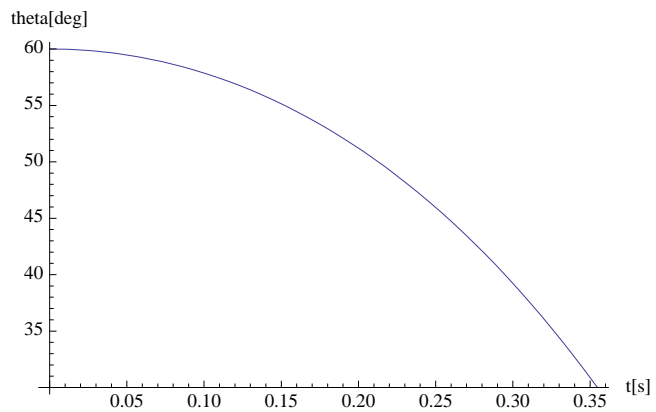
moment equation: $I_O \alpha = \sum M \text{ wrt } O = r_C \times G$

Solution: $\theta''[t] = -14.715 \cos[\theta[t]]$

$t_1 = 0.354702$ [s]

$\theta_1 = 0.523599$ [rad] = 30. [deg]

$\omega_1 = -3.28209$ [rad/s]



Summ of the moments in O at $t=t_1$:

moment equation at t_1 : $I_O \alpha = \sum M \text{ wrt } O = r_C \times G + r_A \times F_e$

Solution: $\theta''[t] = -1.5 \cos[\theta[t]] (-990.19 + 2000. \sin[\theta[t]])$

maximum compression

$t_m = 0.388673$ [s]

$\theta_m = 0.450007$ [rad] = 25.7835 deg

$\omega_m = 0$ [rad/s]

end of restitution

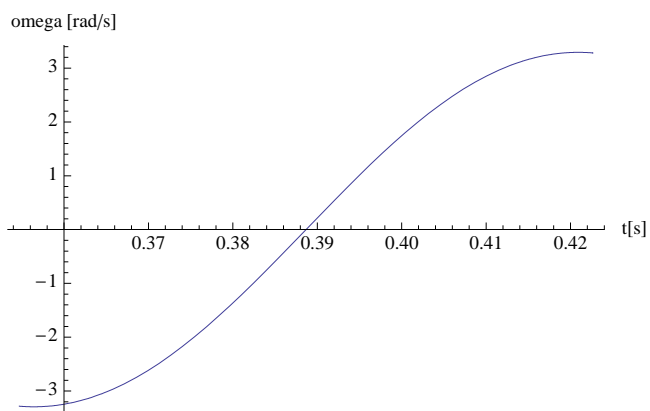
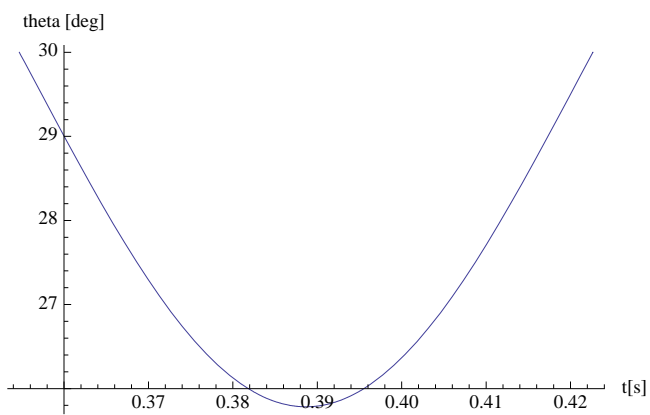
$t_2 = 0.422644$ [s]

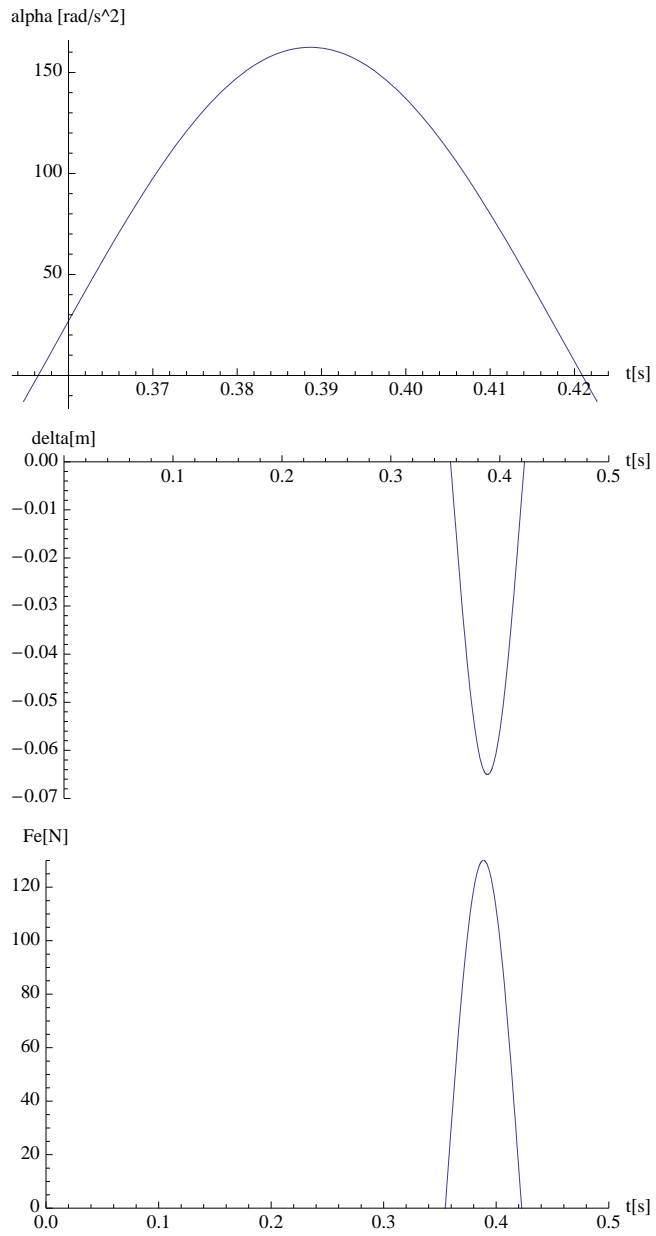
$\theta_2 = 0.523599$ [rad] = 30. deg

$\omega_2 = 3.28209$ [rad/s]

$\Delta x_m = -0.0650279$ [m]

$F_{e_m} = 130.056$ [N]

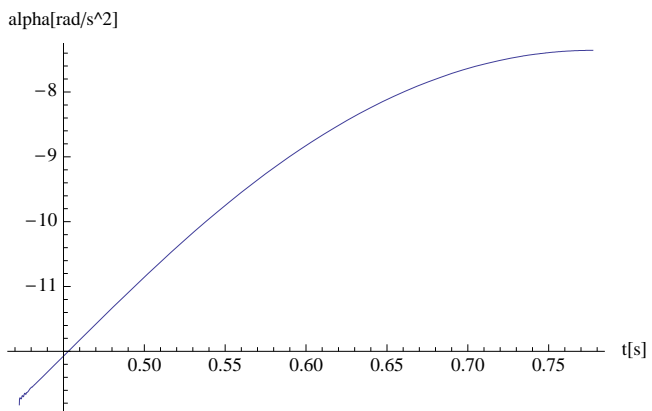
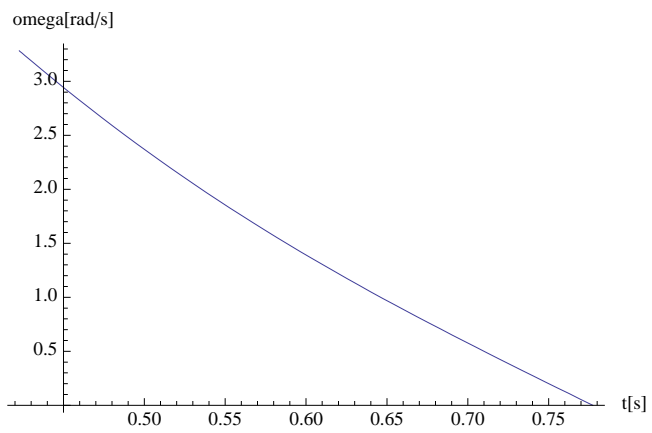
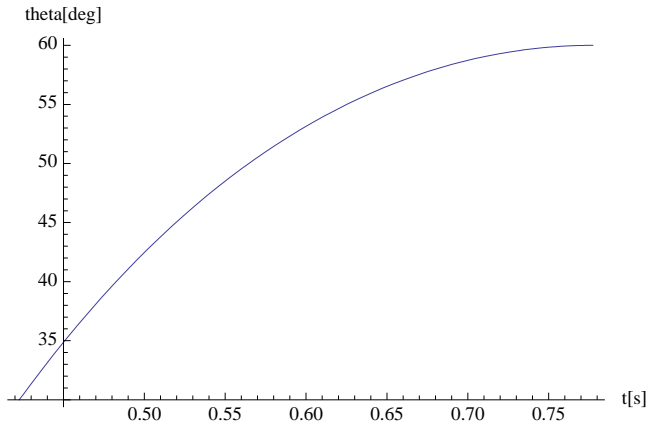




$$t_3 = 0.777346 \text{ [s]}$$

$$\theta_3 = 1.0472 \text{ [rad]} = 30. \text{ deg}$$

$$\omega_3 = -2.41474 \times 10^{-15} \text{ [rad/s]}$$



all the graphics

