1 Problem

The R-RTR mechanism shown in Fig. 1.1 has the following dimensions: $AB = 1$ m, $AC = \sqrt{2}$ m, $AD = 3$ m. The angle ϕ_1 and ϕ_2 are $\phi_1 = 45°$ and $\phi_1 = 135°$. The coordinates of A, B, C, and D are given in [m]:
$x_A = y_A = 0;\quad x_B = y_B = \dfrac{\sqrt{2}}{2};\quad x_C = \sqrt{2},\, y_C = 0;$
$x_D = \dfrac{3\sqrt{2}}{2},\; y_D = -\dfrac{\sqrt{2}}{2}.$
The driver link 1 is rotating with a constant angular speed $\omega = 2$ rad/s. The position vectors of the mass centers of links 1 and 2 are

$$\mathbf{r}_{C_1} = x_{C_1}\mathbf{\imath} + y_{C_1}\mathbf{\jmath} = \frac{x_B}{2}\mathbf{\imath} + \frac{y_B}{2}\mathbf{\jmath} = \frac{\sqrt{2}}{4}\mathbf{\imath} + \frac{\sqrt{2}}{4}\mathbf{\jmath} \text{ m},$$
$$\mathbf{r}_{C_2} = x_{C_2}\mathbf{\imath} + y_{C_2}\mathbf{\jmath} = \frac{x_B + x_D}{2}\mathbf{\imath} + \frac{y_B + y_D}{2}\mathbf{\jmath} = \frac{5\sqrt{2}}{4}\mathbf{\imath} - \frac{\sqrt{2}}{4}\mathbf{\jmath} \text{ m}.$$

The mass center of the slider 3 is at C ($C = C_3$): $\mathbf{r}_{C_3} = \mathbf{r}_C$. The width of the slider 3 is $w_{Slider} = 0.01$ m and the height is $h_{Slider} = 0.01$ m. The masses of the links are $m_1 = m_2 = m_3 = 1$ kg. The gravitational forces are $\mathbf{G}_i = -m_i\, g\, \mathbf{\jmath} = -10\, \mathbf{\jmath}$ N $\quad i = 1, 2, 3$.

Inertia forces and moments

Link 1
The acceleration of C_1 is

$$\mathbf{a}_{C_1} = -\sqrt{2}\mathbf{\imath} - \sqrt{2}\mathbf{\jmath} \text{ m/s}^2.$$

The inertia force on link 1 at C_1 is

$$\mathbf{F}_{in\,1} = -m_1\, \mathbf{a}_{C_1} = \sqrt{2}\mathbf{\imath} + \sqrt{2}\mathbf{\jmath} \text{ N}.$$

The total force on link 1 at the mass center C_1 is

$$\mathbf{F}_1 = \mathbf{F}_{in\,1} + \mathbf{G}_1 = \sqrt{2}\mathbf{\imath} + (\sqrt{2} - 10)\mathbf{\jmath} \text{ N}.$$

The mass moment of inertia is

$$I_{C_1} = \frac{m_1\, AB^2}{12} = \frac{1}{12} \text{ kg} \cdot \text{m}^2.$$

The moment of inertia is

$$\mathbf{M}_1 = \mathbf{M}_{in\,1} = -I_{C_1}\ \boldsymbol{\alpha}_1 = \mathbf{0}.$$

Link 2
The acceleration of C_2 is

$$\mathbf{a}_{C_2} = \sqrt{2}\boldsymbol{\imath} + \sqrt{2}\boldsymbol{\jmath}\ \mathrm{m/s}^2.$$

The inertia force on link 2 at C_2 is

$$\mathbf{F}_{in\,2} = -m_2\,\mathbf{a}_{C_2} = -\sqrt{2}\boldsymbol{\imath} - \sqrt{2}\boldsymbol{\jmath}\ \mathrm{N}.$$

The total force on link 2 at the mass center C_2 is

$$\mathbf{F}_2 = \mathbf{F}_{in\,2} + \mathbf{G}_2 = -\sqrt{2}\boldsymbol{\imath} - (\sqrt{2} + 10)\boldsymbol{\jmath}\ \mathrm{N}.$$

The mass moment of inertia is

$$I_{C_2} = \frac{m_2\ BD^2}{12} = \frac{3}{4}\ \mathrm{kg \cdot m^2}.$$

The moment of inertia is

$$\mathbf{M}_2 = \mathbf{M}_{in\,2} = -I_{C_2}\ \boldsymbol{\alpha}_2 = -3\,\mathbf{k}\ \mathrm{N \cdot m}.$$

where $\boldsymbol{\alpha}_2 = 4\,\mathbf{k}\ \mathrm{rad/s}^2$.
There is an external moment on link 2

$$\mathbf{M}_{2ext} = -100\mathbf{k}\ \mathrm{N \cdot m}.$$

Link 3
The acceleration of $C_3 = C$ is $\mathbf{a}_{C_3} = \mathbf{0}\ \ \mathrm{m/s}^2$. The inertia force on link 3 at C_3 is $\mathbf{F}_{in\,3} = -m_3\,\mathbf{a}_{C_3} = \mathbf{0}$ N. The total force on link 3 at the mass center C_3 is

$$\mathbf{F}_3 = \mathbf{F}_{in\,3} + \mathbf{G}_3 = -10\boldsymbol{\jmath}\ \mathrm{N}.$$

The mass moment of inertia is

$$I_{C_3} = \frac{m_3\ (h^2_{Slider} + w^2_{Slider})}{12} = 0.0000166667\ \mathrm{kg \cdot m^2}.$$

The moment of inertia is

$$\mathbf{M}_3 = \mathbf{M}_{in\,3} = -I_{C_3}\ \boldsymbol{\alpha}_3 = -0.0000666667\,\mathbf{k}\ \mathrm{N \cdot m}.$$

where $\boldsymbol{\alpha}_3 = \boldsymbol{\alpha}_2 = 4\,\mathbf{k}$ rad/s^2.

1. Sketch the free body diagram for each link;
2. Write the Newton-Euler equations of motion for each link;
3. Determine the moment $\mathbf{M}$ required for dynamic equilibrium and the joint forces for the mechanism.

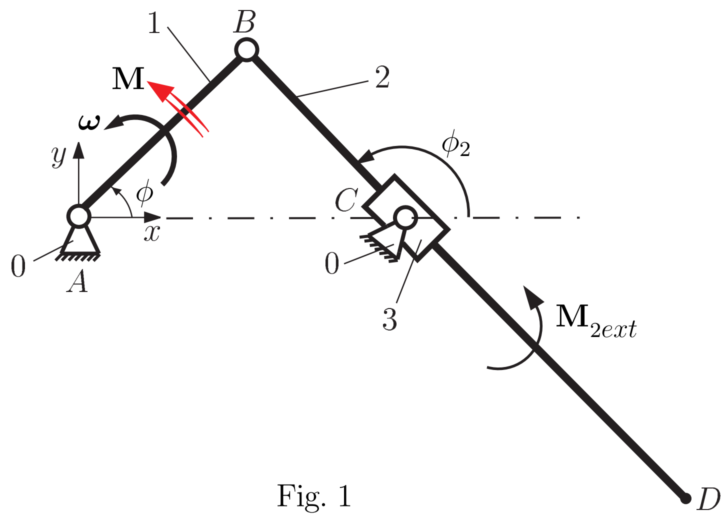

Fig. 1