R-TRR mechanism

The following dimensions are given for the mechanism shown in the Fig. 1(a): $AC = a = 0.100$ m and $BC = 0.300$ m. The angle of the driver link 1 with the horizontal axis is $\phi = \phi_1 = 45°$. The coordinates of joint B are $x_B = y_B = 0.256$ m. The driver link 1 rotates with a constant speed of $n_1 = 30$ rpm. Find the velocities and the accelerations of the mechanism.

Solution

A cartesian reference frame with the origin at A is selected.
The coordinates of joint A are

$$x_A = y_A = 0.$$

the coordinates of the joint C are

$$x_C = AC = 0.100 \text{ m} \quad \text{and} \quad y_C = 0,$$

and the coordinates of joint B are

$$x_B = 0.256 \,\text{m} \quad \text{and} \quad y_B = 0.256 \,\text{m}.$$

The position of joint B was calculated from the equations

$$\tan\phi = \frac{y_B}{x_B} \quad \text{and} \quad (x_B - x_C)^2 + (y_B - y_C)^2 = BC^2.$$

The magnitude of the angular velocity of the driver link 1 is

$$\omega = \omega_1 = \dot{\phi} = \frac{\pi\, n_1}{30} = \frac{\pi\,(30 \text{ rpm})}{30} = 3.141 \text{ rad/s}. \tag{1}$$

The angular velocity of link 1 is

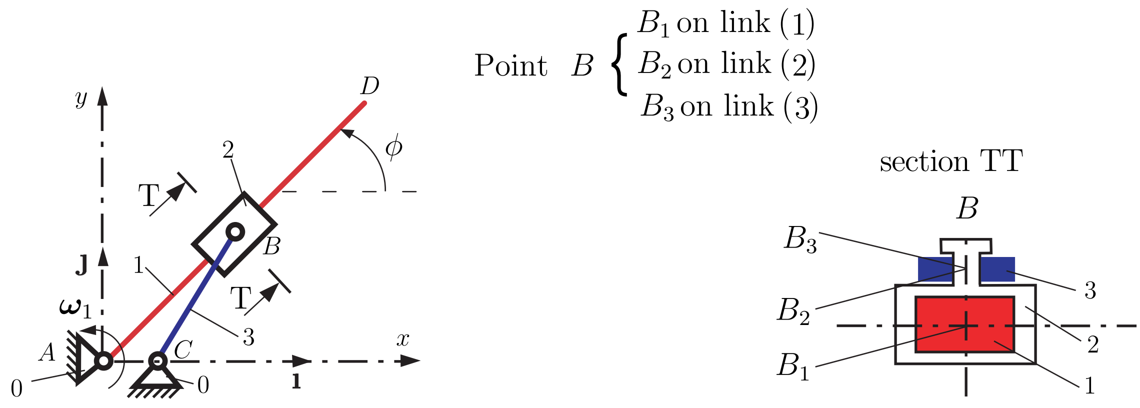
$$\boldsymbol{\omega} = \boldsymbol{\omega}_1 = \omega\,\mathbf{k} = 3.141\,\mathbf{k} \quad \text{rad/s}.$$

The link 2 and the driver link 1 have the same angular velocity $\boldsymbol{\omega}_1 = \boldsymbol{\omega}_2$.
The angular acceleration of link 1 is $\boldsymbol{\alpha}_1 = \dot{\boldsymbol{\omega}}_1 = \mathbf{0}$.
The velocity of the point B_1 on the link 1 is

$$\mathbf{v}_{B_1} = \mathbf{v}_A + \boldsymbol{\omega}_1 \times \mathbf{r}_B = \boldsymbol{\omega}_1 \times \mathbf{r}_B,$$

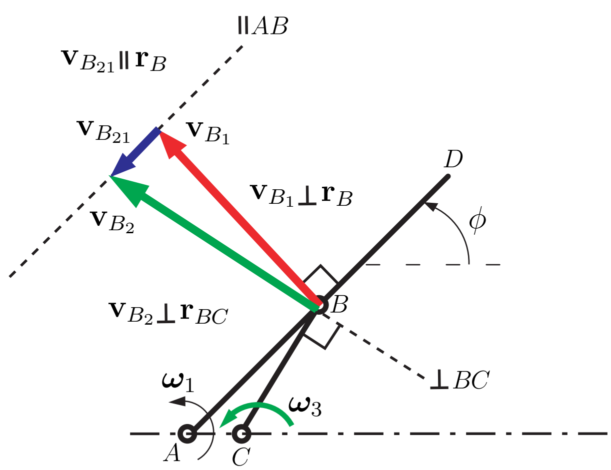
where $\mathbf{v}_A \equiv \mathbf{0}$ is the velocity of the origin $A \equiv O$.
The position vector of point B is

$$\mathbf{r}_B = x_B\boldsymbol{\imath} + y_B\boldsymbol{\jmath} = 0.256\boldsymbol{\imath} + 0.256\boldsymbol{\jmath} \;\; \text{m}.$$

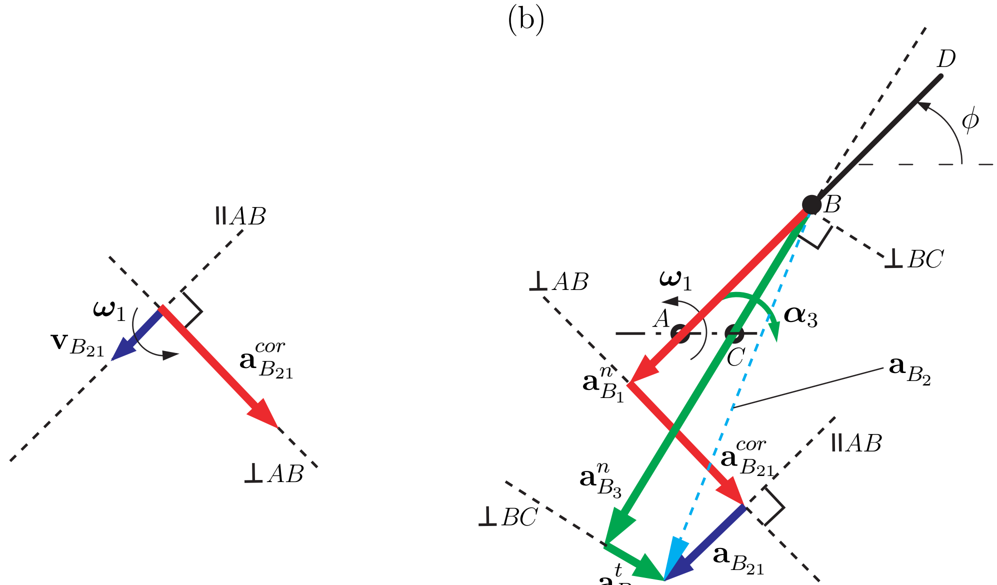


(a)

DRAWING NOT TO SCALE



(b)



(c)

Figure 1

The velocity of B_1, Fig. 1(b), is

$$\mathbf{v}_{B_1} = \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & \omega \\ x_B & y_B & 0 \end{vmatrix} = \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & 3.141 \\ 0.256 & 0.256 & 0 \end{vmatrix} = -0.804\boldsymbol{\imath} + 0.804\boldsymbol{\jmath} \;\; \text{m/s},$$

and has the magnitude is $|\mathbf{v}_{B_1}| = v_{B_1} = 1.138$ m/s.

The acceleration of the point B_1 on the link 1, Fig 1(c), is

$$\mathbf{a}_{B_1} = \mathbf{a}_A + \boldsymbol{\alpha}_1 \times \mathbf{r}_B + \boldsymbol{\omega}_1 \times (\boldsymbol{\omega}_1 \times \mathbf{r}_B) = \boldsymbol{\alpha}_1 \times \mathbf{r}_B - \boldsymbol{\omega}_1^2 \mathbf{r}_B$$
$$= -\boldsymbol{\omega}_1^2 \mathbf{r}_B = -3.141^2(0.256\boldsymbol{\imath} + 0.256\boldsymbol{\jmath}) = -2.528\boldsymbol{\imath} - 2.528\boldsymbol{\jmath} \;\; \text{m/s}^2.$$

For this case when $\mathbf{a}_A = \mathbf{0}$ and $\boldsymbol{\alpha}_1 = \mathbf{0}$, $\mathbf{a}_{B_1} = \mathbf{a}^n_{B_1}$. The magnitude of the acceleration of the point B_1 is
$|\mathbf{a}_{B_1}| = a_{B_1} = a^n_{B_1} = 3.575 \;\; \text{m/s}^2$.

The velocity of the point B_2 on the link 2 is equal to the velocity of the point B_3 on the link 3 (link 2 and link 3 are connected with rotational joint).

The points B_3 and C are on the link 3 and

$$\mathbf{v}_{B_2} = \mathbf{v}_{B_3} = \mathbf{v}_C + \boldsymbol{\omega}_3 \times \mathbf{r}_{CB} = \boldsymbol{\omega}_3 \times (\mathbf{r}_B - \mathbf{r}_C), \tag{2}$$

where $\mathbf{v}_C \equiv \mathbf{0}$ and the angular velocity of link 3 is

$$\boldsymbol{\omega}_3 = \omega_3 \mathbf{k}.$$

The velocity of the point B_2 on the link 2 is calculated in terms of the velocity of the point B_1 on the link 1

$$\mathbf{v}_{B_2} = \mathbf{v}_{B_1} + \mathbf{v}^{rel}_{B_2 B_1} = \mathbf{v}_{B_1} + \mathbf{v}_{B_{21}}, \tag{3}$$

where $\mathbf{v}^{rel}_{B_2 B_1} = \mathbf{v}_{B_{21}}$ is the relative acceleration of B_2 with respect to B_1 on link 1. This relative velocity is parallel to the sliding direction AB, $\mathbf{v}_{B_{21}} || AB$, or

$$\mathbf{v}_{B_{21}} = v_{B_{21}} \cos\phi_1 \boldsymbol{\imath} + v_{B_{21}} \sin\phi_1 \boldsymbol{\jmath}, \tag{4}$$

where $\phi_1 = 45°$. Equations (2), (3), and (4) give

$$\begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & \omega_3 \\ x_B - x_C & y_B - y_C & 0 \end{vmatrix} = \mathbf{v}_{B_1} + v_{B_{21}} \cos\phi_1 \boldsymbol{\imath} + v_{B_{21}} \sin\phi_1 \boldsymbol{\jmath}. \tag{5}$$

Equation (5) represents a vectorial equations with two scalar components on x-axis and y-axis and with two unknowns ω_3 and $v_{B_{21}}$

$$
\begin{aligned}
-\omega_3(y_B - y_C) &= v_{B_{1x}} + v_{B_{21}} \cos\phi_1, \\
\omega_3(x_B - x_C) &= v_{B_{1y}} + v_{B_{21}} \sin\phi_1,
\end{aligned}
$$

or

$$
\begin{aligned}
-\omega_3(0.256 - 0) &= -0.804 + v_{B_{21}} \cos 45^\circ, \\
\omega_3(0.256 - 0.1) &= 0.804 + v_{B_{21}} \sin 45^\circ.
\end{aligned}
$$

It results

$$\omega_3 = 3.903 \text{ rad/s} \text{ and } v_{B_{21}} = -0.276 \text{ m/s},$$

or in vectorial form

$$\boldsymbol{\omega}_3 = 3.903\mathbf{k} \text{ rad/s}$$

and the relative velocity vector of B_2 with respect to B_1 is

$$\mathbf{v}_{B_{21}} = -0.195\,\boldsymbol{\imath} - 0.195\,\boldsymbol{\jmath} \text{ m/s}.$$

The velocity of B_3 (or B_2), Fig 1(b), is

$$\mathbf{v}_{B_2} = \mathbf{v}_{B_3} = \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & 3.903 \\ 0.256 - 0.1 & 0.256 & 0 \end{vmatrix} = -0.999\boldsymbol{\imath} + 0.609\boldsymbol{\jmath} \text{ m/s}.$$

The magnitude of the velocity of the point $B_3 = B_2$ is $|\mathbf{v}_{B_2}| = v_{B_2} = 1.171$ m/s.

The acceleration of the point B_1 on the link 1 is $\mathbf{a}_{B_1} = -2.528\boldsymbol{\imath} - 2.528\boldsymbol{\jmath}$ m/s^2.

The points B_3 and C are on the link 3 and

$$\mathbf{a}_{B_2} = \mathbf{a}_{B_3} = \mathbf{a}_C + \boldsymbol{\alpha}_3 \times \mathbf{r}_{CB} - \omega_3^2\mathbf{r}_{CB} = \boldsymbol{\alpha}_3 \times \mathbf{r}_{CB} - \omega_3^2\mathbf{r}_{CB}, \tag{6}$$

where $\mathbf{a}_C \equiv \mathbf{0}$ and the angular acceleration of link 3 is

$$\boldsymbol{\alpha}_3 = \alpha_3\mathbf{k}.$$

The acceleration of the point B_2 on the link 2 is calculated in terms of the acceleration of the point B_1 on the link 1

$$\mathbf{a}_{B_2} = \mathbf{a}_{B_1} + \mathbf{a}^{rel}_{B_2 B_1} + \mathbf{a}^{cor}_{B_2 B_1} = \mathbf{a}_{B_1} + \mathbf{a}_{B_{21}} + \mathbf{a}^{cor}_{B_{21}}, \tag{7}$$

where $\mathbf{a}^{rel}_{B_2 B_1} = \mathbf{a}_{B_{21}}$ is the relative acceleration of B_2 with respect to B_1 on link 1. This relative acceleration is parallel to the sliding direction AB, $\mathbf{a}_{B_{21}} || AB$, or

$$\mathbf{a}_{B_{21}} = a_{B_{21}} \cos\phi_1 \mathbf{\imath} + a_{B_{21}} \sin\phi_1 \mathbf{\jmath}. \tag{8}$$

The Coriolis acceleration of B_2 realative to B_1, Fig 1(c), is

$$\mathbf{a}^{cor}_{B_{21}} = 2\ \boldsymbol{\omega}_1 \times \mathbf{v}_{B_{21}} = 2\ \boldsymbol{\omega}_2 \times \mathbf{v}_{B_{21}} = 2 \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ 0 & 0 & \omega_1 \\ v_{B_{21}} \cos\phi_1 & v_{B_{21}} \sin\phi_1 & 0 \end{vmatrix} =$$
$$2(-\omega_1 v_{B_{21}} \sin\phi_1 \mathbf{\imath} + \omega_1 v_{B_{21}} \cos\phi_1 \mathbf{\jmath}) =$$
$$2[-3.141(-0.276)\sin 45^\circ \mathbf{\imath} + 3.141(-0.276)\cos 45^\circ \mathbf{\jmath}] =$$
$$1.226\,\mathbf{\imath} - 1.226\,\mathbf{\jmath} \ \ \text{m/s}^2. \tag{9}$$

The magnitude of the Coriolis acceleration of the point B_2 relative to B_1 is $|\mathbf{a}^{cor}_{B_{21}}| = a^{cor}_{B_{21}} = 1.734 \ \ \text{m/s}^2$.

Equations (6), (7), (8), and (9) give

$$\begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ 0 & 0 & \alpha_3 \\ x_B - x_C & y_B - y_C & 0 \end{vmatrix} - \omega_3^2(\mathbf{r}_B - \mathbf{r}_C) =$$
$$\mathbf{a}_{B_1} + a_{B_{21}}(\cos\phi_1 \mathbf{\imath} + \sin\phi_1 \mathbf{\jmath}) + 2\ \boldsymbol{\omega}_1 \times \mathbf{v}_{B_{21}}. \tag{10}$$

Equation (10) represents a vectorial equations with two scalar components on x-axis and y-axis and with two unknowns α_3 and $a_{B_{21}}$

$$-\alpha_3(y_B - y_C) - \omega_3^2(x_B - x_C) = a_{B_{1x}} + a_{B_{21}} \cos\phi_1 - 2\omega_1 v_{B_{21}} \sin\phi_1,$$
$$\alpha_3(x_B - x_C) - \omega_3^2(y_B - y_C) = a_{B_{1y}} + a_{B_{21}} \sin\phi_1 + 2\omega_1 v_{B_{21}} \cos\phi_1,$$

or

$$-\alpha_3(0.256 - 0) - 3.903^2(0.256 - 0.1) =$$
$$-2.528 + a_{B_{21}} \cos 45^\circ + 1.226,$$
$$\alpha_3(0.256 - 0.1) - 3.903^2(0.256 - 0) =$$
$$-2.528 + a_{B_{21}} \sin 45^\circ - 1.226.$$

It results

$$\alpha_3 = -2.252 \text{ rad/s}^2 \quad \text{and} \quad a_{B_{21}} = -0.707 \text{ m/s}^2.$$

The relative acceleration of B_2 with respect to B_1, Fig 1(c), is

$$\mathbf{a}_{B_{21}} = -0.707 \cos 45^\circ \boldsymbol{\imath} - 0.707 \sin 45^\circ \boldsymbol{\jmath} = -0.500\boldsymbol{\imath} - 0.500\boldsymbol{\jmath},$$

and the acceleration of B_3 is

$$\mathbf{a}_{B_2} = \mathbf{a}_{B_3} = \boldsymbol{\alpha}_3 \times \mathbf{r}_{CB} - \omega_3^2 \mathbf{r}_{CB} =$$
$$-2.252\mathbf{k} \times [(0.256 - 0.1)\boldsymbol{\imath} + (0.256 - 0)\boldsymbol{\jmath}] - 3.903^2[(0.256 - 0.1)\boldsymbol{\imath} + (0.256 - 0)\boldsymbol{\jmath}] =$$
$$-1.802\boldsymbol{\imath} - 4.25\boldsymbol{\jmath} \text{ m/s}^2.$$

The magnitude of the acceleration of the point B_3 is
$|\mathbf{a}_{B_3}| = a_{B_2} = 4.620 \text{ m/s}^2$.
The normal acceleration of B_3 is

$$\mathbf{a}^n_{B_2} = \mathbf{a}^n_{B_3} = -\omega_3^2 \mathbf{r}_{CB} = -3.903^2[(0.256 - 0.1)\boldsymbol{\imath} + (0.256 - 0)\boldsymbol{\jmath}]$$
$$= -2.379\boldsymbol{\imath} - 3.903\boldsymbol{\jmath} \text{ m/s}^2, \quad \text{and}$$
$$|\mathbf{a}^n_{B_2}| = |\mathbf{a}^n_{B_3}| = 4.57129 \text{ m/s}^2,$$

The tangential acceleration of B_3 is

$$\mathbf{a}^t_{B_3} = \mathbf{a}^t_{B_3} = \boldsymbol{\alpha}_3 \times \mathbf{r}_{CB} = -2.252\mathbf{k} \times [(0.256 - 0.1)\boldsymbol{\imath} + (0.256 - 0)\boldsymbol{\jmath}]$$
$$= 0.577\,\boldsymbol{\imath} - 0.351\,\boldsymbol{\jmath} \text{ m/s}^2, \quad \text{and}$$
$$|\mathbf{a}^t_{B_3}| = |\mathbf{a}^t_{B_3}| = 0.675 \text{ m/s}^2.$$

The relation between the angular velocities of link 2 and link 3 is

$$\boldsymbol{\omega}_2 = \boldsymbol{\omega}_3 + \boldsymbol{\omega}_{23},$$

and the relative angular velocity of link 2 with respect to link 3 is

$$\boldsymbol{\omega}_{23} = \boldsymbol{\omega}_2 - \boldsymbol{\omega}_3 = 3.141\,\mathbf{k} - 3.903\,\mathbf{k} = -0.762\,\mathbf{k} \quad \text{rad/s}.$$

The relative angular acceleration of link 2 with respect to link 3 is

$$\boldsymbol{\alpha}_{23} = \boldsymbol{\alpha}_2 - \boldsymbol{\alpha}_3 = -\boldsymbol{\alpha}_3 = 2.252\,\mathbf{k} \quad \text{rad/s}^2,$$

where $\boldsymbol{\alpha}_2 = \boldsymbol{\alpha}_1 = \mathbf{0}$.

```
rC = {0.1, 0., 0} m

rB = {0.256155, 0.256155, 0} m

ϕ1= 0.785398 rad = 45. deg

ϕ3= 1.02334 rad = 58.633 deg

ω = ω1 = {0, 0, 3.14159} rad/s

α = α1 = {0, 0, 0} rad/s^2

vB1 = ω1 x rB = {-0.804736, 0.804736, 0.} m/s

|vB1| = 1.13807 m/s

aB1 = α1 x rB - ω1^2 rB = {-2.52815, -2.52815, 0.} m/s^2

|aB1| = 3.57535 m/s^2

ω3 = {0, 0, ω3z}

vB2 = vB3 = vC + ω3 x (rB-rC)

vB21={vB21 Cos[ϕ1],vB21 Sin[ϕ1],0}

vB2 = vB1 + vB21 => ω3z, vB21

ω3 = {0, 0, 3.90354} rad/s

vB21 = -0.276022 m/s

vB21v = {-0.195177, -0.195177, 0} m/s

vB3 = {-0.999913, 0.609559, 0.} m/s

|vB3| = 1.17106 m/s

α3 = {0, 0, α3z}

aB2 = aB3 = aC + α3 x (rB-rC) - ω3.ω3(rB-rC)

aB21={aB21 Cos[ϕ1],aB21 Sin[ϕ1],0}

aB21cor = 2 ω1 x vB21 = {1.22633, -1.22633, 0.} m/s^2

|aB21cor| = 1.7343 m/s^2

aB2 = aB1 + aB21 + aB21cor => α3z, aB21

α3 = {0, 0, -2.25292} rad/s^2

aB21 = -0.707843 m/s^2

aB21v = {-0.500521, -0.500521, 0} m/s^2

aB3 = {-1.80234, -4.25501, 0.} m/s^2

|aB3| = 4.62098 m/s^2

aB3n = {-2.37944, -3.9032, 0} m/s^2

|aB3n| = 4.57129 m/s^2

aB3t = {0.577098, -0.351806, 0.} m/s^2

|aB3t| = 0.675877 m/s^2
```