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(* Problem I.6.8 *)
(* *)

Apply[Clear,Names["Global`*"]];
Off[General::spell];
Off[General::spell1];

(* Input data *)
AB = .11 ;
AC = .055 ;
BD = .22 ;
DE = .3 ;
EF = .175 ;
La = .275 ;
Lb = .065;

n = 2400. ; (* rpm *)
 $\omega = n * N[\text{Pi}] / 30 ;$  (* rad/s *)

initdata = { $\phi[t] \rightarrow N[\text{Pi}] 150 / 180$ };

(* Position of joint A *)
xA = yA = 0; rA={xA,yA,0};

(* Position of joint C *)
xC = -AC ;
yC = 0. ;
rC={xC,yC,0};

(* Position of joint F *)
xF = La ;
yF = Lb ;
rF={xF,yF,0};

(* Position of joint B *)
XB = AB Cos[ $\phi[t]$ ] ;
YB = AB Sin[ $\phi[t]$ ] ;
xBs=XB/.initdata;
yBs=YB/.initdata;
rB={xBs,yBs,0};
ruleB={xB[t] $\rightarrow$ xBs,yB[t] $\rightarrow$ yBs};

phi2 = (ArcTan[(yB[t]-yC)/(xB[t]-xC) ])/.ruleB ;

(* Position of joint D *)
eqnD1 = ( xD[t]- xB[t] )^2 + ( yD[t] - yB[t] )^2 - BD^2 == 0 ;
eqnD2 = ( yD[t] - yB[t] ) / ( xD[t]- xB[t] ) == ( yB[t] - yC ) /
( xB[t] - xC );
solutionD = Solve [ { eqnD1/.ruleB, eqnD2/.ruleB } , { xD[t] ,

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yD[t] } ];
(* Two solutions for D *)
xD1 = xD[t] /. solutionD[[1]];
yD1 = yD[t] /. solutionD[[1]];
xD2 = xD[t] /. solutionD[[2]];
yD2 = yD[t] /. solutionD[[2]];
If [ yD1 < 0 , xDs = xD1 ; yDs = yD1 , xDs = xD2 ; yDs=yD2 ] ;
rD={xDs,yDs,0};
ruleD={xD[t]→xDs,yD[t]→yDs};

(* Position of joint E *)
eqnE1=(xE[t]-xD[t])^2+(yE[t]-yD[t])^2-DE^2==0;
eqnE2=(xE[t]-xF)^2+(yE[t]-yF)^2-EF^2==0;
solutionE=Solve[{eqnE1/.ruleD,eqnE2/.ruleD},{xE[t],yE[t]};
(*Two solutions for E*)
xE1=xE[t]/.solutionE[[1]];
yE1=yE[t]/.solutionE[[1]];
xE2=xE[t]/.solutionE[[2]];
yE2=yE[t]/.solutionE[[2]];
(*Select the correct position for E*)
If[yE1 ≤ 0,xEs=xE1;yEs=yE1,xEs=xE2;yEs=yE2];
rE={xEs,yEs,0};
ruleE={xE[t]→xEs,yE[t]→yEs};

phi4 = (ArcTan[(yD[t]-yE[t])/(xD[t]-xE[t])])/.ruleD/.ruleE;

(*GRAPH OF THE MECHANISM*)

markers=Table[{Point[{xA,yA}],Point[{xBs,yBs}],Point[{xC,yC}],Point[{xDs,yDs}],
Point[{xEs,yEs}],Point[{xF,yF}]}];

name=Table[{
Text["A",{0,0},{-1,1}],
Text["B",{xBs,yBs},{-2,1}],
Text["C",{xC,yC},{-1,-1}],
Text["D",{xDs,yDs},{-1,-1}],
Text["E",{xEs,yEs},{-0.5,1}],
Text["F",{xF,yF},{-2,1}]}];

graph=Graphics[{{RGBColor[1,0,0],Line[{{xA,yA},{xBs,yBs}]}},{RGBColor[0,1,0],
Line[{{xBs,yBs},{xC,yC}]}},{RGBColor[0,1,0],
Line[{{xC,yC},{xDs,yDs}]}},{RGBColor[0,0,1],
Line[{{xDs,yDs},{xEs,yEs}]}},{RGBColor[0,1,1],
Line[{{xEs,yEs},{xF,yF}]}},{RGBColor[0,0,0],PointSize[0.01],
markers},{name}];

Show[Graphics[graph],PlotRange→{All,All},Frame→True,

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    AxesOrigin→{xA,yA},FrameLabel→{"x","y"},
    Axes→{True,True},AspectRatio→Automatic];

φ2 = (ArcTan[ (yB[t]-yC)/(xB[t]-xC) ])/.ruleB ;

(*0-Ar-1-Br-2-Ct-3-Cr-0*)

ω10={0,0,ω};
ω21v={0,0,omega21};
ω03v={0,0,omega03};
vC32v={vC32 Cos[φ2],vC32 Sin[φ2],0};

eqIkv=(ω10+ω21v+ω03v)[[3]]==0;
eqIijv=Cross[rB,ω21v]+Cross[rC,ω03v]+vC32v;
eqIiv=eqIijv[[1]]==0;
eqIjv=eqIijv[[2]]==0;
solIvel=Solve[{eqIkv,eqIiv,eqIjv},{omega21,omega03,vC32}];

omega21s=omega21/.solIvel[[1]];
omega03s=omega03/.solIvel[[1]];
vC32s=vC32/.solIvel[[1]];

ω21={0,0,omega21s};
ω03={0,0,omega03s};
v32={vC32s Cos[φ2],vC32s Sin[φ2],0};

ω20=ω10+ω21;

vB=Cross[ω10,rB];
vD=vB+Cross[ω20,rD-rB];

(*accelerations*)

α10=D[ω10,t];
α21v={0,0,alpha21};
α03v={0,0,alpha03};
aC32v={aC32 Cos[φ2],aC32 Sin[φ2],0};

eqIka=(α10+α21v+α03v)[[3]]==0;
eqIija=Cross[rB,α21v]+Cross[rC,α03v]+
    aC32v-ω10.ω10 rB-ω20.ω20 (rC-
    rB)+2 Cross[ω20,v32];
eqIia=eqIija[[1]]==0;
eqIja=eqIija[[2]]==0;
solIacc=Solve[{eqIka,eqIia,eqIja},{alpha21,alpha03,aC32}];

alpha21s=alpha21/.solIacc[[1]];
alpha03s=alpha03/.solIacc[[1]];
aC32s=aC32/.solIacc[[1]];

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α21={0,0,alpha21s};
α03={0,0,alpha03s};
a32={aC32s Cos[φ2],aC32s Sin[φ2],0};

α20=α10+α21;

aB=Cross[α10,rB]-ω10.ω10 rB;
aD=aB+Cross[α20,rD-rB]-ω20.ω20 (rD-rB);

(*0-Ar-1-Br-2-Dr-4-Er-5-Fr-0*)

ω42v={0,0,omega42};
ω54v={0,0,omega54};
ω05v={0,0,omega05};

eqIIkv=(ω10+ω21+ω42v+ω54v+ω05v)[[3]]==0;
eqIIijv=Cross[rB,ω21]+Cross[rD,ω42v]+Cross[rE,ω54v]+Cross[rF,ω05v];
eqIIiv=eqIIijv[[1]]==0;
eqIIjv=eqIIijv[[2]]==0;
solIIvel=Solve[{eqIIkv,eqIIiv,eqIIjv},{omega42,omega54,omega05}];

omega42s=omega42/.solIIvel[[1]];
omega54s=omega54/.solIIvel[[1]];
omega05s=omega05/.solIIvel[[1]];

ω42={0,0,omega42s};
ω54={0,0,omega54s};
ω05={0,0,omega05s};

ω40=ω20+ω42;
ω50=-ω05;

vE=vD+Cross[ω40,rE-rD];

(*accelerations*)

α42v={0,0,alpha42};
α54v={0,0,alpha54};
α05v={0,0,alpha05};

eqIIka=(α10+α21+α42v+α54v+α05v)[[3]]==0;
eqIIija=Cross[rB,α21]+Cross[rD,α42v]+Cross[rE,α54v]+Cross[rF,α05v]-ω10.ω10 rB-ω20.ω20 (rD-rB)-ω40.ω40 (rE-rD)-ω50.ω50 (rF-rE);
eqIIia=eqIIija[[1]]==0;
eqIIja=eqIIija[[2]]==0;
solIIa=Solve[{eqIIka,eqIIia,eqIIja},{alpha42,alpha54,alpha05}];

alpha42s=alpha42/.solIIa[[1]];

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alpha54s=alpha54/.solIIa[[1]];
alpha05s=alpha05/.solIIa[[1]];

α42={0,0,alpha42s};
α54={0,0,alpha54s};
α05={0,0,alpha05s};

α40=α20+α42;
α50=-α05;

aE=aD+Cross[α40,rE-rD]-ω40.ω40 (rE-rD);

ω5=ω50;

Mext=-Sign[ω5]{0,0,600};
Print["Mext = ",Mext," N"];

α1={0,0,0};
α2=α3=α20;
α4=α40;
α5=α50;

Print["rB = ",rB," m "];
Print["rC = ",rC," m "];
Print["rD = ",rD," m "];
Print["rE = ",rE," m "];
Print["rF = ",rF," m "];

Print["α1 = ",α1," rad/s^2 "];
Print["α2 = ",α2," rad/s^2 "];
Print["α3 = ",α3," rad/s^2 "];
Print["α4 = ",α4," rad/s^2 "];
Print["α5 = ",α5," rad/s^2 "];

h=0.01;
d=0.001;
hSlider=0.02;
wSlider=0.05;
ρ=8000.;
g=9.807;

(*Link 1*)
m1=ρ AB h d;
rC1=rB/2;
aC1=aB/2;
Print["rC1 = ",rC1," m "];
Print["aC1 = ",aC1," m/s^2 "];
Fin1=-m1 aC1;
G1={0,-m1 g,0};

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F1=(Fin1+G1);
IC1=m1 (AB^2+h^2)/12;
M1=Min1=-IC1  $\alpha$ 1;
Print["m1 = ",m1," kg"];
Print["IC1 = ",IC1," kg m^2"];
Print["Fin1 = -m1 aC1 = ",Fin1," N "];
Print["G1 = -m1 g = ",G1," N "];
Print["F1 = -m1 aC1 + G1 = ",F1," N "];
Print["M1 = Min1 = -IC1  $\alpha$ 1 = ",{0,0,0}," N m"];

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(*Link 2*)
m2= $\rho$  BD h d;
rC2=(rB+rD)/2;
aC2=(aB+aD)/2;
Print["rC2 = ",rC2," m "];
Print["aC2 = ",aC2," m/s^2 "];
Fin2=-m2 aC2;
G2={0,-m2 g,0};
F2=(Fin2+G2);
IC2=m2 (BD^2+h^2)/12;
M2=Min2=-IC2  $\alpha$ 2;
Print["m2 = ",m2," kg"];
Print["IC2 = ",IC2," kg m^2"];
Print["Fin2 = -m2 aC2 = ",Fin2," N "];
Print["G2 = -m2 g = ",G2," N "];
Print["F2 = -m2 aC2 + G2 = ",F2," N "];
Print["M2 = Min2 = -IC2  $\alpha$ 2 = ",M2," N m"];

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(*Link 3*)
m3= $\rho$  hSlider wSlider d;
rC3=rC;
aC3={0,0,0};
Print["rC3 = ",rC3," m "];
Print["aC3 = ",aC3," m/s^2 "];
Fin3=-m3 aC3;
G3={0,-m3 g,0};
F3=(Fin3+G3);
IC3=m3 (hSlider^2+wSlider^2)/12;
M3=Min3=-IC3  $\alpha$ 3;
Print["m3 = ",m3," kg"];
Print["IC3 = ",IC3," kg m^2"];
Print["Fin3 = -m3 aC3 = ",Fin3," N "];
Print["G3 = -m3 g = ",G3," N "];
Print["F3 = -m3 aC3 + G3 = ",F3," N "];
Print["M3 = Min3 = -IC3  $\alpha$ 3 = ",M3," N m"];

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(*Link 4*)
m4= $\rho$  DE h d;
rC4=(rD+rE)/2;

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aC4=(aD+aE)/2;
Print["rC4 = ",rC4," m "];
Print["aC4 = ",aC4," m/s^2 "];
Fin4=-m4 aC4;
G4={0,-m4 g,0};
F4=(Fin4+G4);
IC4=m4 (DE^2+h^2)/12;
M4=Min4=-IC4 α4;
Print["m4 = ",m4," kg"];
Print["IC4 = ",IC4," kg m^2"];
Print["Fin4 = -m4 aC4 = ",Fin4," N "];
Print["G4 = -m4 g = ",G4," N "];
Print["F4 = -m4 aC4 + G4 = ",F4," N "];
Print["M4 = Min4 = -IC4 α4 = ",M4," N m"];

(*Link 5*)
m5=ρ EF h d;
rC5=(rE+rF)/2;
aC5=aE/2;
Print["rC5 = ",rC5," m "];
Print["aC5 = ",aC5," m/s^2 "];
Fin5=-m5 aC5;
G5={0,-m5 g,0};
F5=(Fin5+G5);
IC5=m5 (EF^2+h^2)/12;
M5=Min5=-IC5 α5;
Print["m5 = ",m5," kg"];
Print["IC5 = ",IC5," kg m^2"];
Print["Fin5 = -m5 aC5 = ",Fin5," N "];
Print["G5 = -m5 g = ",G5," N "];
Print["F5 = -m5 aC5 + G5 = ",F5," N "];
Print["M5 = Min5 = -IC5 α5 = ",M5," N m"];

Print[" "];
Print[" Fr "];
F05={F05x,F05y,0};
eqME5=(Cross[rF-rE,F05]+Cross[rC5-rE,F5]+M5+Mext)[[3]];
eqMD54=(Cross[rF-rD,F05]+Cross[rC5-rD,F5]+M5+Mext+Cross[rC4-rD,F4]+M4)[[3]];
solF05=Solve[{eqME5==0,eqMD54==0},{F05x,F05y}];
F05s=F05/.solF05[[1]];
Print["F05 = ",F05s," N"];

Print[" "];
Print[" Er "];
F45={F45x,F45y,0};
eqMF5=(Cross[rE-rF,F45]+Cross[rC5-rF,F5]+M5+Mext)[[3]];
eqMD4=(Cross[rE-rD,-F45]+Cross[rC4-rD,F4]+M4)[[3]];
solF45=Solve[{eqMF5==0,eqMD4==0},{F45x,F45y}];
F45s=F45/.solF45[[1]];

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Print["F45 = ",F45s," N"];

Print[" "];
Print[" Dr "];
F24={F24x,F24y,0};
eqME4=(Cross[rD-rE,F24]+Cross[rC4-rE,F4]+M4)[[3]];
eqMF54=(Cross[rD-rF,F24]+Cross[rC4-rF,F4]+M4+Cross[rC5-rF,F5]+M5+
Mext)[[3]];
solF24=Solve[{eqME4==0,eqMF54==0},{F24x,F24y}];
F24s=F24/.solF24[[1]];
Print["F24 = ",F24s," N"];

Print[" "];
Print[" Ct "];
F23={F23x,F23y,0};
(*F23 perpend. on BD*)
eqF23p=F23.(rB-rD)==0;
rQ={xQ,yQ,0};
(*point Q is on BD*)
eqQ=Cross[rB-rC,rC-rQ][[3]]==0;
eqMC3=(Cross[rQ-rC,F23]+M3)[[3]];
eqMB2=(Cross[rQ-rB,-F23]+Cross[rD-rB,-F24s]+Cross[rC2-rB,F2]+M2)[
[3]];
solF23=Solve[{eqF23p,eqQ,eqMC3==0,eqMB2==0},{F23x,F23y,xQ,yQ}];
F23s=F23/.solF23[[1]];
Print["F23 = ",F23s," N"];
rQs=rQ/.solF23[[1]];
Print["rQ = ",rQs," m"];

Print[" "];
Print[" Cr "];
F03={F03x,F03y,0};
eqF3BC=(F03+F3).(rB-rC);
eqMB32=(Cross[rC-rB,F03+F3]+M3+Cross[rD-rB,-F24s]+Cross[rC2-rB,F2
]+M2)[[3]];
solF03=Solve[{eqF3BC==0,eqMB32==0},{F03x,F03y}];
F03s=F03/.solF03[[1]];
Print["F03 = ",F03s," N"];

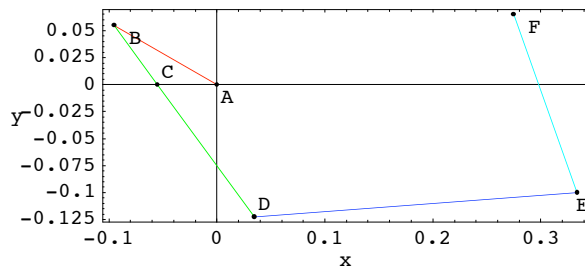
Print[" "];
Print[" Br "];
F12={F12x,F12y,0};
eqF2BC=(F12+F2-F24s).(rB-rC);
eqMC23=(Cross[rB-rC,F12]+Cross[rD-rC,-F24s]+Cross[rC2-rC,F2]+M2+M
3)[[3]];
solF12=Solve[{eqF2BC==0,eqMC23==0},{F12x,F12y}];
F12s=F12/.solF12[[1]];
Print["F12 = ",F12s," N"];

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Print[" "]
(*Link 1*)
F01=-F1+F12s;
Print["F01 = ",F01," N"];
Mm=-Cross[rB,-F12s]-Cross[rC1,F1]-M1;
Print["Meq = ",Mm," N m"]

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Mext = {0, 0, -600} N

rB = {-0.0952628, 0.055, 0} m

rC = {-0.055, 0., 0} m

rD = {0.0346891, -0.122518, 0} m

rE = {0.333829, -0.0998155, 0} m

rF = {0.275, 0.065, 0} m

α1 = {0, 0, 0} rad/s^2

α2 = {0, 0, 80329.7} rad/s^2

α3 = {0, 0, 80329.7} rad/s^2

α4 = {0, 0, -38261.2} rad/s^2

α5 = {0, 0, 55267.7} rad/s^2

rC1 = {-0.0476314, 0.0275, 0} m

aC1 = {3008.66, -1737.05, 0.} m/s^2

m1 = 0.0088 kg

IC1 = 8.94667×10-6 kg m^2

Fin1 = -m1 aC1 = {-26.4762, 15.286, 0.} N

G1 = -m1 g = {0, -0.0863016, 0} N

F1 = -m1 aC1 + G1 = {-26.4762, 15.1997, 0.} N

M1 = Min1 = -IC1 α1 = {0, 0, 0} N m

rC2 = {-0.0302868, -0.0337588, 0} m

aC2 = {4198.29, 13969.9, 0.} m/s^2

m2 = 0.0176 kg

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IC2 = 0.0000711333 kg m^2
Fin2 = -m2 aC2 = {-73.89, -245.871, 0.} N
G2 = -m2 g = {0, -0.172603, 0} N
F2 = -m2 aC2 + G2 = {-73.89, -246.044, 0.} N
M2 = Min2 = -IC2 α2 = {0, 0, -5.71412} N m
rC3 = {-0.055, 0., 0} m
aC3 = {0, 0, 0} m/s^2
m3 = 0.008 kg
IC3 = 1.93333×10-6 kg m^2
Fin3 = -m3 aC3 = {0, 0, 0} N
G3 = -m3 g = {0, -0.078456, 0} N
F3 = -m3 aC3 + G3 = {0, -0.078456, 0} N
M3 = Min3 = -IC3 α3 = {0, 0, -0.155304} N m
rC4 = {0.184259, -0.111167, 0} m
aC4 = {2762.02, 25687.4, 0.} m/s^2
m4 = 0.024 kg
IC4 = 0.0001802 kg m^2
Fin4 = -m4 aC4 = {-66.2885, -616.497, 0.} N
G4 = -m4 g = {0, -0.235368, 0} N
F4 = -m4 aC4 + G4 = {-66.2885, -616.732, 0.} N
M4 = Min4 = -IC4 α4 = {0, 0, 6.89466} N m
rC5 = {0.304414, -0.0174078, 0} m
aC5 = {1572.38, 9980.36, 0.} m/s^2
m5 = 0.014 kg
IC5 = 0.0000358458 kg m^2
Fin5 = -m5 aC5 = {-22.0134, -139.725, 0.} N
G5 = -m5 g = {0, -0.137298, 0} N
F5 = -m5 aC5 + G5 = {-22.0134, -139.862, 0.} N
M5 = Min5 = -IC5 α5 = {0, 0, -1.98112} N m

Fr
F05 = {-3667.41, 142.67, 0} N
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Er

$$F45 = \{3689.42, -2.80758, 0\} \text{ N}$$

Dr

$$F24 = \{3755.71, 613.924, 0\} \text{ N}$$

Ct

$$F23 = \{-9173.2, -6715.25, 0\} \text{ N}$$

$$rQ = \{-0.0550081, 0.000011023, 0\} \text{ m}$$

Cr

$$F03 = \{9173.2, 6715.32, 0\} \text{ N}$$

Br

$$F12 = \{-5343.59, -5855.28, 0\} \text{ N}$$

$$F01 = \{-5317.12, -5870.48, 0.\} \text{ N}$$

$$Meq = \{0., 0., 851.684\} \text{ N m}$$