Velocity and Acceleration Analysis

Contour Method

The mechanism has five moving links and seven full joints. The number of independent contours is

$$n_c = c - n = 7 - 5 = 2,$$

where c is the number of joints and n is the number of moving links.

The mechanism has two independent contours. The first contour I contains the links 0, 1, 2, and 3, while the second contour II contains the links 0, 3, 4, and 5. The diagram of the mechanism is represented in Fig. 8(b). Clockwise paths are chosen for each closed contours I and II.

First contour analysis

Figure 9(a) shows the first independent contour I with

- rotational joint R between the links 0 and 1 (joint A);
- rotational joint R between the links 1 and 2 (joint B);
- translational joint T between the links 2 and 3 (joint B);
- rotational joint R between the links 3 and 0 (joint C).

The angular velocity ω_{10} of the driver link is known:

$$\omega_{10} = \omega_1 = \omega = \frac{50\pi}{30} \text{ rad/s} = 5.235 \text{ rad/s}.$$

The origin of the reference frame is the point $A(0, 0, 0)$.
For the velocity analysis, the following vectorial equations are used:

$$\begin{aligned} &\boldsymbol{\omega}_{10} + \boldsymbol{\omega}_{21} + \boldsymbol{\omega}_{03} = \mathbf{0}, \\ &\mathbf{r}_{AB} \times \boldsymbol{\omega}_{21} + \mathbf{r}_{AC} \times \boldsymbol{\omega}_{03} + \mathbf{v}^r_{B32} = \mathbf{0}, \end{aligned} \tag{1}$$

where $\mathbf{r}_{AB} = x_B\mathbf{\imath} + y_B\mathbf{\jmath}$, $\mathbf{r}_{AC} = x_C\mathbf{\imath} + y_C\mathbf{\jmath}$, and

$$\begin{aligned} &\boldsymbol{\omega}_{10} = \omega_{10}\mathbf{k},\ \boldsymbol{\omega}_{21} = \omega_{21}\mathbf{k},\ \boldsymbol{\omega}_{03} = \omega_{03}\mathbf{k}, \\ &\mathbf{v}^r_{B32} = \mathbf{v}_{32} = v_{32}\cos\phi_2\mathbf{\imath} + v_{32}\sin\phi_2\mathbf{\jmath}. \end{aligned}$$

The sign of the relative angular velocities is selected as positive as shown in Figs. 8(a) and 9(a). The numerical computation will then give the correct orientation of the unknown vectors. The components of the vectors $\mathbf{r}_{AB}$ and

$\mathbf{r}_{AC}$, and the angle ϕ_2 are already known from the position analysis of the mechanism. Equation (1) becomes

$$\omega_{10}\mathbf{k} + \omega_{21}\mathbf{k} + \omega_{03}\mathbf{k} = \mathbf{0},$$
$$\begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_B & y_B & 0 \\ 0 & 0 & \omega_{21} \end{vmatrix} + \begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_C & y_C & 0 \\ 0 & 0 & \omega_{03} \end{vmatrix} + v_{32}\cos\phi_2 \imath + v_{32}\sin\phi_2 \jmath = \mathbf{0}. \qquad (2)$$

Equation (2) represents a system of three equations and with MATLAB/*Mathematica*^{TM} commands gives the following numerical solutions are obtained:

$$\omega_{21} = 0.212 \text{ rad/s},\ \omega_{03} = -5.448 \text{ rad/s},\ \text{and } v_{32} = 0.313 \text{ m/s}.$$

The absolute angular velocities of the links 2 and 3 are

$$\boldsymbol{\omega}_{20} = \boldsymbol{\omega}_{30} = -\boldsymbol{\omega}_{03} = 5.448\ \mathbf{k} \text{ rad/s}.$$

The absolute linear velocities of the joints B and D are

$$\mathbf{v}_B = \mathbf{v}_{B_1} = \mathbf{v}_{B_2} = \mathbf{v}_A + \boldsymbol{\omega}_{10} \times \mathbf{r}_{AB} = -0.366\ \imath + 0.634\ \jmath \text{ m/s},$$
$$\mathbf{v}_D = \mathbf{v}_{D_3} = \mathbf{v}_{D_4} = \mathbf{v}_C + \boldsymbol{\omega}_{30} \times \mathbf{r}_{CD} = 0.067\ \imath - 0.814\ \jmath \text{ m/s},$$

where $\mathbf{v}_A = \mathbf{0}$ and $\mathbf{v}_C = \mathbf{0}$, because the joints A and C are grounded and

$$\mathbf{r}_{CD} = \mathbf{r}_{AD} - \mathbf{r}_{AC}.$$

For the acceleration analysis, the following vectorial equations are used

$$\boldsymbol{\alpha}_{10} + \boldsymbol{\alpha}_{21} + \boldsymbol{\alpha}_{03} = \mathbf{0},$$
$$\mathbf{r}_{AB} \times \boldsymbol{\alpha}_{21} + \mathbf{r}_{AC} \times \boldsymbol{\alpha}_{03} + \mathbf{a}^r_{B32} + \mathbf{a}^c_{B32} - \omega_{10}^2 \mathbf{r}_{AB} - \omega_{20}^2 \mathbf{r}_{BC} = \mathbf{0}. \qquad (3)$$

where

$$\boldsymbol{\alpha}_{10} = \alpha_{10}\mathbf{k},\ \boldsymbol{\alpha}_{21} = \boldsymbol{\alpha}_{21}\mathbf{k},\ \boldsymbol{\alpha}_{03} = \boldsymbol{\alpha}_{03}\mathbf{k},$$
$$\mathbf{a}^r_{B32} = \mathbf{a}_{32} = a_{32}\cos\phi_2 \imath + a_{32}\sin\phi_2 \jmath,$$
$$\mathbf{a}^c_{B32} = \mathbf{a}^c_{32} = 2\boldsymbol{\omega}_{20} \times \mathbf{v}_{32},$$

The driver link has a constant angular velocity and $\alpha_{10} = \dot{\omega}_{10} = 0$.
Equation (3) represents a system of three equations and can be solved using MATLAB/*Mathematica*^{TM}. The following numerical solutions are then obtained

$$\alpha_{21} = 14.568 \text{ rad/s}^2,\ \alpha_{03} = -14.568 \text{ rad/s}^2,\ \text{and } a_{32} = -0.140 \text{ m/s}^2.$$

The absolute angular accelerations of the links 2 and 3 are

$$\boldsymbol{\alpha}_{20} = \boldsymbol{\alpha}_{30} = -\boldsymbol{\alpha}_{03} = 14.568\ \mathbf{k}\ \text{rad/s}^2.$$

The absolute linear accelerations of the joints B and D are obtained from the following equation:

$$\begin{aligned}\mathbf{a}_B &= \mathbf{a}_A + \boldsymbol{\alpha}_{10} \times \mathbf{r}_{AB} - \omega_{10}^2 \mathbf{AB} = -3.323\ \imath - 1.919\ \jmath\ \text{m/s}^2,\\ \mathbf{a}_D &= \mathbf{a}_C + \boldsymbol{\alpha}_{30} \times \mathbf{r}_{CD} - \omega_{30}^2 \mathbf{r}_{CD} = 4.617\ \imath - 1.811\ \jmath\ \text{m/s}^2,\end{aligned}$$

where $\mathbf{a}_A = \mathbf{0}$ and $\mathbf{a}_C = \mathbf{0}$, because the joints A and C are grounded.

Second contour analysis

Figure 10(a) depicts the second independent contour II

- rotational joint R between the links 0 and 3 (joint C);
- rotational joint R between the links 3 and 4 (joint D);
- translational joint T between the links 4 and 5 (joint D);
- rotational joint R between the links 5 and 0 (joint E).

For the velocity analysis, the following vectorial equations are used

$$\begin{aligned}&\boldsymbol{\omega}_{30} + \boldsymbol{\omega}_{43} + \boldsymbol{\omega}_{05} = \mathbf{0},\\ &\mathbf{r}_{AC} \times \boldsymbol{\omega}_{30} + \mathbf{r}_{AD} \times \boldsymbol{\omega}_{43} + \mathbf{r}_{AE} \times \boldsymbol{\omega}_{05} + \mathbf{v}^r_{D54} = \mathbf{0},\end{aligned} \tag{4}$$

where $\mathbf{r}_{AD} = x_D \imath + y_D \jmath$, $\mathbf{r}_{AE} = x_E \imath + y_E \jmath$, and

$$\begin{aligned}&\boldsymbol{\omega}_{30} = \omega_{30}\mathbf{k},\ \boldsymbol{\omega}_{43} = \omega_{43}\mathbf{k},\ \boldsymbol{\omega}_{05} = \omega_{05}\mathbf{k},\\ &\mathbf{v}^r_{D54} = \mathbf{v}_{54} = v_{54}\cos\phi_4 \imath + v_{54}\sin\phi_4 \jmath.\end{aligned}$$

The sign of the relative angular velocities is selected as positive as shown in Figs. 8(a) and 10(a). The numerical computation will then give the correct orientation of the unknown vectors. The components of the vectors $\mathbf{r}_{AD}$ and $\mathbf{r}_{AE}$, and the angle ϕ_4 are already known from the position analysis of the mechanism.

Equation (4) becomes

$$\begin{aligned}&\omega_{30}\mathbf{k} + \omega_{43}\mathbf{k} + \omega_{05}\mathbf{k} = \mathbf{0},\\ &\begin{vmatrix} \imath & \jmath & \mathbf{k}\\ x_C & y_C & 0\\ 0 & 0 & \omega_{30}\end{vmatrix} + \begin{vmatrix} \imath & \jmath & \mathbf{k}\\ x_D & y_D & 0\\ 0 & 0 & \omega_{43}\end{vmatrix} + \begin{vmatrix} \imath & \jmath & \mathbf{k}\\ x_E & y_E & 0\\ 0 & 0 & \omega_{05}\end{vmatrix} +\\ &v_{32}\cos\phi_4 \imath + v_{32}\sin\phi_4 \jmath = \mathbf{0}.\end{aligned} \tag{5}$$

Equation (5) projected onto the "fixed" reference frame $Oxyz$ gives

$$\begin{aligned}
&\omega_{30} + \omega_{43} + \omega_{05} = 0, \\
&y_C\omega_{30} + y_D\omega_{43} + y_E\omega_{05} + v_{54}\cos\phi_4 = 0, \\
&-x_C\omega_{30} - x_D\omega_{43} - x_E\omega_{05} + v_{54}\sin\phi_4 = 0.
\end{aligned} \tag{6}$$

Equation (6) represents an algebraic system of three equations with three unknowns: ω_{43}, ω_{05}, and v_{54}. The system is solved using MATLAB/*Mathematica*^{TM} commands.
The following numerical solutions are obtained:

$$\omega_{43} = -4.531 \text{ rad/s}, \;\; \omega_{05} = -0.917 \text{ rad/s}, \;\; \text{and } v_{54} = 0.757 \text{ m/s}.$$

The absolute angular velocities of the links 4 and 5 are

$$\boldsymbol{\omega}_{40} = \boldsymbol{\omega}_{50} = -\boldsymbol{\omega}_{05} = 0.917 \; \mathbf{k} \text{ rad/s}, \tag{7}$$

For the acceleration analysis, the following vectorial equations are used:

$$\begin{aligned}
&\boldsymbol{\alpha}_{30} + \boldsymbol{\alpha}_{43} + \boldsymbol{\alpha}_{05} = \mathbf{0}, \\
&\mathbf{r}_{AC} \times \boldsymbol{\alpha}_{30} + \mathbf{r}_{AD} \times \boldsymbol{\alpha}_{43} + \mathbf{r}_{AE} \times \boldsymbol{\alpha}_{05} + \mathbf{a}^r_{D54} + \mathbf{a}^c_{B54} - \omega_{30}^2\mathbf{r}_{CD} - \omega_{40}^2\mathbf{r}_{DE} = \mathbf{0}.
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
&\boldsymbol{\alpha}_{30} = \alpha_{30}\mathbf{k}, \;\; \boldsymbol{\alpha}_{43} = \boldsymbol{\alpha}_{43}\mathbf{k}, \;\; \boldsymbol{\alpha}_{05} = \boldsymbol{\alpha}_{05}\mathbf{k}, \\
&\mathbf{a}^r_{B54} = \mathbf{a}_{54} = a_{54}\cos\phi_4\boldsymbol{\imath} + a_{54}\sin\phi_4\boldsymbol{\jmath}, \\
&\mathbf{a}^c_{B54} = 2\boldsymbol{\omega}_{40} \times \mathbf{v}_{54}.
\end{aligned}$$

Equation (8) becomes

$$\begin{aligned}
&\alpha_{30}\mathbf{k} + \alpha_{43}\mathbf{k} + \alpha_{05}\mathbf{k} = \mathbf{0}, \\
&\begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ x_C & y_C & 0 \\ 0 & 0 & \alpha_{30} \end{vmatrix} + \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ x_D & y_D & 0 \\ 0 & 0 & \alpha_{43} \end{vmatrix} + \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ x_E & y_E & 0 \\ 0 & 0 & \alpha_{05} \end{vmatrix} + \\
&a_{54}\cos\phi_4\boldsymbol{\imath} + a_{54}\sin\phi_4\boldsymbol{\jmath} + \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & \omega_{40} \\ v_{54}\cos\phi_4 & v_{54}\sin\phi_4 & 0 \end{vmatrix} - \\
&\omega_{30}^2[(x_D - x_C)\boldsymbol{\imath} + (y_D - y_C)\boldsymbol{\jmath}] - \\
&\omega_{40}^2[(x_E - x_D)\boldsymbol{\imath} + (y_E - y_D)\boldsymbol{\jmath}] = \mathbf{0}.
\end{aligned} \tag{9}$$

Equation (9) can be rewritten as

$$\begin{aligned}
&\alpha_{30} + \alpha_{43} + \alpha_{05} = 0,\\
&y_C\alpha_{30} + y_D\alpha_{43} + y_E\alpha_{05} + a_{54}\cos\phi_4 - 2\omega_{40}v_{54}\sin\phi_4 -\\
&\omega_{30}^2(x_D - x_C) - \omega_{40}^2(x_E - x_D) = 0,\\
&-x_C\alpha_{30} - x_D\alpha_{43} - x_E\alpha_{05} + a_{54}\sin\phi_4 + 2\omega_{40}v_{54}\cos\phi_4 -\\
&\omega_{30}^2(y_D - y_C) - \omega_{40}^2(y_E - y_D) = 0.
\end{aligned} \tag{10}$$

The unknowns in Eq. (10) are α_{43}, α_{05}, and a_{54}. To solve the system MATLAB/*Mathematica*^{TM} is used.
The following numerical solutions are obtained:

$$\alpha_{43} = -20.339 \text{ rad/s}^2,\ \alpha_{05} = 5.771 \text{ rad/s}^2,\ \text{and } a_{54} = 3.411 \text{ m/s}^2.$$

The absolute angular accelerations of the links 4 and 5 are

$$\boldsymbol{\alpha}_{40} = \boldsymbol{\alpha}_{50} = -\boldsymbol{\alpha}_{05} = -5.771 \text{ } \mathbf{k} \text{ rad/s}^2,$$

The *Mathematica*^{TM}/ MATLAB program for the velocity and acceleration analysis using the contour method is given in Program 6.