Slider-Crank Mechanism

Velocity and Acceleration Analysis

The R-RRT (slider-crank) mechanism shown in Fig. 1(a) has the dimensions: $AB = 1$ m and $BC = 1$ m. When the driver link 1 makes an angle $\phi = \phi_1 = \pi/6$ rad with the horizontal axis the instantaneous speed and the angular acceleration of the link 1 are ω=1 rad/s and α=-1 rad/s^2.

Find the velocities and the accelerations of the joints and the the angular velocities and accelerations of the links for the given driver link angle.

Solution

The point A is selected as the origin of the xyz reference frame. The position vectors of the joints B and C are:

$$\mathbf{r}_B = x_B\boldsymbol{\imath} + y_B\boldsymbol{\jmath} = \frac{\sqrt{3}}{2}\,\boldsymbol{\imath} + \frac{1}{2}\,\boldsymbol{\jmath} \text{ m} \quad \text{and} \quad \mathbf{r}_C = x_C\boldsymbol{\imath} + y_C\boldsymbol{\jmath} = \sqrt{3}\,\boldsymbol{\imath} + 0\boldsymbol{\jmath} \text{ m.}$$

Velocity of joint B

The velocity of the point $B = B_1$ on the link 1 is

$$\mathbf{v}_B = \mathbf{v}_{B_1} = \mathbf{v}_A + \mathbf{v}_{BA} = \mathbf{v}_A + \boldsymbol{\omega}_1 \times \mathbf{r}_{AB} = \boldsymbol{\omega}_1 \times \mathbf{r}_B,$$

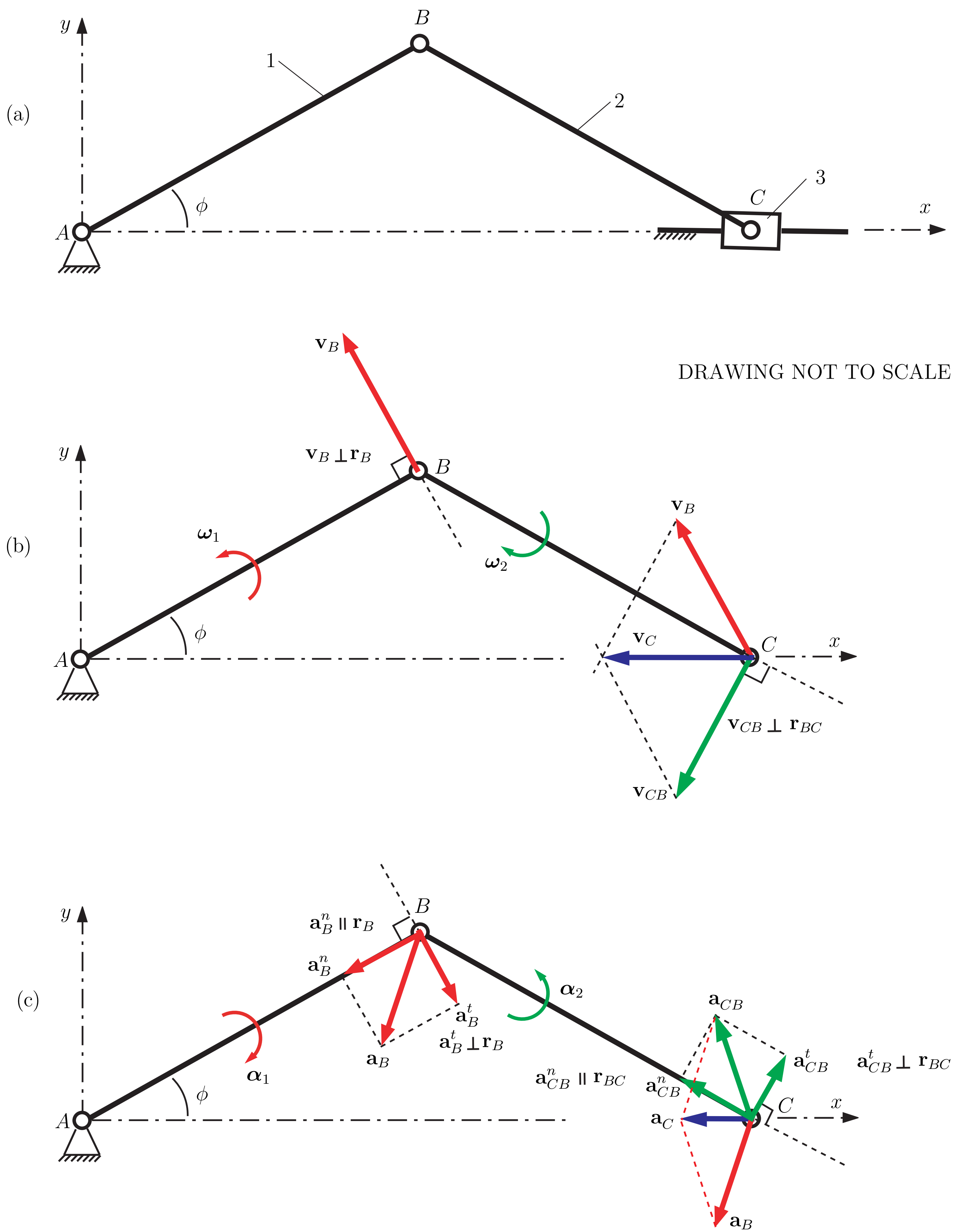
where $\mathbf{v}_A \equiv \mathbf{0}$ is the velocity of the origin $A \equiv O$.
The velocity of point B_2 on the link 2 is $\mathbf{v}_{B_2} = \mathbf{v}_{B_1}$ because the links 1 and 2 are connected at a rotational joint. The velocity of $B_1 = B_2$ is

$$\mathbf{v}_B = \mathbf{v}_{B_1} = \mathbf{v}_{B_2} = \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & \omega_1 \\ x_B & y_B & 0 \end{vmatrix} = \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & 1 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \end{vmatrix} = -\frac{1}{2}\boldsymbol{\imath} + \frac{\sqrt{3}}{2}\boldsymbol{\jmath} \text{ m/s.}$$

The magnitude of the velocity $\mathbf{v}_B$ is

$$|\mathbf{v}_B| = v_B = 1 \text{ m/s.}$$

The velocity $\mathbf{v}_B$ is perpendicular to the position vector $\mathbf{r}_B$ and has the direction given by the angular velocity $\boldsymbol{\omega}_1$ as shown in Fig. 1(b).


(a)
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B
1
2
3
C
x
A
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DRAWING NOT TO SCALE
(b)
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$\mathbf{v}_B \perp \mathbf{r}_B$
B
$\boldsymbol{\omega}_1$
$\boldsymbol{\omega}_2$
$\mathbf{v}_C$
C
x
A
ϕ
$\mathbf{v}_{CB} \perp \mathbf{r}_{BC}$
$\mathbf{v}_{CB}$
(c)
y
B
$\mathbf{a}_B^n \parallel \mathbf{r}_B$
$\mathbf{a}_B^n$
$\mathbf{a}_B^t$
$\mathbf{a}_B^t \perp \mathbf{r}_B$
$\mathbf{a}_B$
$\boldsymbol{\alpha}_1$
$\boldsymbol{\alpha}_2$
$\mathbf{a}_{CB}$
$\mathbf{a}_{CB}^t$
$\mathbf{a}_{CB}^t \perp \mathbf{r}_{BC}$
$\mathbf{a}_{CB}^n \parallel \mathbf{r}_{BC}$
$\mathbf{a}_{CB}^n$
$\mathbf{a}_C$
C
x
A
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$\mathbf{a}_B$

Figure 1

Velocity of joint C

The points B_2 and C_2 are on the link 2 and

$$\mathbf{v}_C = \mathbf{v}_{C_2} = \mathbf{v}_B + \mathbf{v}_{CB} = \mathbf{v}_{B_2} + \boldsymbol{\omega}_2 \times \mathbf{r}_{BC} = \mathbf{v}_B + \boldsymbol{\omega}_2 \times (\mathbf{r}_C - \mathbf{r}_B), \qquad (1)$$

where the angular velocity of link 2 is $\boldsymbol{\omega}_2 = \omega_2 \mathbf{k}$ (ω_2 is unknown).

On the other hand the velocity of C is along the vertical axis (x-axis) because the slider 2 translates along x-axis

$$\mathbf{v}_C = \mathbf{v}_{C_3} = v_C \boldsymbol{\imath}. \qquad (2)$$

Equations (1) and (2) give

$$\mathbf{v}_B + \boldsymbol{\omega}_2 \times (\mathbf{r}_C - \mathbf{r}_B) = v_C \boldsymbol{\imath},$$

or

$$\mathbf{v}_B + \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & \omega_2 \\ x_C - x_B & y_C - y_B & 0 \end{vmatrix} = v_C \boldsymbol{\imath}. \qquad (3)$$

Equation (3) represents a vectorial equation with two scalar components on x-axis and y-axis and with two unknowns ω_2 and v_C

$$v_{Bx} - \omega_2 (y_C - y_B) = v_C,$$
$$v_{Bx} + \omega_2 (x_C - x_B) = 0,$$

or

$$-\frac{1}{2} - \omega_2 \left(0 - \frac{1}{2}\right) = v_C,$$
$$\frac{\sqrt{3}}{2} + \omega_2 \left(\sqrt{3} - \frac{\sqrt{3}}{2}\right) = 0.$$

It results

$$\omega_2 = -1 \text{ rad/s} \ \text{ and } \ v_C = -1 \text{ m/s}.$$

The relative velocity of point C with respect to B is

$$\mathbf{v}_{CB} = \boldsymbol{\omega}_2 \times (\mathbf{r}_C - \mathbf{r}_B) = \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & -1 \\ \sqrt{3} - \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \end{vmatrix} = -\frac{1}{2} \boldsymbol{\imath} - \frac{\sqrt{3}}{2} \boldsymbol{\jmath} \ \text{ m/s}.$$

The relative velocity $\mathbf{v}_{CB}$ is perpendicular to $\mathbf{r}_{BC}$ and has the direction given by the angular velocity $\boldsymbol{\omega}_2$ as shown in Fig. 1(b).

Acceleration of joint B

The acceleration of the point $B = B_1$ on the link 1 is

$$\begin{aligned}
\mathbf{a}_B &= \mathbf{a}_{B_1} = \mathbf{a}_{B_2} = \mathbf{a}_A + \boldsymbol{\alpha}_1 \times \mathbf{r}_B + \boldsymbol{\omega}_1 \times (\boldsymbol{\omega}_1 \times \mathbf{r}_B) = \boldsymbol{\alpha}_1 \times \mathbf{r}_B - \boldsymbol{\omega}_1^2 \mathbf{r}_B \\
&= \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & \alpha_1 \\ x_B & y_B & 0 \end{vmatrix} - \boldsymbol{\omega}_1^2 \mathbf{r}_B = \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & -1 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \end{vmatrix} - 1^2 \left(\frac{\sqrt{3}}{2} \boldsymbol{\imath} + \frac{1}{2} \boldsymbol{\jmath} \right) \\
&= \left(\frac{1}{2} - \frac{\sqrt{3}}{2} \right) \boldsymbol{\imath} - \left(\frac{1}{2} + \frac{\sqrt{3}}{2} \right) \boldsymbol{\jmath} \ \text{ m/s}^2.
\end{aligned}$$

The angular acceleration of link 1 is $\boldsymbol{\alpha}_1$ =-1 $\mathbf{k}$ rad/s^2.
The normal acceleration of the point B is

$$\mathbf{a}_B^n = -\boldsymbol{\omega}_1^2 \, \mathbf{r}_B = -1^2 \left(\frac{\sqrt{3}}{2} \boldsymbol{\imath} + \frac{1}{2} \boldsymbol{\jmath} \right) = -\frac{\sqrt{3}}{2} \boldsymbol{\imath} - \frac{1}{2} \boldsymbol{\jmath} \ \text{ m/s}^2.$$

The normal acceleration $\mathbf{a}_B^n$ is parallel to the vector $\mathbf{r}_B$ and the orientation is toward the center of rotation A (from B to A) as shown in Fig. 1(c). The tangential acceleration of the point B is

$$\mathbf{a}_B^t = \boldsymbol{\alpha}_1 \times \mathbf{r}_B = \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & \alpha_1 \\ x_B & y_B & 0 \end{vmatrix} = \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & -1 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \end{vmatrix} = \frac{1}{2} \boldsymbol{\imath} - \frac{\sqrt{3}}{2} \boldsymbol{\jmath} \quad \text{m/s}^2.$$

The tangential acceleration $\mathbf{a}_B^t$ is perpendicular to the vector $\mathbf{r}_B$ and the orientation given by the vector $\boldsymbol{\alpha}_1$ as shown in Fig. 1(c).

Acceleration of joint C

The points C_2 and B_2 are on the link 2 and

$$\mathbf{a}_C = \mathbf{a}_{C_2} = \mathbf{a}_{B_2} + \boldsymbol{\alpha}_2 \times \mathbf{r}_{BC} - \omega_2^2 \, \mathbf{r}_{BC} = \mathbf{a}_B + \boldsymbol{\alpha}_2 \times (\mathbf{r}_C - \mathbf{r}_B) - \omega_2^2 \, (\mathbf{r}_C - \mathbf{r}_B), \quad (4)$$

where the angular acceleration of link 2 is $\boldsymbol{\alpha}_2 = \alpha_2 \mathbf{k}$ (α_2 is unknown).

The slider C has a translational motion along x-axis and

$$\mathbf{a}_C = \mathbf{a}_{C_3} = a_C \boldsymbol{\imath}. \quad (5)$$

Equations (4) and (5) give

$$\mathbf{a}_B + \boldsymbol{\alpha}_2 \times (\mathbf{r}_C - \mathbf{r}_B) - \omega_2^2\ (\mathbf{r}_C - \mathbf{r}_B) = a_C \mathbf{\imath},$$

or

$$\mathbf{a}_B + \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ 0 & 0 & \alpha_2 \\ x_C - x_B & y_C - y_B & 0 \end{vmatrix} - \omega_2^2 \left[(x_C - x_B)\mathbf{\imath} + (y_C - y_B)\mathbf{\jmath}\right] = a_C \mathbf{\imath}. \quad (6)$$

Equation (6) represents a vectorial equation with two scalar components on x-axis and y-axis and with two unknowns α_2 and α_3

$$a_{Bx} - \alpha_2(y_C - y_B) - \omega_2^2(x_C - x_B) = a_C,$$
$$a_{By} + \alpha_2(x_C - x_B) - \omega_2^2(y_C - y_B) = 0,$$

or

$$\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right) - \alpha_2\left(0 - \frac{1}{2}\right) - (-1)^2\left(\sqrt{3} - \frac{\sqrt{3}}{2}\right) = a_C,$$
$$-\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) + \alpha_2\left(\sqrt{3} - \frac{\sqrt{3}}{2}\right) - (-1)^2\left(0 - \frac{1}{2}\right) = 0.$$

It results

$$\alpha_2 = 1 \text{ rad/s}^2 \quad \text{and} \quad a_C = 1 - \sqrt{3} \text{ m/s}^2.$$

The normal relative acceleration of point C with respect to B is

$$\begin{aligned} \mathbf{a}_{CB}^n &= -\boldsymbol{\omega}_2^2\, \mathbf{r}_{BC} = -\omega_2^2\ (\mathbf{r}_C - \mathbf{r}_B) \\ &= -\omega_2^2\ (\mathbf{r}_C - \mathbf{r}_B) = -\omega_2^2\ [(x_C - x_B)\mathbf{\imath} + (y_C - y_B)\mathbf{\jmath}] \\ &= -(-1)^2 \left[\left(\sqrt{3} - \frac{\sqrt{3}}{2}\right)\mathbf{\imath} + \left(0 - \frac{1}{2}\right)\mathbf{\jmath}\right] \\ &= -\frac{\sqrt{3}}{2}\mathbf{\imath} + \frac{1}{2}\mathbf{\jmath} \ \text{ m/s}^2. \end{aligned}$$

The normal relative acceleration of point C with respect to B, $\mathbf{a}_{CB}^n$, is parallel to the vector $\mathbf{r}_{BC}$ and the orientation is toward the center of rotation B (from C to B) as shown in Fig. 1(c). The tangential relative acceleration of the point C with respect to B is

$$\begin{aligned}\mathbf{a}^t_{CB} &= \boldsymbol{\alpha}_2 \times \mathbf{r}_{BC} = \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & \alpha_2 \\ x_C - x_B & y_C - y_B & 0 \end{vmatrix} \\ &= \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & 1 \\ \sqrt{3} - \frac{\sqrt{3}}{2} & 0 - \frac{1}{2} & 0 \end{vmatrix} = \frac{1}{2}\boldsymbol{\imath} + \frac{\sqrt{3}}{2}\boldsymbol{\jmath} \quad \text{m/s}^2.\end{aligned}$$

The tangential relative acceleration $\mathbf{a}^t_{CB}$ is perpendicular to the vector $\mathbf{r}_{BC}$ and the orientation given by the vector $\boldsymbol{\alpha}_2$ as shown in Fig. 1(c).

$$rB = \left\{\frac{\sqrt{3}}{2}, \frac{1}{2}, 0\right\} \text{ [m]}$$

$$rC = \left\{\sqrt{3}, 0, 0\right\} \text{ [m]}$$

omega1 = {0, 0, 1} [rad/s]

$$vB = \text{omega1 x rB} = \left\{-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right\} = \{-0.5, 0.866025, 0.\} \text{ [m/s]}$$

|vB| = 1 = 1. [m/s]

vC = vB + omega2 x (rC-rB) = {vCx,0,0} => omega2, vCx

vC = {-1, 0, 0} = {-1., 0., 0.} [m/s]

omega2 = {0, 0, -1} = {0., 0., -1.} [rad/s]

$$vCB = \text{omega2 x (rC-rB)} = \left\{-\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0\right\} = \{-0.5, -0.866025, 0.\} \text{ [m/s]}$$

|vCB| = 1 = 1. [m/s]

alpha1 = {0, 0, -1} [rad/s^2]

$$aB = \text{alpha1 x rB - omega1\^{}2 rB} = \left\{\frac{1}{2} - \frac{\sqrt{3}}{2}, -\frac{1}{2} - \frac{\sqrt{3}}{2}, 0\right\} = \{-0.366025, -1.36603, 0.\} \text{ [m/s\^{}2]}$$

$$|aB| = \sqrt{\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)^2} = 1.41421 \text{ [m/s\^{}2]}$$

$$aBn = \text{- omega1\^{}2 rB} = \left\{-\frac{\sqrt{3}}{2}, -\frac{1}{2}, 0\right\} = \{-0.866025, -0.5, 0.\} \text{ [m/s\^{}2]}$$

$$aBt = \text{alpha1 x rB} = \left\{\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0\right\} = \{0.5, -0.866025, 0.\} \text{ [m/s\^{}2]}$$

aC = aB + alpha2 x (rC-rB) - omega2.omega2 (rC-rB) = {aCx,0,0} => alpha2, aCx

$$aC = \left\{1 - \sqrt{3}, 0, 0\right\} = \{-0.732051, 0., 0.\} \text{ [m/s\^{}2]}$$

alpha2 = {0, 0, 1} = {0., 0., 1.} [rad/s^2]

aCB = alpha2 x (rC-rB) - omega2.omega2 (rC-rB) =

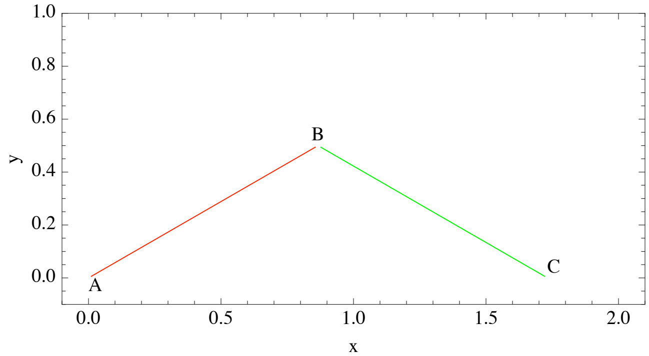
$$\left\{\frac{1}{2} - \frac{\sqrt{3}}{2}, \frac{1}{2} + \frac{\sqrt{3}}{2}, 0\right\} = \{-0.366025, 1.36603, 0.\} \text{ [m/s\^{}2]}$$

$$aCBn = \text{- omega2.omega2 (rC-rB)} = \left\{-\frac{\sqrt{3}}{2}, \frac{1}{2}, 0\right\} = \{-0.866025, 0.5, 0.\} \text{ [m/s\^{}2]}$$

|aCBn| = 1 = 1. [m/s^2]

$$aCBt = \text{alpha2 x (rC-rB)} = \left\{\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right\} = \{0.5, 0.866025, 0.\} \text{ [m/s\^{}2]}$$

|aCBt| = 1 = 1. [m/s^2]


1.0
0.8
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y
B
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C
0.0
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1.0
1.5
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x