

Velocity and Acceleration Analysis

Problem 1.4.4

The planar mechanism considered is shown in Fig. 3.19. The following data are given: $AB=0.150$ m, $BC=0.400$ m, $CD=0.370$ m, $CE=0.230$ m, $EF=CE$, $L_a=0.300$ m, $L_b=0.450$ m, and $L_c=CD$. The constant angular speed of the driver link 1 is 60 rpm. Find the velocities and the accelerations of the mechanism for $\phi = \phi_1 = 30^\circ$.

The joints have the the coordinates [m]:

$$\begin{aligned}x_A &= y_A = 0; \\x_D &= 0.3, \quad y_D = 0.45; \\x_B &= 0.129904, \quad y_B = 0.075; \\x_C &= -0.0689445, \quad y_C = 0.422073; \\x_E &= -0.298288, \quad y_E = 0.404712; \\x_F &= -0.37, \quad y_F = 0.186177.\end{aligned}$$

Velocity of joint B

The velocity of the point $B = B_1$ on the link 1 is

$$\mathbf{v}_B = \mathbf{v}_{B_1} = \mathbf{v}_A + \boldsymbol{\omega}_1 \times \mathbf{r}_{AB} = \boldsymbol{\omega}_1 \times \mathbf{r}_B,$$

where $\mathbf{v}_A \equiv \mathbf{0}$ is the velocity of the origin $A \equiv O$.

The angular velocity of link 1 is

$$\boldsymbol{\omega}_1 = \omega_1 \mathbf{k} = \frac{\pi n}{30} \mathbf{k} = \frac{\pi(60)}{30} \mathbf{k} = 6.28319 \mathbf{k} \text{ rad/s.}$$

the position vector of point B is

$$\mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A = \mathbf{r}_B = x_B \mathbf{i} + y_B \mathbf{j} + z_B \mathbf{k} = 0.129904 \mathbf{i} + 0.075 \mathbf{j} \text{ m.}$$

The velocity of point B_2 on the link 2 is $\mathbf{v}_{B_2} = \mathbf{v}_{B_1}$ because between the links 1 and 2 there is a rotational joint.

The velocity of $B_1 = B_2$ is

$$\mathbf{v}_B = \mathbf{v}_{B_1} = \mathbf{v}_{B_2} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega \\ x_B & y_B & 0 \end{vmatrix} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 6.28319 \\ 0.129904 & 0.075 & 0 \end{vmatrix} = -0.471239 \mathbf{i} + 0.81621 \mathbf{j} \text{ m/s.}$$

Velocity of joint C

The points B_2 and C_2 are on the link 2 and

$$\mathbf{v}_{C_2} = \mathbf{v}_{B_2} + \boldsymbol{\omega}_2 \times \mathbf{r}_{BC} = \mathbf{v}_B + \boldsymbol{\omega}_2 \times (\mathbf{r}_C - \mathbf{r}_B), \quad (1)$$

where the angular velocity of link 2 is $\boldsymbol{\omega}_2 = \omega_2 \mathbf{k}$ (ω_2 is unknown).

The points D_3 and C_3 are on the link 3 and

$$\mathbf{v}_{C_3} = \mathbf{v}_{D_3} + \boldsymbol{\omega}_3 \times \mathbf{r}_{DC} = \boldsymbol{\omega}_3 \times (\mathbf{r}_C - \mathbf{r}_D), \quad (2)$$

where $\mathbf{v}_D = \mathbf{v}_{D_3} \equiv \mathbf{0}$ and the angular velocity of link 3 is

$\boldsymbol{\omega}_3 = \omega_3 \mathbf{k}$ (ω_3 is unknown).

Equations (1) and (2) give ($\mathbf{v}_{C_2} = \mathbf{v}_{C_3}$)

$$\mathbf{v}_B + \boldsymbol{\omega}_2 \times (\mathbf{r}_C - \mathbf{r}_B) = \boldsymbol{\omega}_3 \times (\mathbf{r}_C - \mathbf{r}_D),$$

or

$$\mathbf{v}_B + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_2 \\ x_C - x_B & y_C - y_B & 0 \end{vmatrix} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_3 \\ x_C - x_D & y_C - y_D & 0 \end{vmatrix} \quad (3)$$

Equation (3) represents a vectorial equation with two scalar components on x -axis and y -axis and with two unknowns ω_2 and ω_3

$$\begin{aligned} v_{Bx} - \omega_2(y_C - y_B) &= -\omega_3(y_C - y_D), \\ v_{By} + \omega_2(x_C - x_B) &= \omega_3(x_C - x_D), \end{aligned}$$

or

$$\begin{aligned} -0.471239 - \omega_2(0.422073 - 0.075) &= -\omega_3(0.422073 - 0.45), \\ 0.81621 + \omega_2(-0.0689445 - 0.129904) &= \omega_3(-0.0689445 - 0.3). \end{aligned}$$

It results

$$\omega_2 = -1.1307 \text{ rad/s} \quad \text{and} \quad \omega_3 = -2.82169 \text{ rad/s}.$$

The velocity of C is

$$\begin{aligned} \mathbf{v}_C &= \boldsymbol{\omega}_3 \times (\mathbf{r}_C - \mathbf{r}_D) = -\omega_3(y_C - y_D)\mathbf{i} + \omega_3(x_C - x_D)\mathbf{j} \\ &= -(-2.82169)(0.422073 - 0.45)\mathbf{i} + (-2.82169)(-0.0689445 - 0.3)\mathbf{j} \\ &= -0.0788027\mathbf{i} + 1.04105\mathbf{j} \text{ m/s}. \end{aligned}$$

Velocity of joint E

The points E_3 and D_3 are on the link 3 and

$$\begin{aligned}
 \mathbf{v}_E &= \mathbf{v}_{E_3} = \mathbf{v}_{D_3} + \boldsymbol{\omega}_3 \times \mathbf{r}_{DE} = \boldsymbol{\omega}_3 \times (\mathbf{r}_E - \mathbf{r}_D) \\
 &= -\omega_3(y_E - y_D)\mathbf{i} + \omega_3(x_E - x_D)\mathbf{j} \\
 &= -(-2.82169)(0.404712 - 0.45)\mathbf{i} + (-2.82169)(-0.298288 - 0.3)\mathbf{j} \\
 &= -0.127788\mathbf{i} + 1.68819\mathbf{j} \text{ m/s.}
 \end{aligned}$$

Velocity of joint F

The points F_4 and E_4 are on the link 4 and

$$\mathbf{v}_F = \mathbf{v}_{F_4} = \mathbf{v}_{E_4} + \boldsymbol{\omega}_4 \times \mathbf{r}_{EF} = \mathbf{v}_E + \boldsymbol{\omega}_4 \times (\mathbf{r}_F - \mathbf{r}_E), \quad (4)$$

where the angular velocity of link 4 is $\boldsymbol{\omega}_4 = \omega_4\mathbf{k}$ (ω_4 is unknown).

On the other hand the velocity of F is along the vertical axis (y -axis) because slider 5 translates along y -axis

$$\mathbf{v}_F = \mathbf{v}_{F_5} = \mathbf{v}_{F_4} = v_F\mathbf{j}. \quad (5)$$

Equations (4) and (5) give

$$\mathbf{v}_E + \boldsymbol{\omega}_4 \times (\mathbf{r}_F - \mathbf{r}_E) = v_F\mathbf{j},$$

or

$$\mathbf{v}_E + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_4 \\ x_F - x_E & y_F - y_E & 0 \end{vmatrix} = v_F\mathbf{j} \quad (6)$$

Equation (6) represents a vectorial equation with two scalar components on x -axis and y -axis and with two unknowns ω_4 and v_F

$$\begin{aligned}
 v_{Ex} - \omega_4(y_F - y_E) &= 0, \\
 v_{Ey} + \omega_4(x_F - x_E) &= v_F,
 \end{aligned}$$

or

$$\begin{aligned}
 -0.127788 - \omega_4(0.186177 - 0.404712) &= 0, \\
 1.68819 + \omega_4(-0.37 + 0.298288) &= v_F.
 \end{aligned}$$

It results

$$\omega_4 = 0.58475 \text{ rad/s and } v_F = 1.64625 \text{ m/s.}$$

Acceleration of joint B

The acceleration of the point $B = B_1$ on the link 1 is

$$\begin{aligned}\mathbf{a}_B &= \mathbf{a}_{B_1} = \mathbf{a}_{B_2} = \mathbf{a}_A + \boldsymbol{\alpha}_1 \times \mathbf{r}_B + \boldsymbol{\omega}_1 \times (\boldsymbol{\omega}_1 \times \mathbf{r}_B) = \boldsymbol{\alpha}_1 \times \mathbf{r}_B - \omega_1^2 \mathbf{r}_B \\ &= -(6.28319)^2(0.129904\mathbf{i} + 0.075\mathbf{j}) = -5.1284\mathbf{i} - 2.96088\mathbf{j} \text{ m/s}^2.\end{aligned}$$

The angular acceleration of link 1 is $\boldsymbol{\alpha}_1 = \dot{\boldsymbol{\omega}}_1 = \mathbf{0}$.

Acceleration of joint C

The points C_2 and B_2 are on the link 2 and

$$\mathbf{a}_{C_2} = \mathbf{a}_{B_2} + \boldsymbol{\alpha}_2 \times \mathbf{r}_{BC} - \omega_2^2 \mathbf{r}_{BC} = \mathbf{a}_B + \boldsymbol{\alpha}_2 \times (\mathbf{r}_C - \mathbf{r}_B) - \omega_2^2 (\mathbf{r}_C - \mathbf{r}_B), \quad (7)$$

where the angular acceleration of link 2 is $\boldsymbol{\alpha}_2 = \alpha_2 \mathbf{k}$ (α_2 is unknown).

The points C_3 and D_3 are on the link 3 and

$$\mathbf{a}_{C_3} = \mathbf{a}_{D_3} + \boldsymbol{\alpha}_3 \times \mathbf{r}_{DC} - \omega_3^2 \mathbf{r}_{DC} = \boldsymbol{\alpha}_3 \times (\mathbf{r}_C - \mathbf{r}_D) - \omega_3^2 (\mathbf{r}_C - \mathbf{r}_D), \quad (8)$$

where $\mathbf{a}_D = \mathbf{a}_{D_3} \equiv \mathbf{0}$ and the angular velocity of link 3 is

$\boldsymbol{\alpha}_3 = \alpha_3 \mathbf{k}$ (α_3 is unknown).

Equations (7) and (8) give

$$\mathbf{a}_B + \boldsymbol{\alpha}_2 \times (\mathbf{r}_C - \mathbf{r}_B) - \omega_2^2 (\mathbf{r}_C - \mathbf{r}_B) = \boldsymbol{\alpha}_3 \times (\mathbf{r}_C - \mathbf{r}_D) - \omega_3^2 (\mathbf{r}_C - \mathbf{r}_D),$$

or

$$\begin{aligned}\mathbf{a}_B + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_2 \\ x_C - x_B & y_C - y_B & 0 \end{vmatrix} - \omega_2^2 [(x_C - x_B)\mathbf{i} + (y_C - y_B)\mathbf{j}] = \\ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_3 \\ x_C - x_D & y_C - y_D & 0 \end{vmatrix} - \omega_3^2 [(x_C - x_D)\mathbf{i} + (y_C - y_D)\mathbf{j}]. \quad (9)\end{aligned}$$

Equation (9) represents a vectorial equation with two scalar components on x -axis and y -axis and with two unknowns α_2 and α_3

$$\begin{aligned}a_{Bx} - \alpha_2(y_C - y_B) - \omega_2^2(x_C - x_B) &= -\alpha_3(y_C - y_D) - \omega_3^2(x_C - x_D), \\ a_{By} + \alpha_2(x_C - x_B) - \omega_2^2(y_C - y_B) &= \alpha_3(x_C - x_D) - \omega_3^2(y_C - y_D),\end{aligned}$$

or

$$\begin{aligned}
& -5.1284 - \alpha_2(0.422073 - 0.075) - (-1.1307)^2(-0.0689445 - 0.129904) \\
& = -\alpha_3(0.422073 - 0.45) - (-2.82169)^2(-0.0689445 - 0.3), \\
& -2.96088 + \alpha_2(-0.0689445 - 0.129904) - (-1.1307)^2(0.422073 - 0.075) \\
& = \alpha_3(-0.0689445 - 0.3) - (-2.82169)^2(0.422073 - 0.45).
\end{aligned}$$

It results

$$\alpha_2 = -22.33 \text{ rad/s}^2 \quad \text{and} \quad \alpha_3 = -2.20443 \text{ rad/s}^2.$$

The acceleration of C is

$$\begin{aligned}
\mathbf{a}_C &= \boldsymbol{\alpha}_3 \times (\mathbf{r}_C - \mathbf{r}_D) - \omega_3^2 (\mathbf{r}_C - \mathbf{r}_D) \\
&= [-\alpha_3(y_C - y_D) - \omega_3^2(x_C - x_D)]\mathbf{i} + [\alpha_3(x_C - x_D) - \omega_3^2(y_C - y_D)]\mathbf{j} \\
&= [-(-2.20443)(0.422073 - 0.45) - (-2.82169)^2(-0.0689445 - 0.3)]\mathbf{i} \\
&+ [(-2.20443)(-0.0689445 - 0.3) - (-2.82169)^2(0.422073 - 0.45)]\mathbf{j} \\
&= 2.87595\mathbf{i} + 1.03567\mathbf{j} \text{ m/s}^2.
\end{aligned}$$

Acceleration of joint E

The points E and D are on the link 3 and the acceleration of E is

$$\begin{aligned}
\mathbf{a}_E &= \mathbf{a}_D + \boldsymbol{\alpha}_3 \times \mathbf{r}_{DE} - \omega_3^2 \mathbf{r}_{DE} = \boldsymbol{\alpha}_3 \times (\mathbf{r}_E - \mathbf{r}_D) - \omega_3^2 (\mathbf{r}_E - \mathbf{r}_D) \\
&= [-\alpha_3(y_E - y_D) - \omega_3^2(x_E - x_D)]\mathbf{i} + [\alpha_3(x_E - x_D) - \omega_3^2(y_E - y_D)]\mathbf{j} \\
&= [-(-2.20443)(0.404712 - 0.45) - (-2.82169)^2(-0.298288 - 0.3)]\mathbf{i} \\
&+ [(-2.20443)(-0.298288 - 0.3) - (-2.82169)^2(0.404712 - 0.45)]\mathbf{j} \\
&= 4.66371\mathbf{i} + 1.67947\mathbf{j} \text{ m/s}^2.
\end{aligned}$$

Acceleration of joint F

The points F and E are on the link 4 and

$$\mathbf{a}_F = \mathbf{a}_E + \boldsymbol{\alpha}_4 \times \mathbf{r}_{EF} - \omega_4^2 \mathbf{r}_{EF} = \mathbf{a}_E + \boldsymbol{\alpha}_4 \times (\mathbf{r}_F - \mathbf{r}_E) - \omega_4^2 (\mathbf{r}_F - \mathbf{r}_E), \quad (10)$$

where $\boldsymbol{\alpha}_4 = \alpha_4 \mathbf{k}$ (α_4 is unknown) is the angular acceleration of link 4.

The slider 5 moves along the y -axis and the acceleration of F is

$$\mathbf{a}_F = \mathbf{a}_{F_5} = a_F \mathbf{J}. \quad (11)$$

Equations (10) and (11) yield

$$\mathbf{a}_E + \boldsymbol{\alpha}_4 \times (\mathbf{r}_F - \mathbf{r}_E) - \omega_4^2 (\mathbf{r}_F - \mathbf{r}_E) = a_F \mathbf{J},$$

or

$$\mathbf{a}_F + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_2 \\ x_F - x_E & y_F - y_E & 0 \end{vmatrix} - \omega_4^2 [(x_F - x_E)\mathbf{i} + (y_F - y_E)\mathbf{j}] = a_F \mathbf{J}. \quad (12)$$

Equation (12) represents a vectorial equation with two scalar components on x -axis and y -axis and with two unknowns α_4 and a_F

$$\begin{aligned} a_{Ex} - \alpha_4(y_F - y_E) - \omega_4^2(x_F - x_E) &= 0, \\ a_{Ey} + \alpha_4(x_F - x_E) - \omega_4^2(y_F - y_E) &= v_F, \end{aligned}$$

or

$$\begin{aligned} 4.66371 - \alpha_4(0.186177 - 0.404712) - (0.58475)^2(-0.37 + 0.298288) &= 0, \\ 1.67947 + \alpha_4(-0.37 + 0.298288) - (0.58475)^2(0.186177 - 0.404712) &= v_F. \end{aligned}$$

The following results are obtained

$$\alpha_4 = -21.453 \text{ rad/s}^2 \quad \text{and} \quad a_F = 3.29262 \text{ m/s}^2.$$

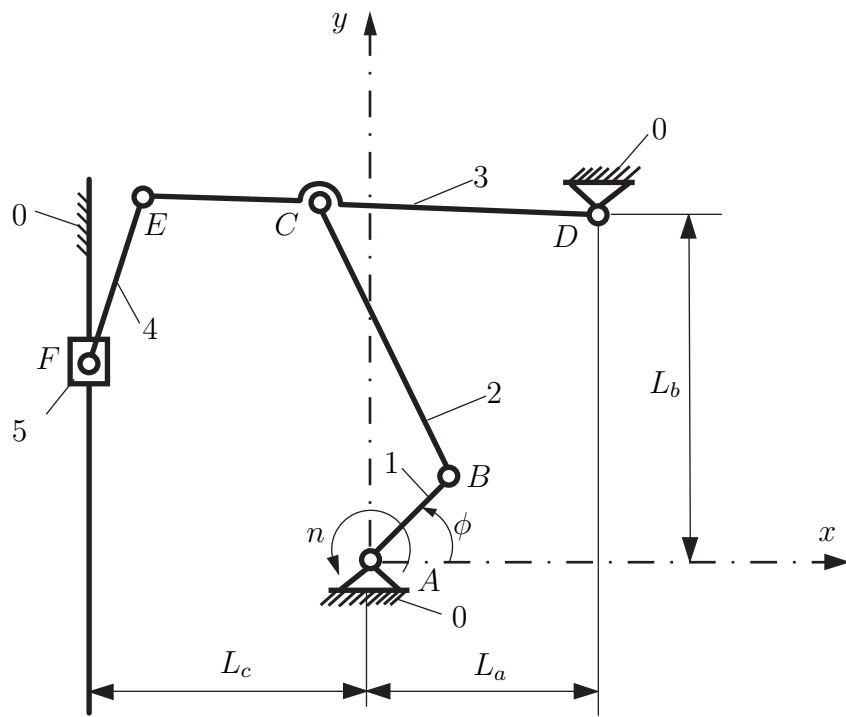


Fig. 19

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(*Problem I.4.4*)
(*velocity and acceleration analysis*)
Apply[Clear,Names["Global`*"]];
Off[General::spell];
Off[General::spell1];

(* Input data *)
AB = 0.15 ;          (* m *)
BC = 0.40 ;          (* m *)
CD = 0.37 ;          (* m *)
Lc = CD ;
CE = 0.23 ;          (* m *)
EF = CE ;
La = 0.30 ;          (* m *)
Lb = 0.45 ;          (* m *)

xD = La ;
yD = Lb ;
xF = - Lc ;

fi = Pi / 6 ;

(* Position of joint A(0,0) *)

(* Position of joint B *)

xB = AB Cos [ fi ] ;
yB = AB Sin [ fi ] ;

Print["xB= ",xB," [m]"];
Print["yB= ",yB," [m]"];

(* Position of joint C *)

eqCa = ( xc - xB )^2 + ( yc - yB )^2 - BC^2 ;
eqCb = ( xc - xD )^2 + ( yc - yD )^2 - CD^2 ;
solution = Solve [ { eqCa == 0 , eqCb == 0 } ,
                  { xc , yc } ] ;
(* Two solutions for C *)
xc1 = xc /. solution [ [ 1 ] ] ;
yc1 = yc /. solution [ [ 1 ] ] ;
xc2 = xc /. solution [ [ 2 ] ] ;
yc2 = yc /. solution [ [ 2 ] ] ;
(* Select the correct position for C *)
If [ xc1 < xD , xC = xc1 ; yC = yc1 ,
    xC = xc2 ; yC = yc2 ] ;
Print["xC= ",xC," [m]"];
Print["yC= ",yC," [m]"];

(* Position of joint E *)

(* Parameters m and n of line CD: y=n x + n *)
m = ( yC - yD ) / ( xC - xD ) ;
n = yC - m xC ;
eqEa = ( xe - xC )^2 + ( ye - yC )^2 - CE^2 ;
eqEb = ye - m xe - n ;
solutionE = Solve [ { eqEa == 0 , eqEb == 0 } ,
                  { xe , ye } ] ;
(* Two solutions for E *)
xe1 = xe /. solutionE [ [ 1 ] ] ;
ye1 = ye /. solutionE [ [ 1 ] ] ;
xe2 = xe /. solutionE [ [ 2 ] ] ;
ye2 = ye /. solutionE [ [ 2 ] ] ;

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(* Select the correct position for E *)
If [ xe1 <= xC , xE = xe1 ; yE = ye1 ,
    xE = xe2 ; yE = ye2 ];

Print["xE= ",xE," [m]"];
Print["yE= ",yE," [m]"];

(* Position of joint F *)

eqF = ( xE - xF )^2 + ( yE - yf )^2 - EF^2 ;
solutionF = Solve [ { eqF == 0 } , { yf } ] ;
(* Two solutions for yF *)
yf1 = yf /. solutionF [ [ 1 ] ] ;
yf2 = yf /. solutionF [ [ 2 ] ] ;
(* Select the correct position for yF *)
If [ yf1 < yE , yF = yf1 ,
    yF = yf2 ] ;

Print["xF= ",xF," [m]"];
Print["yF= ",yF," [m]"];

(*****)
(*Link 2*)
(*****)
phi2=ArcTan[(yB-yC)/(xB-xC)];
"phi2=ArcTan[(yB-yC)/(xB-xC)]"
Print["phi2 = ",phi2," rad = ",phi2*180/N[Pi]," deg "];

(*****)
(*Link 3*)
(*****)
phi3=ArcTan[(yC-yD)/(xC-xD)];
"phi3=ArcTan[(yC-yD)/(xC-xD)]"
Print["phi3 = ",phi3," rad = ",phi3*180/N[Pi]," deg "];

(*****)
(*Link 4*)
(*****)
phi4=ArcTan[(yF-yE)/(xF-xE)];
"phi4=ArcTan[(yF-yE)/(xF-xE)]"
Print["phi4 = ",phi4," rad = ",phi4*180/N[Pi]," deg "];

markers = Table [ {
    Point [ { 0 , 0 } ] ,
    Point [ { xB , yB } ] ,
    Point [ { xC , yC } ] ,
    Point [ { xD , yD } ] ,
    Point [ { xE , yE } ] ,
    Point [ { xF , yF } ] ,
    Point [ { (xF+xE)/2 , (yF+yE)/2 } ] ,
    Point [ { (xB+xC)/2 , (yB+yC)/2 } ]
} ] ;

name = Table [ {
    Text [ "A" , {0 , 0 } , { 3 , 0 } ] ,
    Text [ "B" , {xB , yB } , {-2 , 0 } ] ,
    Text [ "C" , {xC , yC } , {-2 , -2 } ] ,
    Text [ "D" , {xD , yD } , {-3 , 0 } ] ,
    Text [ "E" , {xE , yE } , {3 , 0 } ] ,
    Text [ "F" , {xF , yF } , {3 , 0 } ] ,
    Text [ "C4" , {(xF+xE)/2 , (yF+yE)/2 } ,
        {-2 , 0 } ] ,
    Text [ "C2" , {(xB+xC)/2 , (yB+yC)/2 } ,
        { 2 , 0 } ]
} ]

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    } ] ;

graph = Graphics [
  { { RGBColor [ 1 , 0 , 0 ] ,
      Line [ { {0,0},{xB,yB} } ] } ,
    { RGBColor [ 0 , 0 , 1 ] ,
      Line [ { {xB,yB} , {xC,yC} } ] } ,
    { RGBColor [ 1 , 0 , 1 ] ,
      Line [ { {xD,yD} , {xE,yE} } ] } ,
    { RGBColor [ 0 , 0 , 0 ] ,
      Line [ { {xE,yE},{xF,yF} } ] } ,
    { RGBColor [ 0 , 1 , 1 ] ,
      PointSize [ 0.01 ] , markers } ,
    { name } } ] ;

Show [ Graphics [ graph ] ,
  PlotRange -> { { -0.5 , 0.5 } ,
                { -0.2 , 0.7 } } ,
  Frame -> True,
  AxesOrigin -> {0,0},
  FrameLabel->{"x","y"},
  Axes -> {True,True},
  AspectRatio -> Automatic ] ;

rB={xB,yB,0};
rC={xC,yC,0};
rD={xD,yD,0};
rE={xE,yE,0};
rF={xF,yF,0};
Print["rB = ", rB, " m" ];
Print["rC = ", rC, " m" ];
Print["rD = ", rD, " m" ];
Print["rE = ", rE, " m" ];
Print["rF = ", rF, " m" ];

(*****)

Print["Velocities" ];

n=60.;(*rpm*)
omega=n*N[Pi]/30;(*rad/s*)
Print[" $\omega = \omega_1 =$ ", omega, " rad/s" ];

omega1={0,0,omega};
vB=Cross[omega1,rB];
Print["vB =  $\omega_1 \times rB =$ ",vB, " m/s" ];

omega2={0,0,omega2z};
omega3={0,0,omega3z};

eqvC=vB+Cross[omega2,rC-rB]-Cross[omega3,rC-rD];
Print["vC = vB +  $\omega_2 \times (rC-rB) = \omega_3 \times (rC-rD) \Rightarrow \omega_2, \omega_3$ "];
solutionomega23 =
  Solve [ { eqvC[[1]]==0, eqvC[[2]]==0 } , {omega2z , omega3z } ];
omega2 = {0,0, omega2z/.solutionomega23[[1]]};
omega3 = {0,0,omega3z/.solutionomega23[[1]]};

Print[" $\omega_2 =$ ", omega2, " rad/s" ];
Print[" $\omega_3 =$ ", omega3, " rad/s" ];

vC=Cross[omega3,rC-rD];
Print["vC =  $\omega_3 \times (rC-rD) =$ ",vC, " m/s" ];

vE=Cross[omega3,rE-rD];

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Print["vE =  $\omega_3 \times (rE-rD)$  = ", vE, " m/s" ];

 $\omega_4 = \{0, 0, \omega_{4z}\}$ ;
vF = {0, vFy, 0};
eqvF = vE + Cross[ $\omega_4, rF-rE$ ] - vF;
Print["vF = {0, vFy, 0} = vE +  $\omega_4 \times (rF-rE) \Rightarrow \omega_4, vFy$ "];
solutionvF = Solve [ { eqvF[[1]]==0, eqvF[[2]]==0 } , { $\omega_{4z}, vFy$ } ];
 $\omega_4 = \{0, 0, \omega_{4z} /. solutionvF[[1]]\}$ ;
VF = {0, vFy /. solutionvF[[1]], 0};
Print[" $\omega_4 =$ ",  $\omega_4$  , " rad/s" ];
Print["vF = ", VF , " m/s" ];

(*****)

Print["Accelerations"];

alpha1 = D[ $\omega_{g1}, t$ ];
aB = Cross[alpha1, rB] -  $\omega_{g1} . \omega_{g1} rB$ ;
Print["aB =  $\alpha_1 \times rB - \omega_1^2 rB$  = ", aB, " m/s^2" ];

 $\alpha_2 = \{0, 0, \alpha_{2z}\}$ ;
 $\alpha_3 = \{0, 0, \alpha_{3z}\}$ ;

eqaC2 = aB + Cross[ $\alpha_2, rC-rB$ ] -  $\omega_{g2} . \omega_{g2} (rC-rB)$ ;
eqaC3 = Cross[ $\alpha_3, rC-rD$ ] -  $\omega_{g3} . \omega_{g3} (rC-rD)$ ;
eqaC = eqaC2 - eqaC3;

Print["aC2 = aB +  $\alpha_2 \times (rC-rB) - \omega_2^2 (rC-rB)$ "];
Print["aC3 =  $\alpha_3 \times (rC-rD) - \omega_3^2 (rC-rD)$ "];
Print["aC = aC2 = aC3  $\Rightarrow \alpha_2, \alpha_3$ "];

solution $\alpha_{23}$  =
  Solve [ { eqaC[[1]]==0, eqaC[[2]]==0 } , { $\alpha_{2z}, \alpha_{3z}$ } ];
alpha2 = {0, 0,  $\alpha_{2z} /. solution\alpha_{23}[[1]]$ };
alpha3 = {0, 0,  $\alpha_{3z} /. solution\alpha_{23}[[1]]$ };

Print[" $\alpha_2 =$ ", alpha2 , " rad/s^2" ];
Print[" $\alpha_3 =$ ", alpha3 , " rad/s^2" ];

aC = Cross[alpha3, rC-rD] -  $\omega_{g3} . \omega_{g3} (rC-rD)$ ;
Print["aC =  $\alpha_3 \times (rC-rD) - \omega_3^2 (rC-rD)$  = ", aC , " m/s^2" ];

aE = Cross[alpha3, rE-rD] -  $\omega_{g3} . \omega_{g3} (rE-rD)$ ;
Print["aE =  $\alpha_3 \times (rE-rD) - \omega_3^2 (rE-rD)$  = ", aE , " m/s^2" ];

 $\alpha_4 = \{0, 0, \alpha_{4z}\}$ ;
aF = {0, aFy, 0};
eqaF = aE + Cross[ $\alpha_4, rF-rE$ ] -  $\omega_{g4} . \omega_{g4} (rF-rE) - aF$ ;
Print["aF = {0, aFy, 0} = aE +  $\alpha_4 \times (rF-rE) - \omega_4^2 (rF-rE) \Rightarrow \alpha_4, aFy$ "];
solutionaF = Solve [ { eqaF[[1]]==0, eqaF[[2]]==0 } , { $\alpha_{4z}, aFy$ } ];
alpha4 = {0, 0,  $\alpha_{4z} /. solutionaF[[1]]$ };
AF = {0, aFy /. solutionaF[[1]], 0};
Print[" $\alpha_4 =$ ", alpha4 , " rad/s^2" ];
Print["aF = ", AF , " m/s^2" ];

xB = 0.129904 [m]

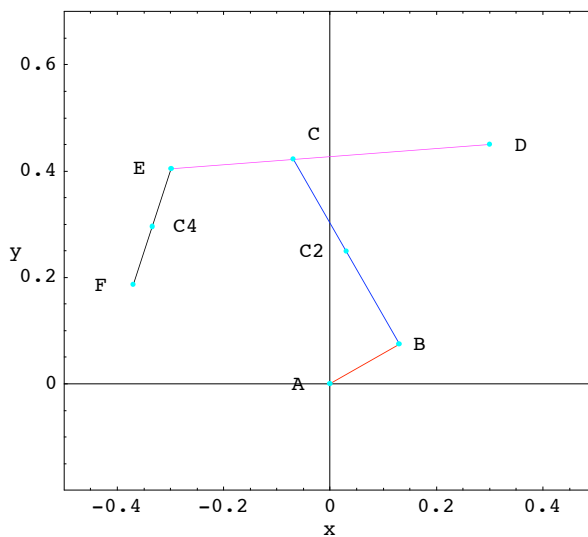
yB = 0.075 [m]

```

```

xC= -0.0689445 [m]
yC= 0.422073 [m]
xE= -0.298288 [m]
yE= 0.404712 [m]
xF= -0.37 [m]
yF= 0.186177 [m]
φ2=ArcTan[ (yB-yC) / (xB-xC) ]
φ2 = -1.05052 rad = -60.1903 deg
φ3=ArcTan[ (yC-yD) / (xC-xD) ]
φ3 = 0.0755515 rad = 4.32878 deg
φ4=ArcTan[ (yF-yE) / (xF-xE) ]
φ4 = 1.25372 rad = 71.8329 deg

```



```
rB = {0.129904, 0.075, 0} m
```

```
rC = {-0.0689445, 0.422073, 0} m
```

```
rD = {0.3, 0.45, 0} m
```

```
rE = {-0.298288, 0.404712, 0} m
```

```
rF = {-0.37, 0.186177, 0} m
```

Velocities

```
ω = ω1 = 6.28319 rad/s
```

```
vB = ω1 x rB = {-0.471239, 0.81621, 0.} m/s
```

```
vC = vB + ω2 x (rC-rB) = ω3 x (rC-rD) => ω2, ω3
```

$$\omega_2 = \{0, 0, -1.1307\} \text{ rad/s}$$

$$\omega_3 = \{0, 0, -2.82169\} \text{ rad/s}$$

$$v_C = \omega_3 \times (r_C - r_D) = \{-0.0788027, 1.04105, 0.\} \text{ m/s}$$

$$v_E = \omega_3 \times (r_E - r_D) = \{-0.127788, 1.68819, 0.\} \text{ m/s}$$

$$v_F = \{0, v_{Fy}, 0\} = v_E + \omega_4 \times (r_F - r_E) \Rightarrow \omega_4, v_{Fy}$$

$$\omega_4 = \{0, 0, 0.58475\} \text{ rad/s}$$

$$v_F = \{0, 1.64625, 0\} \text{ m/s}$$

Accelerations

$$a_B = \alpha_1 \times r_B - \omega_1^2 r_B = \{-5.1284, -2.96088, 0.\} \text{ m/s}^2$$

$$a_{C2} = a_B + \alpha_2 \times (r_C - r_B) - \omega_2^2 (r_C - r_B)$$

$$a_{C3} = \alpha_3 \times (r_C - r_D) - \omega_3^2 (r_C - r_D)$$

$$a_C = a_{C2} = a_{C3} \Rightarrow \alpha_2, \alpha_3$$

$$\alpha_2 = \{0, 0, -22.33\} \text{ rad/s}^2$$

$$\alpha_3 = \{0, 0, -2.20443\} \text{ rad/s}^2$$

$$a_C = \alpha_3 \times (r_C - r_D) - \omega_3^2 (r_C - r_D) = \{2.87595, 1.03567, 0.\} \text{ m/s}^2$$

$$a_E = \alpha_3 \times (r_E - r_D) - \omega_3^2 (r_E - r_D) = \{4.66371, 1.67947, 0.\} \text{ m/s}^2$$

$$a_F = \{0, a_{Fy}, 0\} = a_E + \alpha_4 \times (r_F - r_E) - \omega_4^2 (r_F - r_E) \Rightarrow \alpha_4, a_{Fy}$$

$$\alpha_4 = \{0, 0, -21.453\} \text{ rad/s}^2$$

$$a_F = \{0, 3.29262, 0\} \text{ m/s}^2$$