Contents

3 Position Analysis

3.1 Absolute Cartesian Method

The position analysis of a kinematic chain requires the determination of the joint positions and/or the position of the center of gravity (CG) of the link. A planar link with the end nodes A and B is considered in Fig. 3.1. Let $(x_A,\, y_A)$ be the coordinates of the joint A with respect to the reference frame xOy, and $(x_B,\, y_B)$ be the coordinates of the joint B with the same reference frame. Using Pythagoras the following relation can be written

$$(x_B - x_A)^2 + (y_B - y_A)^2 = AB^2 = L_{AB}^2, \tag{3.1}$$

where L_{AB} is the length of the link AB.
Let ϕ be the angle of the link AB with the horizontal axis Ox. Then, the slope m of the link AB is defined as

$$m = \tan\phi = \frac{y_B - y_A}{x_B - x_A}. \tag{3.2}$$

Let n be the intercept of AB with the vertical axis Oy. Using the slope m and the intercept n, the equation of the straight link, in the plane, is

$$y = m\,x + n, \tag{3.3}$$

where x and y are the coordinates of any point on this link.

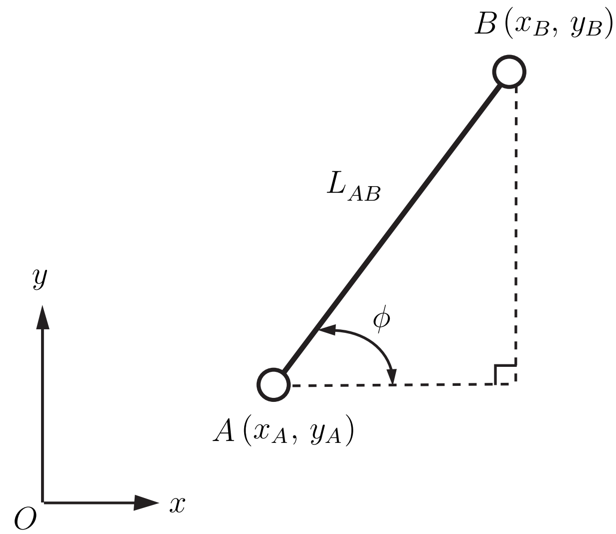


Figure 3.1

3.2 Examples

3.2.1 Slider-Crank (R-RRT) Mechanism

The R-RRT (slider-crank) mechanism shown in Fig. 3.2(a) has the dimensions: $AB = 0.1$ m and $BC = 1$ m. The driver link 1 makes an angle $\phi = \phi_1 = 45°$ with the horizontal axis. Find the positions of the joints and the angles of the links with the horizontal axis.

Solution
Position of joint A
A Cartesian reference frame xOy is selected. The joint A is in the origin of the reference frame, that is, $A \equiv O$,

$$x_A = 0, \;\; y_A = 0,$$

Position of joint B
The unknowns are the coordinates of the joint B, x_B and y_B. Because the joint A is fixed and the angle ϕ is known, the coordinates of the joint B are computed from the following expressions

$$\begin{aligned} x_B &= AB\cos\phi = (0.5)\;\cos 45° = 0.353553\,\text{m}, \\ y_B &= AB\sin\phi = (0.5)\;\sin 45° = 0.353553\,\text{m}. \end{aligned} \tag{3.4}$$

Position of joint C
The unknowns are the coordinates of the joint C, x_C and y_C. The joint C is located on the horizontal axis $y_C = 0$. The length of the segment BC is constant

$$(x_B - x_C)^2 + (y_B - y_C)^2 = BC^2, \tag{3.5}$$

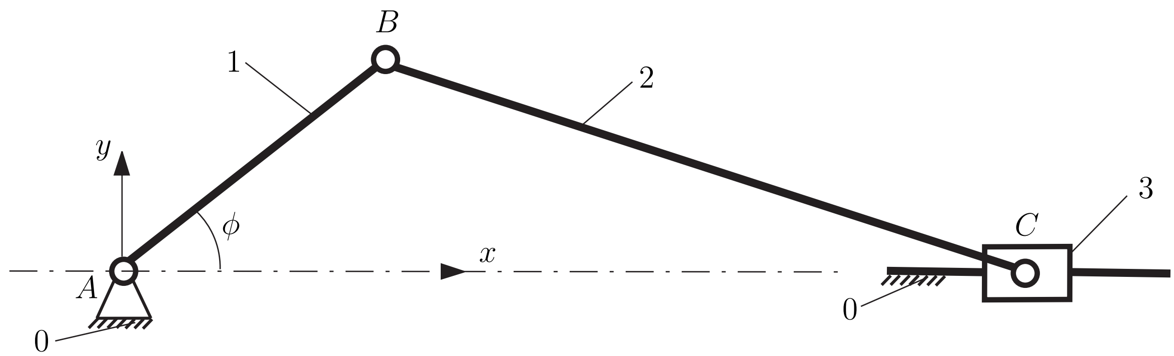
or

$$(0.353553 - x_C)^2 + (0.353553 - 0)^2 = 1^2.$$

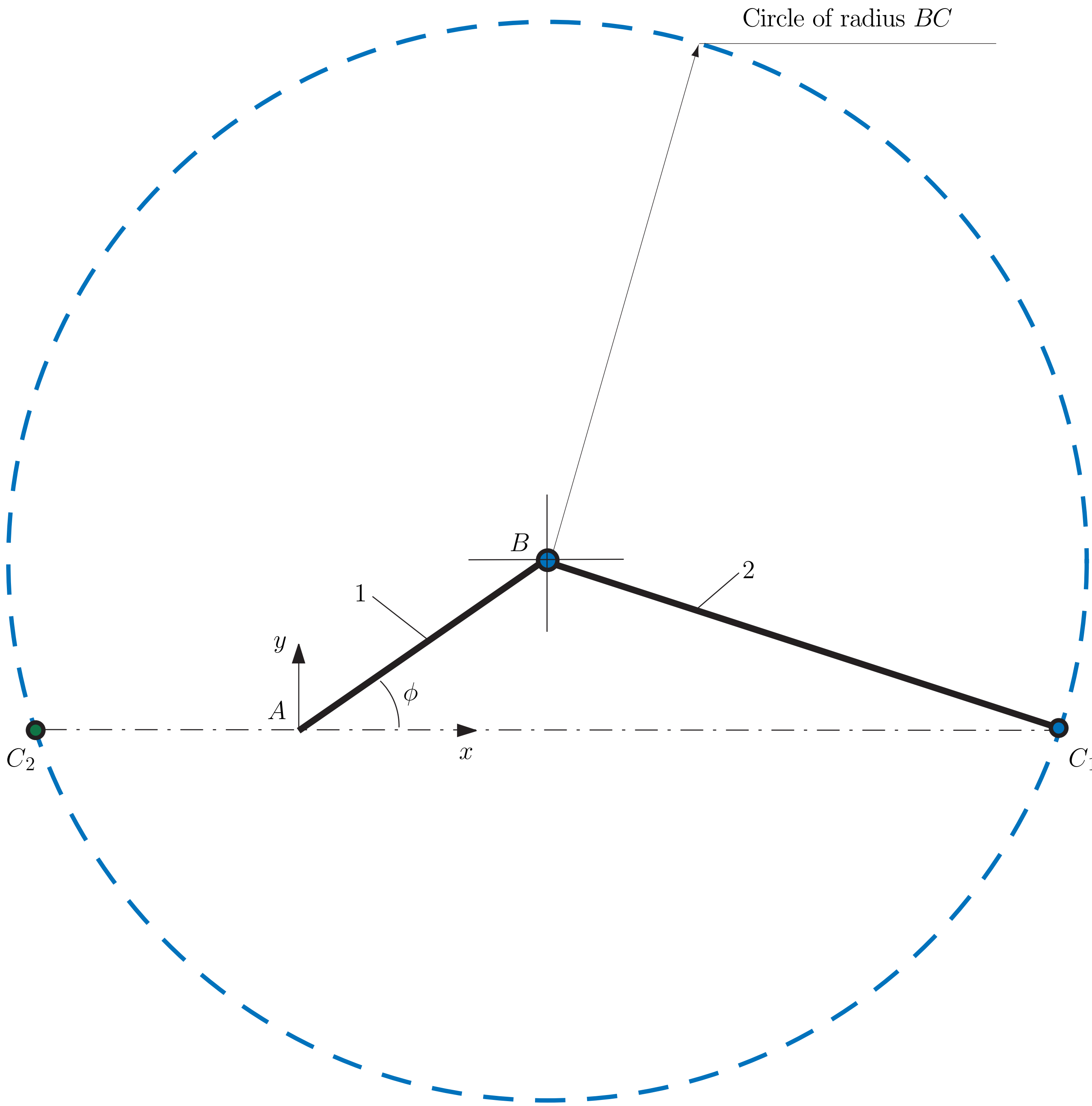
Because it is a quadratic equation two solutions are found for the position of C.

These two solutions for x_C are located at the intersection of the horizontal axis $0x$ with the circle centered in B and radius CB, as shown in Fig. 3.2(b), and they have the following numerical values:

$$x_{C_1} = 1.2890 \text{ m} \quad \text{and} \quad x_{C_2} = -0.5819 \text{ m}.$$



(a)



(b)

Figure 3.2

To determine the correct position of the joint C for the mechanism, an additional condition is needed. For the first quadrant, $0 \leq \phi \leq 90°$, the condition is $x_C > xB$.

The angle of the link 2 (link BC) with the horizontal is

$$\phi_2 = \arctan \frac{y_B - y_C}{x_B - x_C},$$

The results are

$$x_C = 1.28897 \text{ m}, \quad y_C = 0 \text{ m}, \quad \& \quad \phi_2 = -20.7048 \text{ degrees}.$$

The MATLAB program for the positions is given in Program 3.1.

3.2.2 R-RRR-RRT Mechanism

The considered planar R-RRR-RRT mechanism is shown in Fig. 3.3. The driver link is the rigid link 1 (the element AB). The following data are given: AB=0.150 m, BC=0.400 m, CD=0.370 m, CE=0.230 m, EF=CE, L_a=0.300 m, L_b=0.450 m, and L_c=CD. The angle of the driver link 1 with the horizontal axis is $\phi = \phi_1 = 45°$.
Find the positions of the joints and the angles of the links with the horizontal axis.

Solution
Position of joint A
A cartesian reference frame $xOyz$ with the versors $[\mathbf{\imath}, \mathbf{\jmath}, \mathbf{k}]$ is selected, Fig. 3.3. Since the joint A is in the origin of the reference system $A \equiv O$ the coordinates of A are $x_A = 0$, $y_A = 0$ and the position vector of A is $\mathbf{r}_A = x_A\,\mathbf{\imath} + y_A\,\mathbf{\jmath}$.
Position of joint D
The coordinates of the joint D are $x_D = L_a$, $y_D = L_b$ and the position vector of D is $\mathbf{r}_D = x_D\,\mathbf{\imath} + y_D\,\mathbf{\jmath}$.
Position of joint B
The unknowns are the coordinates of the joint B, x_B and y_B. Because the joint A is fixed and the angle ϕ is known, the coordinates of the joint B are computed from the following expressions
$x_B = AB\cos\phi = 0.150\cos 45° = 0.106\,\text{m}$,
$y_B = AB\sin\phi = 0.150\sin 45° = 0.106\,\text{m}$.
The position vector of B is $\mathbf{r}_B = x_B\,\mathbf{\imath} + y_B\,\mathbf{\jmath}$.
Position of joint C
The unknowns are the coordinates of the joint C, x_C and y_C. Knowing the positions of the joints B and D, the position of the joint C can be computed using the fact that the lengths of the links BC and CD are constants

$$(x_C - x_B)^2 + (y_C - y_B)^2 = BC^2,$$
$$(x_C - x_D)^2 + (y_C - y_D)^2 = CD^2, \quad (3.6)$$

or

$$(x_C - 0.106)^2 + (y_C - 0.106)^2 = 0.400^2,$$
$$(x_C - 0.300)^2 + (y_C - 0.450)^2 = 0.370^2, \quad (3.7)$$

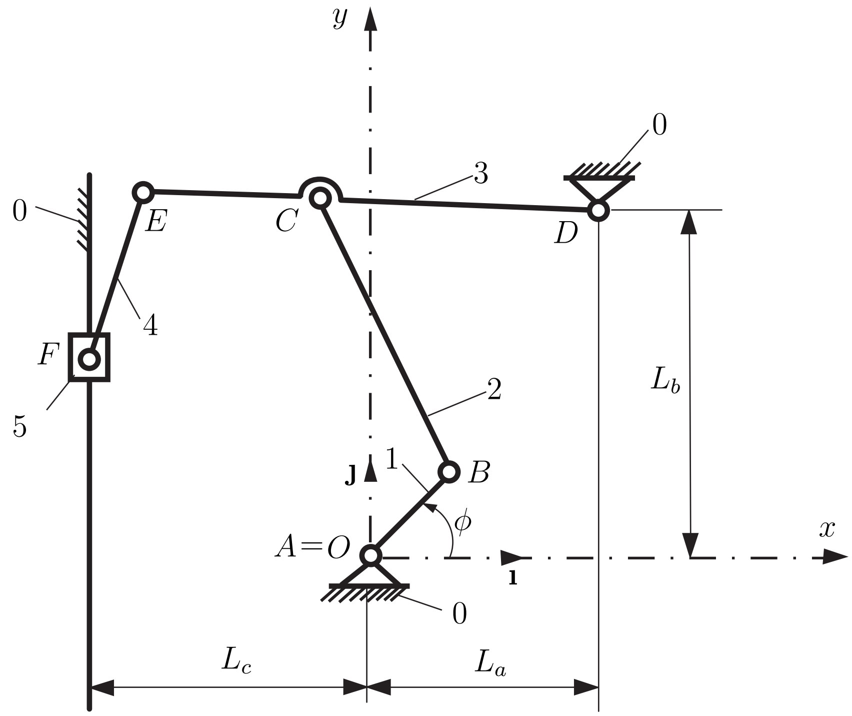


Figure 3.3

Equations (3.7) consist of two quadratic equations. Solving this system of equations, two sets of solutions are found for the position of the joint C. These solutions are

$$\begin{aligned} x_{C_1} &= -0.069 \text{ m}, \quad y_{C_1} = 0.465 \text{ m}, \\ x_{C_2} &= 0.504 \text{ m}, \quad y_{C_2} = 0.141 \text{ m}. \end{aligned} \tag{3.8}$$

The points C_1 and C_2 are the intersections of the circle of radius BC (with its center at B) with the circle of radius CD (with its center at D), as shown in Fig. 3.4. To determine the position of the joint C for this mechanism, an additional constraint condition is needed: $x_C < x_D$. Because $x_D = 0.300$ m, the coordinates of joint C have the following numerical values

$$x_C = x_{C_1} = -0.069 \text{ m}, \qquad y_C = y_{C_1} = 0.465 \text{ m}. \tag{3.9}$$

Position of joint E

The unknowns are the coordinates of the joint E, x_E and y_E. The position of the joint E is determined from the equation

$$(x_E - x_C)^2 + (y_E - y_C)^2 = CE^2, \tag{3.10}$$

or

$$(x_E + 0.069)^2 + (y_E - 0.465)^2 = 0.230^2.$$

The joints D, C and E are located on the same straight element DE. For these joints, the following equation can be written

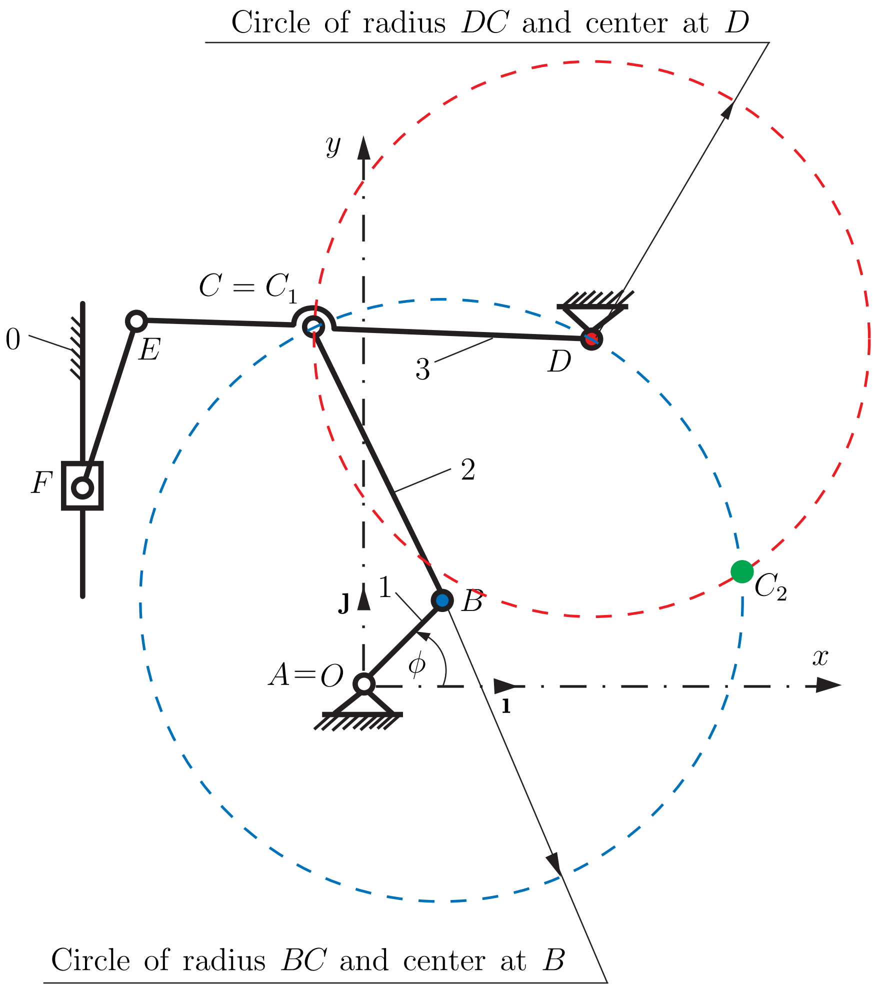
$$\frac{y_D - y_C}{x_D - x_C} = \frac{y_E - y_C}{x_E - x_C}, \tag{3.11}$$

or

$$\frac{0.450 - 0.465}{0.300 + 0.069} = \frac{y_E - 0.465}{x_E + 0.069}.$$

Equations (3.10) and (3.11) form a system from which the coordinates of the joint E can be computed. Two solutions are obtained, Fig. 3.5, and the numerical values are

$$\begin{aligned} x_{E_1} &= -0.299 \text{ m}, \quad y_{E_1} = 0.474 \text{ m}, \\ x_{E_2} &= 0.160 \text{ m}, \quad y_{E_2} = 0.455 \text{ m}. \end{aligned} \tag{3.12}$$



Circle of radius DC and center at D
y
C = C1
0
E
3
D
F
2
C2
J
1
B
A=O
ϕ
x
ı
Circle of radius BC and center at B

Figure 3.4

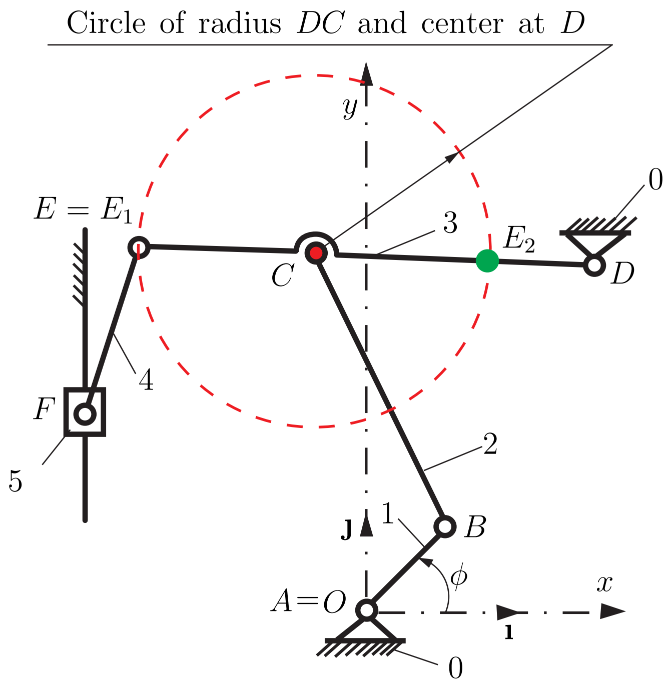

Circle of radius DC and center at D
y
0
$E = E_1$
3
E_2
C
D
4
F
5
2
1
B
$\mathbf{J}$
ϕ
x
$A\!=\!O$
$\mathbf{I}$
0

Figure 3.5

For continuous motion of the mechanism, a constraint condition is needed, $x_E < x_C$. Using this condition, the coordinates of the joint E are

$$x_E = x_{E_1} = -0.300\,\text{m},\;\; y_E = y_{E_1} = 0.475\,\text{m}.$$

Position of joint F

The joint F is restricted to move in a vertical direction, i.e. $x_F = -L_c = 0.370$ m. The coordinate y_F of the joint F can be calculated from the following quadratic equation

$$(x_F - x_E)^2 + (y_F - y_E)^2 = EF^2, \tag{3.13}$$

or

$$(0.370 + 0.300)^2 + (y_F - 0.475)^2 = 0.230^2,$$

The solutions of Eq. (3.13) are

$$y_{F_1} = 0.256\,\text{m}, \qquad y_{F_2} = 0.693\,\text{m}. \tag{3.14}$$

The points F_1 and F_2 are the intersections between the circle of radius EF (centered at E) and the vertical line with $x = x_F$, Fig. 3.6. For the mechanism depicted in Fig. 3.3, with $\theta = \pi/4$ the y coordinate of the joint F should be smaller that the y coordinate of the joint E, $y_F < y_E$. The y coordinate of the joint F is

$$y_F = y_{F_1} = 0.256\,\text{m}. \tag{3.15}$$

The angles of the links 2, 3, and 4 with the horizontal are

$$\phi_2 = \arctan\frac{y_B - y_C}{x_B - x_C}, \quad \phi_3 = \arctan\frac{y_D - y_C}{x_D - x_C}, \quad \phi_4 = \arctan\frac{y_F - y_E}{x_F - x_E},$$

The MATLAB program for the positions and the results is given in Program 3.2.

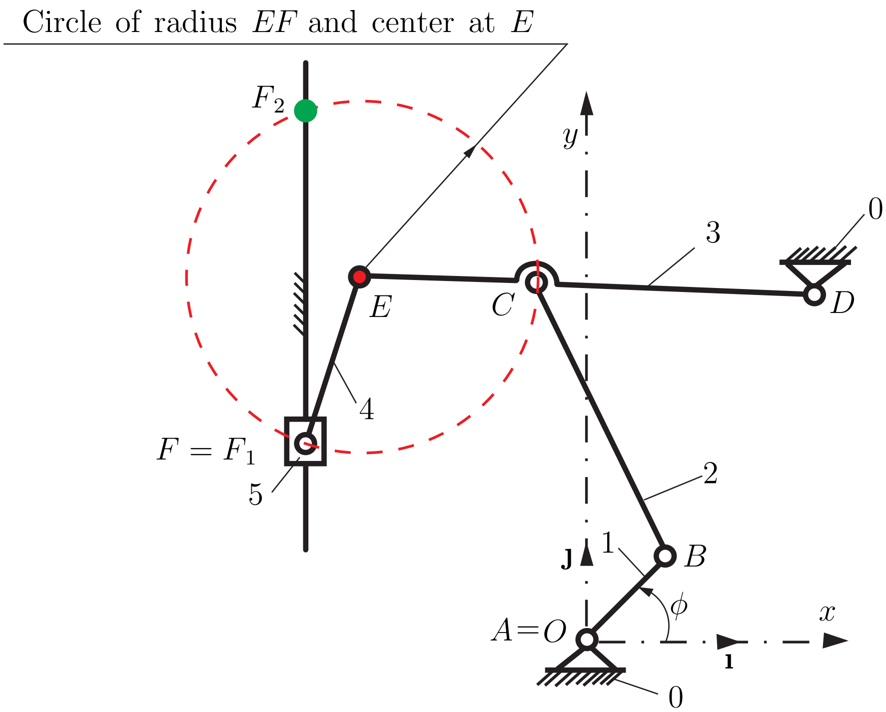

Circle of radius EF and center at E
F_2
y
0
3
E
C
D
4
$F = F_1$
5
2
J
1
B
ϕ
x
A=O
ı
0

Figure 3.6

3.2.3 R-RTR-RTR Mechanism

The planar R-RTR-RTR mechanism considered is shown in Fig. 3.7. The driver link is the rigid link 1 (the link AB). The following numerical data are given: $AB = 0.140$ m, $AC = 0.060$ m, $AE = 0.250$ m, $CD = 0.150$ m. The angle of the driver link 1 with the horizontal axis is $\phi = 30°$.

Solution

A Cartesian reference frame xOy is selected. The joint A is in the origin of the reference frame, that is, $A \equiv O$, $x_A = 0$, $y_A = 0$.

Position of joint C

The position vector of C is $\mathbf{r}_C = x_C\mathbf{\imath} + y_C\mathbf{\jmath} = 0.060\ \mathbf{\jmath}$ m.

Position of joint E

The position vector of E is $\mathbf{r}_E = x_E\mathbf{\imath} + y_E\mathbf{\jmath} = -0.250\ \mathbf{\jmath}$ m.

Position of joint B

The unknowns are the coordinates of the joint B, x_B and y_B. Because the joint A is fixed and the angle ϕ is known, the coordinates of the joint B are computed from the following expressions

$$x_B = AB\cos\phi = 0.140\cos 30° = 0.121\,\text{m},$$
$$y_B = AB\sin\phi = 0.140\sin 30° = 0.070\,\text{m}, \tag{3.16}$$

and $\mathbf{r}_B = x_B\mathbf{\imath} + y_B\mathbf{\jmath}$.

Position of joint D

The unknowns are the coordinates of the joint D, x_D and y_D. The length of the segment CD is constant:

$$(x_D - x_C)^2 + (y_D - y_C)^2 = CD^2, \tag{3.17}$$

or

$$(x_D - 0)^2 + (y_D - 0.060)^2 = 0.150^2.$$

The points B, C, and D are on the same straight line with the slope

$$m = \frac{(y_B - y_C)}{(x_B - x_C)} = \frac{(y_D - y_C)}{(x_D - x_C)}, \tag{3.18}$$

or

$$\frac{(0.070 - 0)}{(0.121 - 0.060)} = \frac{(y_D - 0)}{(x_D - 0.060)}.$$

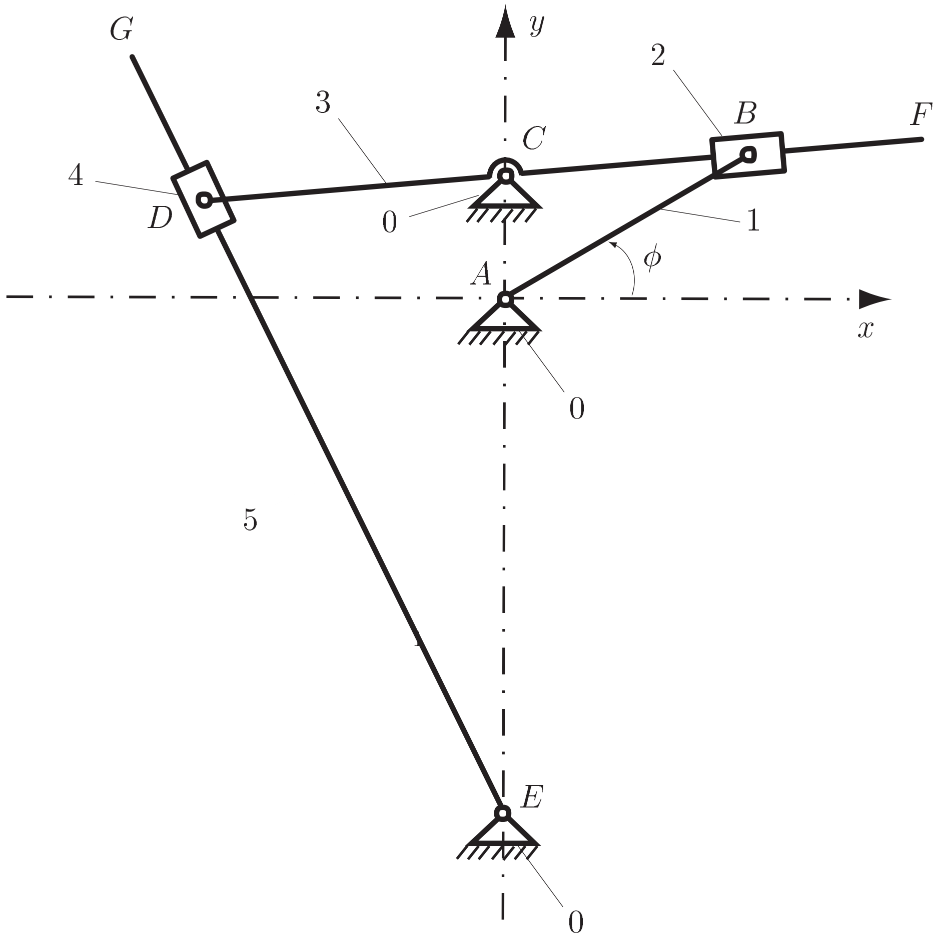


Figure 3.7

Equations (3.17) and (3.18) form a system from which the coordinates of the joint D can be computed. These solutions D_1 and D_2 are located at the intersection of the line BC with the circle centered in C and radius CD (Fig. 3.8), and they have the following numerical values:

$$x_{D1} = -0.149 \text{ m}, \ y_{D1} = 0.047 \text{ m},$$
$$x_{D2} = 0.149 \text{ m}, \ y_{D2} = 0.072 \text{ m}.$$

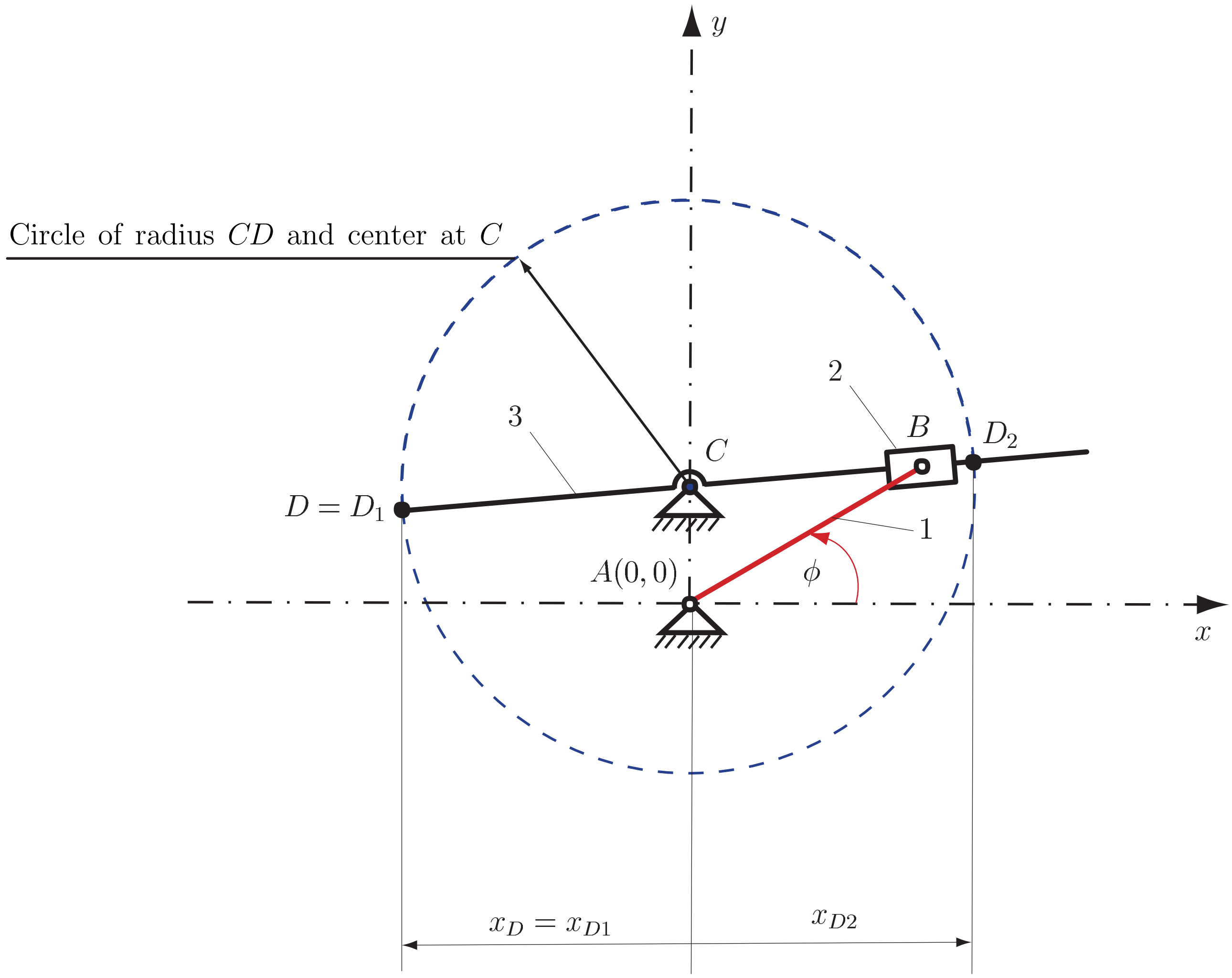
To determine the correct position of the joint D for the mechanism, an additional condition is needed. For the first quadrant, $0 \leq \phi \leq 90°$, the condition is $x_D \leq x_C$.
Because $x_C = 0$, the coordinates of the joint D are

$$x_D = x_{D1} = -0.149 \, \text{m} \quad \text{and} \quad y_D = y_{D1} = 0.047 \, \text{m}.$$

The angles of the links 2, 3, and 4 with the horizontal are

$$\phi_2 = \arctan \frac{y_B - y_C}{x_B - x_C}, \quad \phi_3 = \phi_2, \quad \phi_4 = \arctan \frac{y_D - y_E}{x_D - x_E} + \pi, \quad \phi_5 = \phi_4,$$

The MATLAB program for the positions and the results for the R-RTR-RTR mechanism is given in Program 3.3.



y
Circle of radius CD and center at C
2
3
B
D_2
C
$D = D_1$
1
A(0, 0)
ϕ
x
$x_D = x_{D1}$
x_{D2}

Figure 3.8

3.3 Problems

Problem 3.1: R-TRR mechanism

The following dimensions are given for the mechanism shown in Fig. P3.1: AC=a=0.100 m and BC=0.300 m. The angle of the driver link 1 with the horizontal axis is ϕ=ϕ_1=45°. Find the positions of the joints and the angles of the links with the horizontal axis. Write a MATLAB program for the positions of the mechanism.

Results

$x_B = y_B$=0.256 m, ϕ_3=1.023 rad=58.633°.

Problem 3.2

Figure P3.2 shows a quick-return shaper mechanism. Given the lengths AB=0.20 m, AD =0.40 m, CD=0.70 m, CE=0.30 m, and the input angle ϕ=ϕ_1=45°, obtain the positions of all the joints. The distance from the slider 5 to the horizontal axis Ax is y_E=0.35 m.

Results

$x_B = y_B$=0.141 m, x_C=0.17 m, y_C=0.26 m, x_E=-0.114 m, ϕ_3=75.36°, ϕ_4=165.9°.

Problem 3.3: R-RTR-RRT mechanism

The planar R-RTR-RRT mechanism is considered in Fig. P3.3. The driver is the rigid link 1 (the element AB) and makes an angle ϕ=ϕ_1=$\pi/6$ with the horizontal. The length of the links are AB=0.02 m, BC=0.03 m, and CD=0.06 m. The following dimensions are given: AE=0.05 m and L_a=0.02 m. Find the positions of the joints and the angles of the links with the horizontal axis.

Results

x_B=0.017 m, y_B=0.010 m, x_C= 0.046 m, y_C=0.014 m, x_D=0.02 m, y_D=-0.039 m, $\phi_2 = \phi_3$=0.147 rad=8.449°, ϕ_4=1.104 rad=63.261°.

Problem 3.4: R-TRR-RRT mechanism

The mechanism is shown in Fig. P3.4. The following data are given: AC=0.100 m, BC=0.300 m, BD=0.900 m, and L_a=0.100 m. If the angle of link 1 with the horizontal axis is ϕ=45°, find the position of joint D.

Results

x_C=0.100 m, y_C=0, x_B=0.256 m, y_B=0.256 m, x_D=1.142 m, y_D=0.100 m.

Problem 3.5: R-RRR-RTT mechanism

The R-RRR-RTT mechanism is shown in Fig. P3.5. The following data are given: AB=0.080 m, BC=0.350 m, CE=0.200 m, CD=0.150 m, L_a=0.200 m, L_b=0.350 m, and L_c=0.040 m. The angle of the driver link with the horizontal axis is ϕ=155°. Find the positions of the joints and the angles of the links with the horizontal axis.

Results

x_B=-0.0725046 m, y_B=0.0338095 m, x_C= 0.254847 m, y_C=0.157668 m,
x_D=0.295983 m, y_D=0.0134181 m, x_F=0.295983 m, y_F=-0.04 m,
ϕ_2=0.361716 rad=20.7248°, ϕ_3=-1.293 rad=-74.0835°.

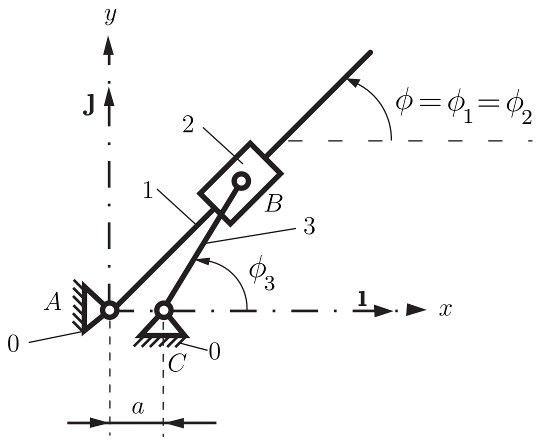


Figure P3.1

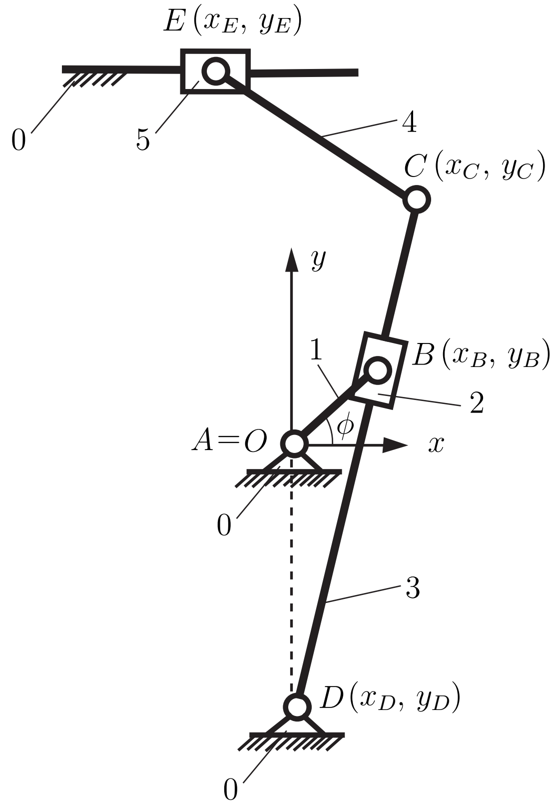


Figure P3.2

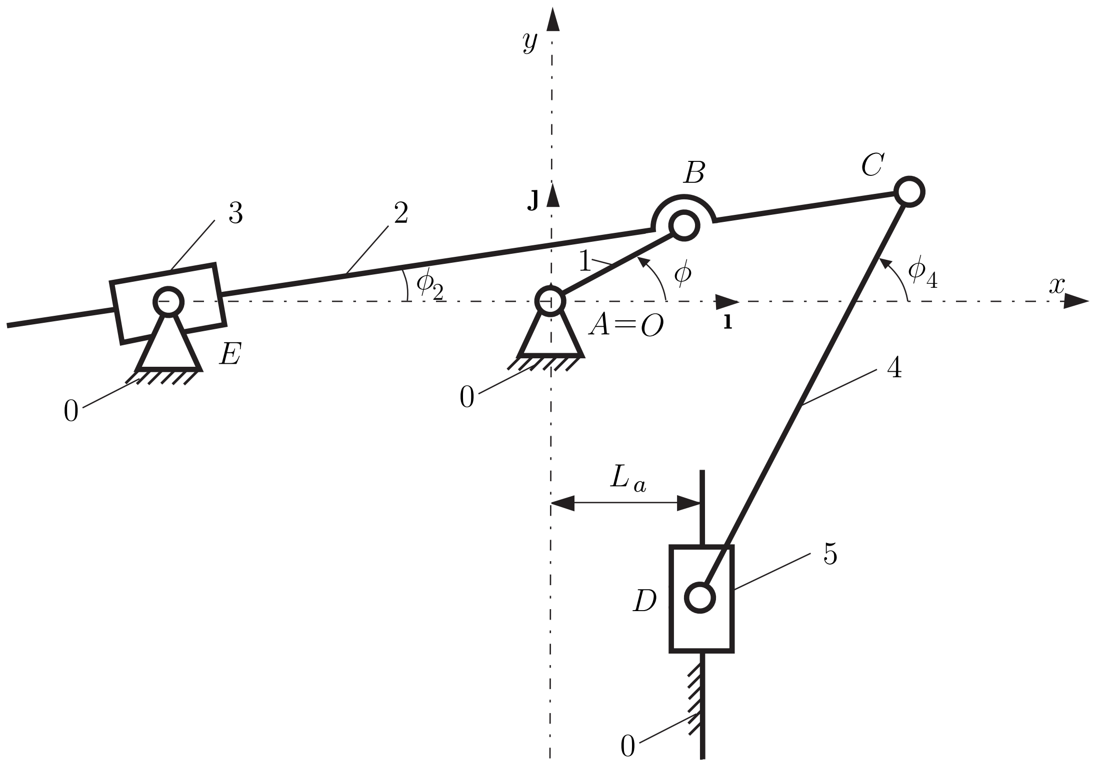


Figure P3.3

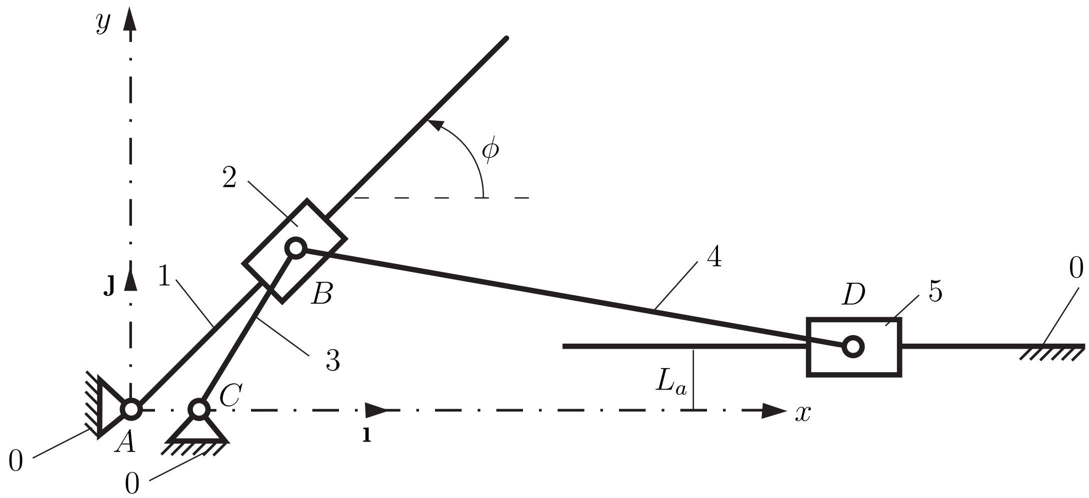


Figure P3.4

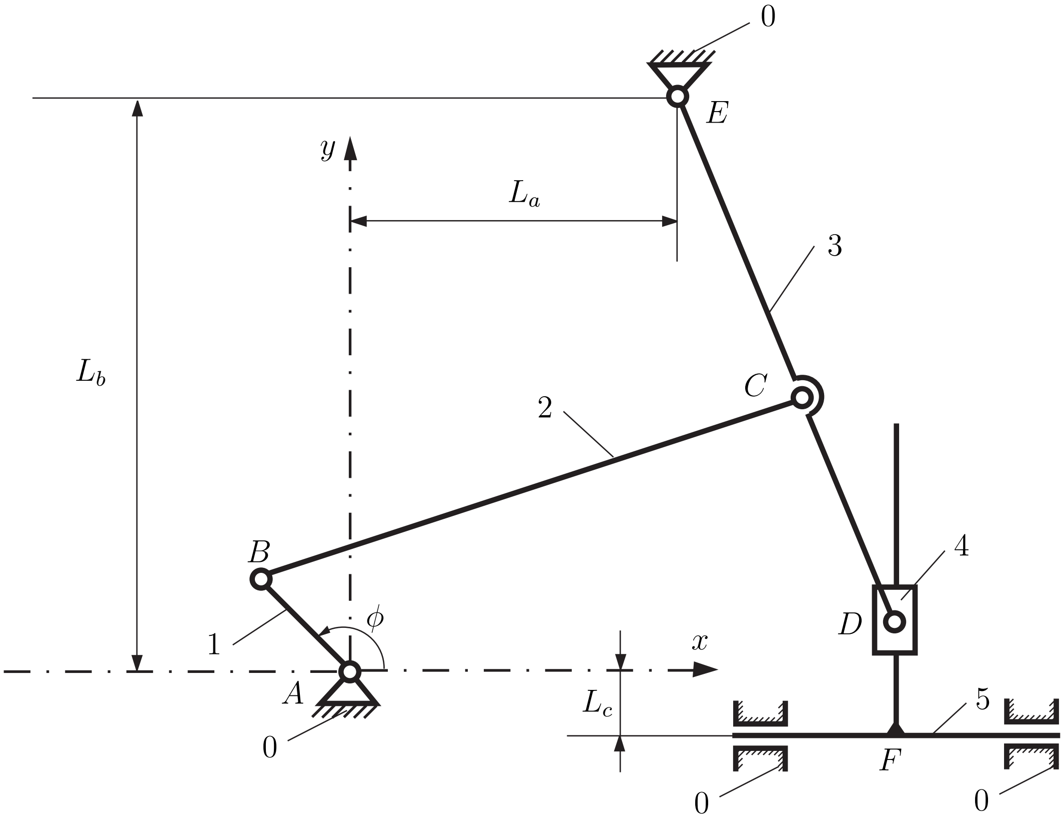


Figure P3.5