

## Homework 12: Compound Pendulum

The figure depicts a uniform rod of mass  $m$  and length  $L$ . The figure is a “snapshot“ of the rod taken while the rod is moving. The “snapshot“ has been taken at an instant when the rod length makes an angle  $\theta(t)$  with the horizontal. The rod is connected to the ground by a pin joint. The pin is connected to the rod at a point that is a distance  $D$  from one end of the rod. The rod is free to swing in a vertical plane. A resisting frictional torque  $\mathbf{M}$  is acting at the pin and is proportional to the angular velocity of the rod  $\boldsymbol{\omega}(t)$ :  $\mathbf{M} = -k\boldsymbol{\omega}$ , where  $k$  is a constant. The local acceleration of gravity is  $g$ .

1) If the rod is released from rest at a position in which the angle  $\theta$  is equal to zero radians (rod length is horizontal), use the expressions developed above to determine the angular acceleration at the instant after the rod is released in terms of the rod mass,  $m$ , the rod length,  $L$  the pin location,  $D$ , the local acceleration of gravity,  $g$ , and  $k$ . Use the expressions developed above to determine the horizontal and vertical components of the force exerted by the pin on the rod at the instant after the rod is released in terms of the rod mass,  $m$ , the rod length,  $L$ , the pin position,  $D$ , the local acceleration of gravity,  $g$ , and  $k$ .

2) Find and solve the differential equation or equations describing the motion of the rod if the rod is released from rest at  $\theta = 0$ . This equation or these equations should involve the angle  $\theta(t)$ , various time derivatives of that angle, and the parameters  $L$ ,  $D$ ,  $g$ ,  $m$ , and  $k$ .

Numerical values:  $L = 1$  m,  $D = 0.01$  m,  $g = 10$  m/s<sup>2</sup>,  $m = 1$  kg, and  $k = 10$  kg m<sup>2</sup>/s.

