Velocity and Acceleration Analysis

Derivative Method

The angular velocity of link 1 is constant and has the value

$$n = 50 \text{ rpm and } \omega = \dot{\phi} = \frac{\pi n}{30} = 5.235 \text{ rad/s},$$

The velocity is obtained taking the derivative of the position with respect to time, t. The symbolic variable t is introduced in MATLAB with the statement sym

```
t = sym('t','real');
```

The coordinates of the joint B are

$$x_B(t) = AB\cos\phi(t) \quad \text{and} \quad y_B(t) = AB\sin\phi(t),$$

and the position vector of B is $\mathbf{r}_B = x_B\,\mathbf{\imath} + x_B\,\mathbf{\jmath}$.
To calculate symbolically the position of the joint B, the following MATLAB commands are used

```
xB = AB*cos(sym('phi(t)')); yB = AB*sin(sym('phi(t)'));
rB = [ xB yB 0 ];
```

The statement `sym('phi(t)')` represents the mathematical function $\phi(t)$ and is introduced with the command `sym` that constructs symbolic numbers, variables and objects. The function `phi` has one argument, the time `t`.
To calculate numerically the position of the joint B, the symbolic variables need to be substituted with the input data. To apply a transformation rule to a particular expression `expr`, type `subs(expr,lhs,rhs)`. The statement `subs(expr,lhs,rhs)` replaces `lhs` with `rhs` in the symbolic expression `expr`.

For the mechanism, the numerical values for the joint B are

```
xBn = subs(xB, 'phi(t)', pi/6); yBn = subs(yB, 'phi(t)', pi/6);
rBn = subs(rB, 'phi(t)', pi/6);
```

The linear velocity vector of $B_1 = B_2$ is

$$\mathbf{v}_{B_1} = \mathbf{v}_{B_2} = \dot{x}_B\mathbf{\imath} + \dot{y}_B\mathbf{\jmath},$$

where

$$\dot{x}_B = \frac{dx_B}{dt} = -AB\dot{\phi}\sin\phi \quad \text{and} \quad \dot{y}_B = \frac{dy_B}{dt} = AB\dot{\phi}\cos\phi,$$

are the components of the velocity vector of $B_1 = B_2$.
To calculate symbolically the components of the velocity vector using the MATLAB the command `diff(f,t)` is used, which gives the derivative of `f` with respect to `t`.
The symbolical expression of the velocity vector of $B_1 = B_2$ is obtain with the statement

```
vB = diff(rB,t);
```

The numerical values for the components of the velocity of $B_1 = B_2$ are

$$\dot{x}_B = -0.140\ (5.235) \sin 30^\circ = -0.366\ \mathrm{m/s},$$
$$\dot{y}_B = 0.140\ (5.235) \cos 30^\circ = 0.634\ \mathrm{m/s}.$$

To obtain the numerical values in MATLAB first `diff('phi(t)',t)` is replaced with `omega` and then `phi(t)` is replaced with `pi/6`

```
vBnn = subs(vB,diff('phi(t)',t),omega); vBn = subs(vBnn,'phi(t)',pi/6);
```

Instead of replacing `diff('phi(t)',t)` with `omega` and then replacing `'phi(t)'` with `pi/6`, a list with the symbolical variable `'phi(t)'`, `diff('phi(t)',t)`, and `diff('phi(t)',t,2)` is created

```
slist = {'phi(t)', diff('phi(t)',t), diff('phi(t)',t,2)};
```

Next a list with the numerical values for `slist` is introduced

```
nlist = {pi/6, omega, 0};  % numerical values for slist
```

The velocities and accelerations need to be calculated at the moment when driver link makes and angle $\phi(t) = \pi/6$ with the horizontal and $\dot{\phi}(t) = \omega$ and $\ddot{\phi}(t) = \dot{\omega} = 0$. To obtain the numerical value for the symbolic vector `rB` the following statements are introduced

```
vBn = subs(vB,slist,nlist); VB = double(vBn);
```

The statement `double(S)` converts the symbolic object `S` to a numeric object.

The linear acceleration vector of $B_1 = B_2$ is

$$\mathbf{a}_{B_1} = \mathbf{a}_{B_2} = \ddot{x}_B \mathbf{\imath} + \ddot{y}_B \mathbf{\jmath},$$

where

$$\ddot{x}_B = \frac{d\dot{x}_B}{dt} = -AB\dot{\phi}^2 \cos\phi - AB\ddot{\phi}\sin\phi,$$
$$\ddot{y}_B = \frac{d\dot{y}_B}{dt} = -AB\dot{\phi}^2 \sin\phi + AB\ddot{\phi}\cos\phi.$$

For the considered mechanism the angular acceleration of the link 1 is $\ddot{\phi} = \dot{\omega} = 0$. The numerical values of the acceleration of B are

$$\ddot{x}_B = -0.140\ (5.235)^2 \cos 30^\circ = -3.323\,\mathrm{m/s^2},$$
$$\ddot{y}_B = -0.140\ (5.235)^2 \sin 30^\circ = -1.919\,\mathrm{m/s^2}.$$

The MATLAB command used to calculate symbolically the acceleration vector is

```
aB = diff(vB,t);
```

The numerical value for the vector `aB` is obtained with

```
aBn = double(subs(aB,slist,nlist));
```

The coordinates of the joint D are x_D and y_D. The MATLAB commands used to calculate the position of D are

```
eqnD1 = '( xDsol - xC )^2 + ( yDsol - yC )^2 = CD^2 ';
eqnD2 = '(yB - yC) / (xB - xC) = (yDsol - yC) / (xDsol - xC)';
solD = solve(eqnD1, eqnD2, 'xDsol, yDsol');
```

Two sets of solutions are found for the position of the joint D that are functions of the angle $\phi(t)$ (i.e., functions of time):

```
xDpositions = eval(solD.xDsol); yDpositions = eval(solD.yDsol);
xD1 = xDpositions(1); xD2 = xDpositions(2);
yD1 = yDpositions(1); yD2 = yDpositions(2);
```

To determine the correct position of the joint D for the mechanism, an additional condition is needed. For the first quadrant, $0 \le \phi \le 90°$, the condition is $x_D \le x_C$.
This condition using the MATLAB command is:

```
xD1n = subs(xD1,'phi(t)',pi/6); % xD1 for phi(t)=pi/6
if xD1n < xC
   xD = xD1; yD = yD1;
else
   xD = xD2; yD = yD2;
end
rD = [ xD yD 0 ]; % position vector of D in term of phi(t)
```

The linear velocity vector of the joint $D_3 = D_4$ (on link 3 or link 4) is

$$\mathbf{v}_{D_3} = \mathbf{v}_{D_4} = \dot{x}_D \boldsymbol{\imath} + \dot{y}_D \boldsymbol{\jmath},$$

where

$$\dot{x}_D = \frac{dx_D}{dt} \quad \text{and} \quad \dot{y}_D = \frac{dy_D}{dt},$$

are the components of the velocity vector of the joint D, respectively, on the x-axis and the y-axis.
To calculate symbolically the components of this velocity vector the following MATLAB commands are used

```
vD = diff(rD,t);
```

The numerical solutions using MATLAB is

```
vDn = double(subs(vD,slist,nlist));
```

For the considered mechanism the numerical values are

$$\dot{x}_D = 0.067\,\text{m/s and } \dot{y}_D = -0.814\,\text{m/s}.$$

The linear acceleration vector of $D_3 = D_4$ is

$$\mathbf{a}_{D_3} = \mathbf{a}_{D_4} = \ddot{x}_D \boldsymbol{\imath} + \ddot{y}_D \boldsymbol{\jmath},$$

where

$$\ddot{x}_D = \frac{d\dot{x}_D}{dt} \quad \text{and} \quad \ddot{y}_D = \frac{d\dot{y}_D}{dt}.$$

To calculate symbolically the components of the acceleration vector the following MATLAB commands are used:

```
aD = diff(vD,t);
```

The numerical values of the acceleration of $D_3 = D_4$ are

$$\ddot{x}_D = 4.617\,\text{m/s}^2 \quad \text{and} \quad \ddot{y}_D = -1.811\,\text{m/s}^2,$$

and can be printed using MATLAB

```
aDn = double(subs(aD,slist,nlist)); % numerical value for aD
fprintf('aD3 = aD4 = [ %g, %g, %g ] (m/s^2) \n', aDn);
```

The angle $\phi_2(t) = \phi_3(t)$ is determined as a function of time t from the equation of the slope of the line BC:

$$\tan\phi_2(t) = \tan\phi_3(t) = \frac{y_B(t) - y_C}{x_B(t) - x_C}.$$

The MATLAB function `atan(z)` gives the arc tangent of the number `z` and the angle ϕ_2 is calculated symbolically

```
phi2 = atan((yB-yC)/(xB-xC));
```

The numerical value is given by

```
phi2n = subs(phi2,'phi(t)',pi/6);
```

The angular velocity $\omega_2(t) = \omega_3(t)$ is the derivative with respect to time of the angle $\phi_2(t)$

$$\omega_2 = \frac{d\phi_2(t)}{dt}.$$

Symbolically, the angular velocity $\omega_2 = \omega_3$ is calculated using MATLAB

```
dphi2 = diff(phi2,t);
```

and is the numerical values are printed using the MATLAB statements

```
dphi2nn = subs(dphi2,diff('phi(t)',t),omega);
dphi2n = subs(dphi2nn,'phi(t)',pi/6);
fprintf('omega2 = omega3 = %g (rad/s) \n', dphi2n);
```

The angular acceleration $\alpha_2(t) = \alpha_3(t)$ is the derivative with respect to time of the angular velocity $\omega_2(t)$:

$$\alpha_2(t) = \frac{d\omega_2(t)}{dt}.$$

Symbolically, using MATLAB, the angular acceleration α_2 is

```
ddphi2 = diff(dphi2,t);
```

The numerical values of the angles, angular velocities, and angular accelerations for the links 2 and 3 are

```
ddphi2n = double(subs(ddphi2,slist,nlist));
```

The results are

$$\phi_3 = \phi_2 = 0.082 \text{ rad}, \quad \omega_3 = \omega_2 = 5.448 \text{ rad/s}, \quad \alpha_3 = \alpha_2 = 14.568 \text{ rad/s}^2.$$

The angle $\phi_4(t) = \phi_5(t)$ is determined as a function of time t from the following equation:

$$\tan\phi_4(t) = \tan\phi_5(t) = \frac{y_D(t) - y_E}{x_D(t) - x_E},$$

and symbolically using MATLAB

```
ddphi4 = diff(dphi4,t);
```

The angular velocity $\omega_4(t) = \omega_5(t)$ is the derivative with respect to time of the angle $\phi_4(t)$

$$\omega_4 = \frac{d\phi_4(t)}{dt}.$$

To calculate symbolically the angular velocity ω_4 using MATLAB, the following command is used

```
dphi4 = diff(phi4,t);
```

The angular acceleration $\alpha_4(t) = \alpha_5(t)$ is the derivative with respect to time of the angular velocity $\omega_4(t)$:

$$\alpha_4(t) = \frac{d\omega_4(t)}{dt},$$

and it is calculated symbolically with MATLAB

```
ddphi4 = diff(dphi4,t);
```

The numerical values of the angles, angular velocities, and angular accelerations for the links 5 and 4 are

$$\phi_5 = \phi_4 = 2.036 \text{ rad}, \quad \omega_5 = \omega_4 = 0.917 \text{ rad/s}, \quad \alpha_5 = \alpha_4 = -5.771 \text{ rad/s}^2.$$

The numerical solutions printed with MATLAB are

```
dphi4n = double(subs(dphi4,slist,nlist)); % numeric omega4
fprintf('omega4 = omega5 = %g (rad/s) \n', dphi4n);
ddphi4n = double(subs(ddphi4,slist,nlist)); % numeric alpha4
fprintf('alpha4 = alpha5 = %g (rad/s^2) \n', ddphi4n);
```