

1 Dynamic Force Analysis

1.1 Joint Forces Analysis

The R-RTR mechanism shown in Fig. 1.1 has the dimensions: $AB = 0.14$ m, $AC = 0.06$ m, and $CF = 0.2$ m. The driver link 1 makes an angle $\phi = \phi_1 = \frac{\pi}{3}$ rad with the horizontal axis and rotates with a constant speed of $n = n_1 = 30\pi$ rpm. The position vectors of the points A , B , C , and F are

$$\begin{aligned}\mathbf{r}_A &= 0\mathbf{i} + 0\mathbf{j} \text{ m,} \\ \mathbf{r}_B = \mathbf{r}_{C_2} &= x_B\mathbf{i} + y_B\mathbf{j} = 0.07\mathbf{i} + 0.121\mathbf{j} \text{ m,} \\ \mathbf{r}_C &= x_C\mathbf{i} + y_C\mathbf{j} = 0\mathbf{i} + 0.06\mathbf{j} \text{ m,} \\ \mathbf{r}_F &= x_F\mathbf{i} + y_F\mathbf{j} = 0.150\mathbf{i} + 0.191\mathbf{j} \text{ m.}\end{aligned}$$

The position vectors of the mass centers of links 1 and 3 are

$$\begin{aligned}\mathbf{r}_{C_1} &= x_{C_1}\mathbf{i} + y_{C_1}\mathbf{j} = \frac{x_B}{2}\mathbf{i} + \frac{y_B}{2}\mathbf{j} = 0.035\mathbf{i} + 0.06\mathbf{j} \text{ m,} \\ \mathbf{r}_{C_3} &= x_{C_3}\mathbf{i} + y_{C_3}\mathbf{j} = \frac{x_C + x_F}{2}\mathbf{i} + \frac{y_C + y_F}{2}\mathbf{j} = 0.075\mathbf{i} + 0.125\mathbf{j} \text{ m.}\end{aligned}$$

The mass center of the slider 2 is at B ($B = C_2$): $\mathbf{r}_{C_2} = \mathbf{r}_B$.

The height of the links 1 and 3 is $h = 0.01$ m. The width of the slider 2 is $w_{Slider} = 0.05$ m, and the height is $h_{Slider} = 0.020$ m. All three moving links are rectangular prisms with the depth $d = 0.01$ m.

The density of the material is $\rho_{Steel} = \rho = 8000$ kg/m³. The gravitational acceleration is $g = 9.807$ m/s².

Forces and moments on each link

Link 1

The mass of the link 1 is

$$m_1 = \rho AB h d = 0.112 \text{ kg.}$$

The acceleration of C_1 is

$$\mathbf{a}_{C_1} = -3.40932\mathbf{i} - 5.90511\mathbf{j} \text{ m/s}^2.$$

The term $m_1 \mathbf{a}_{C_1}$ is

$$m_1 \mathbf{a}_{C_1} = -0.381844\mathbf{i} - 0.661373\mathbf{j} \text{ N.}$$

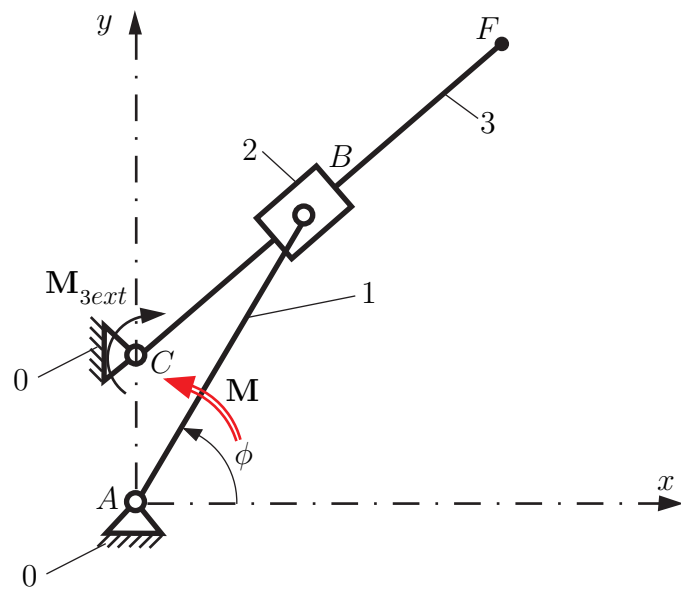


Fig. 1

The gravitational force on link 1 is

$$\mathbf{G}_1 = -m_1 g \mathbf{j} = -1.09838 \mathbf{j} \text{ N.}$$

The mass moment of inertia of link 1 about C_1 is

$$I_{C_1} = m_1 (AB^2 + h^2)/12 = 0.000183867 \text{ kg} \cdot \text{m}^2.$$

The term $I_{C_1} \boldsymbol{\alpha}_1$ is

$$I_{C_1} \boldsymbol{\alpha}_1 = \mathbf{0}.$$

Link 2

The mass of the link 2 is

$$m_2 = \rho_2 h_{Slider} w_{Slider} d = 0.08 \text{ kg.}$$

The acceleration of $C_2 = B$ is

$$\mathbf{a}_{C_2} = -6.81864 \mathbf{i} - 11.8102 \mathbf{j} \text{ m/s}^2.$$

The term $m_2 C_2$ is

$$m_2 \mathbf{a}_{C_2} = -0.545491 \mathbf{i} - 0.944818 \mathbf{j} \text{ N.}$$

The gravitational force is

$$\mathbf{G}_2 = -m_2 g \mathbf{j} = -0.78456 \mathbf{j} \text{ N.}$$

The mass moment of inertia of link 2 about C_2 is

$$I_{C_2} = m_2 (h_{Slider}^2 + w_{Slider}^2)/12 = 0.0000193333 \text{ kg} \cdot \text{m}^2.$$

The term $I_{C_2} \boldsymbol{\alpha}_2$ is

$$I_{C_2} \boldsymbol{\alpha}_2 = 0.00169109 \mathbf{k} \text{ N} \cdot \text{m.}$$

where $\boldsymbol{\alpha}_2 = 87.47 \mathbf{k} \text{ rad/s}^2$.

Link 3

The mass of the link 3 is

$$m_3 = \rho CF h d = 0.112 \text{ kg.}$$

The acceleration of C_3 is

$$\mathbf{a}_{C_3} = -20.6416 \mathbf{i} - 6.4373 \mathbf{j} \text{ m/s}^2.$$

The term $m_3 \mathbf{a}_{C_3}$ is

$$m_3 \mathbf{a}_{C_3} = -3.30266 \mathbf{i} - 1.02997 \mathbf{j} \text{ N}.$$

The gravitational force is

$$\mathbf{G}_3 = -m_3 g \mathbf{j} = -1.56912 \mathbf{j} \text{ N}.$$

The mass moment of inertia of link 3 about C_3 is

$$I_{C_3} = m_3 (CF^2 + h^2)/12 = 0.000534667 \text{ kg} \cdot \text{m}^2.$$

The term $I_{C_3} \boldsymbol{\alpha}_3$ is

$$I_{C_3} \boldsymbol{\alpha}_3 = 0.0467673 \mathbf{k} \text{ N} \cdot \text{m}.$$

where $\boldsymbol{\alpha}_3 = \boldsymbol{\alpha}_2 = 87.47 \mathbf{k} \text{ rad/s}^2$.

The external moment on link 3 is

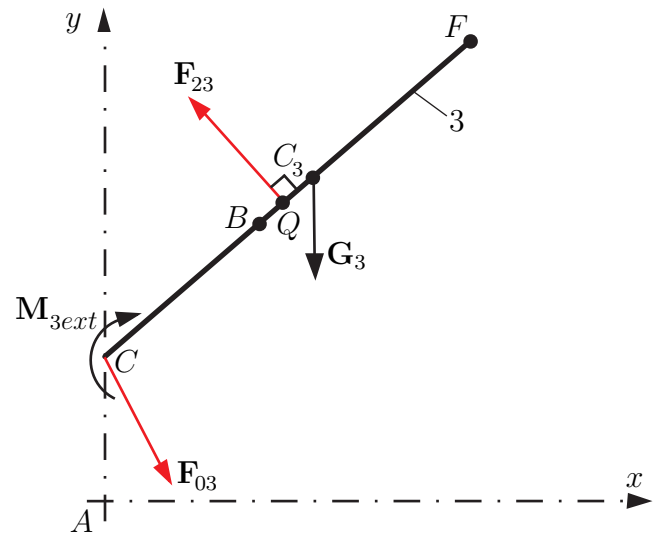
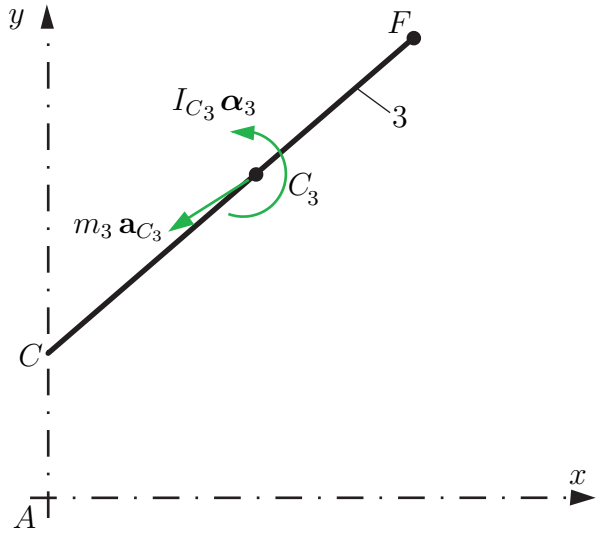
$$\mathbf{M}_{3ext} = -\text{Sign}(\boldsymbol{\omega}_3) |M_{ext}| \mathbf{k} = -1000 \mathbf{k} \text{ N} \cdot \text{m},$$

where $\boldsymbol{\omega}_3 = 14.0619 \mathbf{k} \text{ rad/s}$ and $M_{ext} = |M_{ext}| = 1000 \text{ N}$

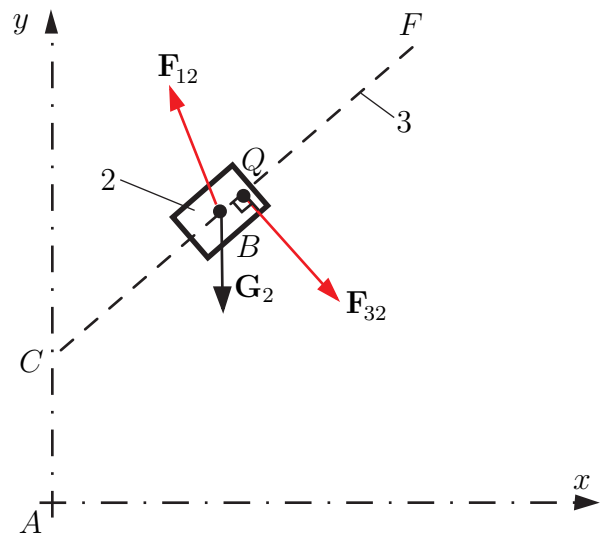
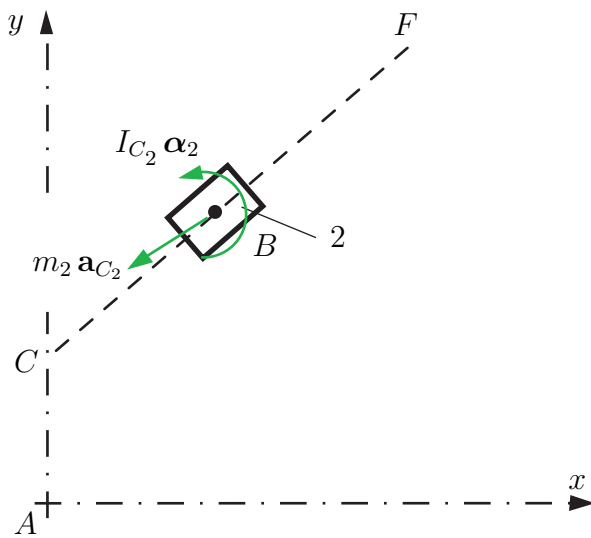
Determine the moment \mathbf{M} that acts on link 1 required for dynamic equilibrium and the joint forces for the mechanism.

NEWTON-EULER

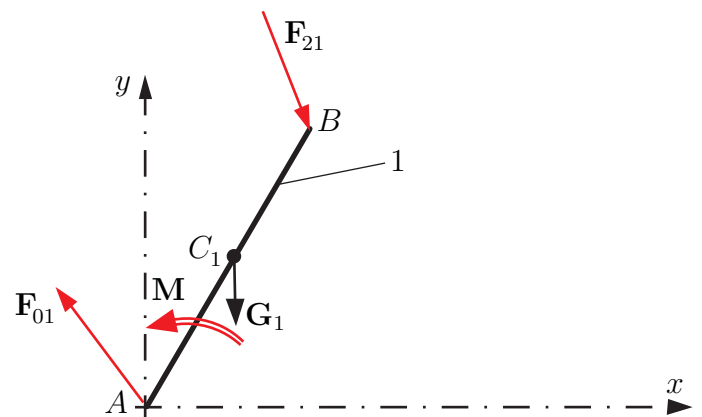
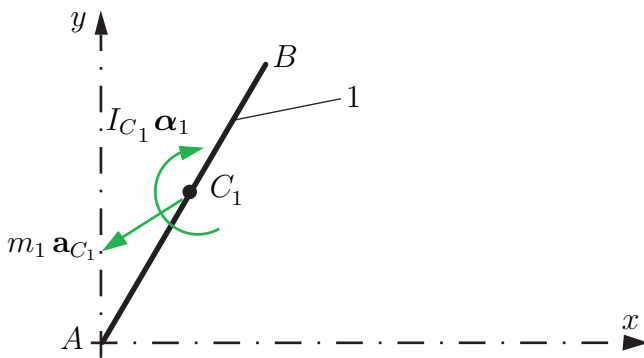
FBD



(a)



(b)



(c)

Fig. 2

Newton-Euler equations of motion

For each moving link the Newton-Euler equations of motion are written

$$m_i \mathbf{a}_{C_i} = \sum \mathbf{F}^{(i)} \quad \text{and} \quad I_{C_i} \boldsymbol{\alpha}_i = \sum \mathbf{M}_{C_i}^{(i)},$$

where C_i is the center of mass of the link i .

The position vectors are

$$\begin{aligned} \mathbf{r}_A &= 0\mathbf{i} + 0\mathbf{j} \text{ m,} \\ \mathbf{r}_B &= \mathbf{r}_{C_2} = x_B\mathbf{i} + y_B\mathbf{j} = 0.07\mathbf{i} + 0.121\mathbf{j} \text{ m,} \\ \mathbf{r}_C &= x_C\mathbf{i} + y_C\mathbf{j} = 0\mathbf{i} + 0.06\mathbf{j} \text{ m,} \\ \mathbf{r}_F &= x_F\mathbf{i} + y_F\mathbf{j} = 0.150\mathbf{i} + 0.191\mathbf{j} \text{ m,} \\ \mathbf{r}_{C_1} &= x_{C_1}\mathbf{i} + y_{C_1}\mathbf{j} = 0.035\mathbf{i} + 0.06\mathbf{j} \text{ m,} \\ \mathbf{r}_{C_3} &= x_{C_3}\mathbf{i} + y_{C_3}\mathbf{j} = 0.075\mathbf{i} + 0.125\mathbf{j} \text{ m.} \end{aligned}$$

The terms $m_i \mathbf{a}_{C_i}$ are

$$\begin{aligned} m_1 \mathbf{a}_{C_1} &= -0.381844\mathbf{i} - 0.661373\mathbf{j} \text{ N,} \\ m_2 \mathbf{a}_{C_2} &= -0.545491\mathbf{i} - 0.944818\mathbf{j} \text{ N,} \\ m_3 \mathbf{a}_{C_3} &= -3.30266\mathbf{i} - 1.02997\mathbf{j} \text{ N.} \end{aligned}$$

The terms $I_{C_i} \boldsymbol{\alpha}_i$ are

$$\begin{aligned} I_{C_1} \boldsymbol{\alpha}_1 &= \mathbf{0} \text{ N} \cdot \text{m,} \\ I_{C_2} \boldsymbol{\alpha}_2 &= 0.00169109\mathbf{k} \text{ N} \cdot \text{m,} \\ I_{C_3} \boldsymbol{\alpha}_3 &= 0.0467673\mathbf{k} \text{ N} \cdot \text{m.} \end{aligned}$$

The force analysis will start with link 3 because the moment \mathbf{M}_{3ext} is known.

Link 3

For the link 3, Fig. 1.2(a), the equations of motion for planar motion give

$$\begin{aligned} m_3 \mathbf{a}_{C_3} &= \sum \mathbf{F}^{(3)} = \mathbf{F}_{03} + \mathbf{G}_3 + \mathbf{F}_{23}, \\ I_{C_3} \boldsymbol{\alpha}_3 &= \sum \mathbf{M}_{C_3}^{(3)} = \mathbf{r}_{C_3C} \times \mathbf{F}_{03} + \mathbf{r}_{C_3Q} \times \mathbf{F}_{23} + \mathbf{M}_{3ext}, \end{aligned}$$

or

$$\begin{aligned}
m_3 a_{C_3x} &= F_{03x} + F_{23x}, \\
m_3 a_{C_3y} &= F_{03y} - m_3 g + F_{23y}, \\
I_{C_3} \alpha_3 \mathbf{k} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_C - x_{C3} & y_C - y_{C3} & 0 \\ F_{03x} & F_{03y} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_Q - x_{C3} & y_Q - y_{C3} & 0 \\ F_{23x} & F_{23y} & 0 \end{vmatrix} \\
&+ M_{3ext} \mathbf{k},
\end{aligned}$$

or

$$\begin{aligned}
m_3 a_{C_3x} &= F_{03x} + F_{23x}, \\
m_3 a_{C_3y} &= F_{03y} - m_3 g + F_{23y}, \\
I_{C_3} \alpha_3 &= (x_C - x_{C3})F_{03y} - (y_C - y_{C3})F_{03x} \\
&+ (x_Q - x_{C3})F_{23y} - (y_Q - y_{C3})F_{23x} + M_{3ext},
\end{aligned}$$

where the unknowns are

$$\mathbf{F}_{03} = F_{03x} \mathbf{i} + F_{03y} \mathbf{j}, \quad \mathbf{F}_{23} = F_{23x} \mathbf{i} + F_{23y} \mathbf{j},$$

and the position vector $\mathbf{r}_Q = x_Q \mathbf{i} + y_Q \mathbf{j}$ of the application point of the joint force \mathbf{F}_{23} .

Numerically the previous system of equations becomes

$$-3.30266 = F_{03x} + F_{23x}, \quad (1.1)$$

$$-1.02997 = -1.56912 + F_{03y} + F_{23y}, \quad (1.2)$$

$$\begin{aligned}
0 &= -0.065F_{03x} - 0.075F_{03y} + 0.125F_{23x} - 0.075F_{23y} + \\
&F_{23y}x_Q - F_{23x}y_Q - 1000.
\end{aligned} \quad (1.3)$$

The application point Q of the joint force \mathbf{F}_{23} is on the line BC :

$$\frac{y_B - y_C}{x_B - x_C} = \frac{y_Q - y_C}{x_Q - x_C} \quad \text{or} \quad 0.874 - \frac{y_Q - 0.06}{x_Q} = 0. \quad (1.4)$$

The joint force \mathbf{F}_{23} is perpendicular to the sliding direction BC :

$$\mathbf{F}_{23} \cdot \mathbf{r}_{BC} = 0 \quad \text{or} \quad -0.07F_{23x} - 0.061F_{23y} = 0. \quad (1.5)$$

There are five scalar equations, Eqs. (1.1) through (1.5), and six unknowns, F_{03x} , F_{03y} , F_{23x} , F_{23y} , x_Q , y_Q . The force analysis will continue with link 2.

Link 2

The Newton-Euler equations for slider 2, Fig. 1.2(b), give

$$\begin{aligned} m_2 \mathbf{a}_{C_2} &= \sum \mathbf{F}^{(2)} = \mathbf{F}_{12} + \mathbf{G}_2 + \mathbf{F}_{32}, \\ I_{C_2} \boldsymbol{\alpha}_2 &= \sum \mathbf{M}_{C_2}^{(2)} = \mathbf{r}_{BQ} \times \mathbf{F}_{32}, \end{aligned}$$

or

$$\begin{aligned} m_2 a_{C_2x} &= F_{12x} - F_{23x}, \\ m_2 a_{C_2y} &= F_{12y} - m_2 g - F_{23y}, \\ I_{C_2} \alpha_2 \mathbf{k} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_Q - x_B & y_Q - y_B & 0 \\ -F_{23x} & -F_{23y} & 0 \end{vmatrix}, \end{aligned}$$

or

$$\begin{aligned} m_2 a_{C_2x} &= F_{12x} - F_{23x}, \\ m_2 a_{C_2y} &= F_{12y} - m_2 g - F_{23y}, \\ I_{C_2} \alpha_2 &= (x_Q - x_B)(-F_{23y}) - (y_Q - y_B)(-F_{23x}), \end{aligned}$$

where the new unknown is introduced (the reaction of link 1 on link 2):

$$\mathbf{F}_{12} = F_{12x} \mathbf{i} + F_{12y} \mathbf{j}.$$

Numerically, the previous equations becomes

$$-0.545491 = F_{12x} - F_{23x}, \quad (1.6)$$

$$-0.944818 = -0.78456 + F_{12y} - F_{23y}, \quad (1.7)$$

$$0.00169109 = -0.121244 F_{23x} + 0.07 F_{23y} - x_Q F_{23y} + y_Q F_{23x} = 0. \quad (1.8)$$

Now there is a system of eight scalar equations, Eqs. (1.1) through (1.8), eight unknowns, and the solution is

$$\begin{aligned} \mathbf{F}_{03} &= F_{03x} \mathbf{i} + F_{03y} \mathbf{j} = 7078.41 \mathbf{i} - 8093.7 \mathbf{j} \quad \text{N}, \\ \mathbf{F}_{23} &= F_{23x} \mathbf{i} + F_{23y} \mathbf{j} = -7081.72 \mathbf{i} + 8094.24 \mathbf{j} \quad \text{N}, \\ \mathbf{F}_{12} &= F_{12x} \mathbf{i} + F_{12y} \mathbf{j} = -7082.26 \mathbf{i} + 8094.08 \mathbf{j} \quad \text{N}, \\ \mathbf{r}_Q &= x_Q \mathbf{i} + y_Q \mathbf{j} = 0.069 \mathbf{i} + 0.121 \mathbf{j} \quad \text{m}. \end{aligned}$$

Link 1

The force equation for the driver link 1, Fig. 1.2(c), gives

$$m_1 \mathbf{a}_{C_1} = \sum \mathbf{F}^{(1)} = \mathbf{F}_{21} + \mathbf{G}_1 + \mathbf{F}_{01}.$$

The reaction of the ground 0 on the link 1 is

$$\mathbf{F}_{01} = m_1 \mathbf{a}_{C_1} + \mathbf{F}_{12} - \mathbf{G}_1 = -7082.64 \mathbf{i} + 8094.52 \mathbf{j} \text{ N}.$$

The sum of the moments about the mass center C_1 for link 1 gives the equilibrium moment

$$I_{C_1} \alpha_1 \mathbf{k} = \sum \mathbf{M}_{C_1}^{(1)} = \mathbf{r}_{C_1B} \times \mathbf{F}_{21} + \mathbf{r}_{C_1A} \times \mathbf{F}_{01} + \mathbf{M},$$

or

$$\begin{aligned} \mathbf{M} &= \mathbf{r}_{C_1B} \times \mathbf{F}_{12} - \mathbf{r}_{C_1A} \times \mathbf{F}_{01} \\ &= 712.632 \mathbf{k} + 712.671 \mathbf{k} = 1425.3 \mathbf{k} \text{ N} \cdot \text{m}. \end{aligned}$$

Another way of calculating the moment \mathbf{M} required for dynamic equilibrium is to write the moment equation of motion for link 1 about the fixed point A

$$I_A \alpha_1 \mathbf{k} = \sum \mathbf{M}_A^{(1)} = \mathbf{r}_{AC_1} \times \mathbf{G}_1 + \mathbf{r}_{AB} \times \mathbf{F}_{21} + \mathbf{M} \implies \mathbf{M} = \mathbf{r}_B \times \mathbf{F}_{12} - \mathbf{r}_{C_1} \times \mathbf{G}_1,$$

where $I_A = I_{C_1} + m_1 (AB/2)^2$.