4 Homework: Velocity and Acceleration Analysis

Motion of a point A that moves relative to a rigid body

A point A is not assumed to be a point of the rigid body, $A \notin (RB)$.

Show the mathematical proof that the acceleration of the point A relative to the primary reference frame $(x_0y_0z_0)$ is

$$\mathbf{a}_A = \mathbf{a}_O + \mathbf{a}^{rel}_{A(xyz)} + 2\,\boldsymbol{\omega} \times \mathbf{v}^{rel}_{A(xyz)} + \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}),$$

where

(xyz) is a body fixed (mobile or rotating) reference frame with its origin at a point O of the rigid body ($O \in (RB)$), and is a moving reference frame relative to the primary reference;

$\mathbf{a}_O$ is the acceleration of O relative to the primary reference;

$\mathbf{r} = \mathbf{r}_{OA} = x\boldsymbol{\imath} + y\boldsymbol{\jmath} + z\mathbf{k}$ is the position vector of A relative to the origin O, of the body fixed reference frame, and x, y, and z are the coordinates of A in terms of the body fixed reference frame.

$\mathbf{v}^{rel}_{A(xyz)} = \dfrac{^{(xyz)}d\,\mathbf{r}}{dt} = \dfrac{dx}{dt}\boldsymbol{\imath} + \dfrac{dy}{dt}\boldsymbol{\jmath} + \dfrac{dz}{dt}\mathbf{k}$, is the velocity of A relative to the body fixed reference frame or relative to the rigid body;

$\mathbf{a}^{rel}_{A(xyz)} = \dfrac{^{(xyz)}d^2\,\mathbf{r}}{dt^2} = \dfrac{d^2x}{dt^2}\boldsymbol{\imath} + \dfrac{d^2y}{dt^2}\boldsymbol{\jmath} + \dfrac{d^2z}{dt^2}\mathbf{k}$, is the acceleration of A relative to the body fixed reference frame or relative to the rigid body;

$\boldsymbol{\omega}$ is the angular velocity vector of the rigid body;

$\boldsymbol{\alpha}$ is the angular acceleration vector of the rigid body;

$\mathbf{a}^{cor}_{A(xyz)} = 2\,\boldsymbol{\omega} \times \mathbf{v}^{rel}_{A(xyz)}$ is the Coriolis acceleration.