```
Apply[Clear, Names["Global`*"]];
Off[General::spell];
Off[General::spell1];

Print["Kinematics"];
omega[t] = {0, 0, theta'[t]};
Print["angular velocity of RB: omega=", omega[t]];
alpha[t] = {0, 0, theta''[t]};
Print["angular acceleration of RB: alpha=", alpha[t]];
xC = (L / 2) * Cos[theta[t]];
yC = (L / 2) * Sin[theta[t]];
rC = {xC, yC, 0};
Print["position vector of C: rC=", rC];
vC = D[rC, t];
Print["velocity of C: vC=d(rC)/dt=", vC];
aC = D[vC, t];
Print["acceleration of C: aC=d(vC)/dt=", Simplify[aC]];
Print["another way of calculating vC and aC"];
vC = Cross[omega[t], rC];
Print["vC=omega x rC=", vC];
aC = Cross[alpha[t], rC] - omega[t].omega[t] * rC;
Print["aC=alpha x rC - omega.omega rC=", Simplify[aC]];
Print["Forces"];
FO = {FOx, FOy, 0};
Print["reaction force at pin joint O: FO=", FO];
G = {0, m g, 0};
Print["gravitational force at C: G=", G];
rCO = -rC;
IC = m * L^2 / 12;
Print["mass moment of inertia wrt C: ICz=", IC];
IO = IC + m * (L / 2)^2;
Print["mass moment of inertia wrt O: IOz=ICz+m*(L/2)^2=",
  Simplify[IO]];
Print["Method I"];
eqI = Simplify[IO * alpha[t] - Cross[rC, G]];
solutionI = Solve[eqI[[3]] == 0, theta''[t]][[1]];
Print["moment equation: IO alpha = sum M wrt O = rC x G"];
Print[eqI[[3]], "=0"];
Print["Solution: theta''[t]=", theta''[t] /. Simplify[solutionI]];
Print["Method II"];
eqIIF = Simplify[m * aC - (FO + G)];
Print["force equation: m aC = sum F = FO + G"];
Print["projection on x:"];
Print[eqIIF[[1]], "=0", " (1)"];
Print["projection on y:"];
Print[eqIIF[[2]], "=0", " (2)"];
eqIIM = Simplify[IC * alpha[t] - Cross[rCO, FO]];
Print["moment eqation: IC alpha = sum M wrt C = -rC x FO"];
Print["projection on z: "]
Print[eqIIM[[3]], "=0", " (3)"];
solFOx = Solve[eqIIF[[1]] == 0, FOx][[1]];
Fx = FOx /. solFOx;
Print["from Eq.(1) => FOx = ", Fx];
solFOy = Solve[eqIIF[[2]] == 0, FOy][[1]];
Fy = FOy /. solFOy;
Print["from Eq.(2) => FOy = ", Fy];
solutionII = Solve[(eqIIM[[3]] /. solFOx /. solFOy) == 0, theta''[t]][[1]];
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ddtheta = theta''[t] /. Simplify[solutionII];
Print["from Eqs.(1)(2)(3) => theta''[t] = ", ddtheta];
Print["Initial Conditions"];
Print["at t=0: theta[0]=0, theta'[0]=omega[0]=0"];
ic = {theta[0] → 0, theta'[0] → 0};
ddtheta0 = ddtheta /. {t → 0} /. ic;
Print["numerical data: m=12/32.2,L=3,g=32.2"];
data = {m → 12 / 32.2, L → 3, g → 32.2};
Print["theta''[0]=alpha[0]= ", ddtheta0, "=", ddtheta0 /. data, " rad/s^2"];
V = Simplify[Fy /. {t → 0} /. ic];
V0 = Simplify[V /. theta''[0] -> ddtheta0];
Print["V=FOy[0] = ", V, "=", V0, "=", V0 /. data, " lb"];
H = Simplify[Fx /. {t → 0} /. ic];
Print["H=FOx[0] = ", H, " lb"];
(* Numerical solution of differential equation *)
soldif =
  NDSolve[{(eqI[[3]] /. data) == 0, theta[0] == 0, theta'[0] == 0}, theta[t], {t, 0, 2}];
Plot[Evaluate[theta[t] /. soldif] * 180 / Pi, {t, 0, 2}, AxesLabel → {"t[s]", "theta[deg]"}];
Plot[Evaluate[D[theta[t] /. soldif, t]], {t, 0, 2}, AxesLabel → {"t[s]", "omega[rad/s]"}];
Plot[Evaluate[D[theta[t] /. soldif, {t, 2}]],
  {t, 0, 2}, AxesLabel → {"t[s]", "alpha[rad/s]"}];

DSolve[theta''[t] + C Sin[theta[t]] == 0, theta[t], t]
```

Kinematics

angular velocity of RB: omega={0, 0, theta′[t]}

angular acceleration of RB: alpha={0, 0, theta″[t]}

position vector of C: rC= $\left\{\frac{1}{2} L \,\mathrm{Cos}[\mathrm{theta}[t]],\ \frac{1}{2} L \,\mathrm{Sin}[\mathrm{theta}[t]],\ 0\right\}$

velocity of C: vC=d(rC)/dt= $\left\{-\frac{1}{2} L \,\mathrm{Sin}[\mathrm{theta}[t]]\,\mathrm{theta}'[t],\ \frac{1}{2} L \,\mathrm{Cos}[\mathrm{theta}[t]]\,\mathrm{theta}'[t],\ 0\right\}$

acceleration of C: aC=d(vC)/dt= $\left\{-\frac{1}{2} L \left(\mathrm{Cos}[\mathrm{theta}[t]]\,\mathrm{theta}'[t]^2 + \mathrm{Sin}[\mathrm{theta}[t]]\,\mathrm{theta}''[t]\right),\ \frac{1}{2}\left(-L \,\mathrm{Sin}[\mathrm{theta}[t]]\,\mathrm{theta}'[t]^2 + L \,\mathrm{Cos}[\mathrm{theta}[t]]\,\mathrm{theta}''[t]\right),\ 0\right\}$

another way of calculating vC and aC

vC=omega x rC= $\left\{-\frac{1}{2} L \,\mathrm{Sin}[\mathrm{theta}[t]]\,\mathrm{theta}'[t],\ \frac{1}{2} L \,\mathrm{Cos}[\mathrm{theta}[t]]\,\mathrm{theta}'[t],\ 0\right\}$

aC=alpha x rC - omega.omega rC= $\left\{-\frac{1}{2} L \left(\mathrm{Cos}[\mathrm{theta}[t]]\,\mathrm{theta}'[t]^2 + \mathrm{Sin}[\mathrm{theta}[t]]\,\mathrm{theta}''[t]\right),\ \frac{1}{2}\left(-L \,\mathrm{Sin}[\mathrm{theta}[t]]\,\mathrm{theta}'[t]^2 + L \,\mathrm{Cos}[\mathrm{theta}[t]]\,\mathrm{theta}''[t]\right),\ 0\right\}$

Forces

reaction force at pin joint O: FO={FOx, FOy, 0}

gravitational force at C: G={0, g m, 0}

mass moment of inertia wrt C: ICz= $\frac{L^2 m}{12}$

mass moment of inertia wrt O: IOz=ICz+m*(L/2)^2= $\frac{L^2 m}{3}$

Method I

moment equation: IO alpha = sum M wrt O = rC x G

$\frac{1}{6}$ L m (-3 g Cos[theta[t]] + 2 L theta''[t]) = 0

Solution: theta''[t] = $\frac{3\,g\,\mathrm{Cos}[\mathrm{theta}[t]]}{2\,L}$

Method II

force equation: m aC = sum F = FO + G

projection on x:

$\frac{1}{2}$ (-2 FOx - L m Cos[theta[t]] theta'[t]^2 - L m Sin[theta[t]] theta''[t]) = 0 (1)

projection on y:

$\frac{1}{2}$ (-2 (FOy + g m) - L m Sin[theta[t]] theta'[t]^2 + L m Cos[theta[t]] theta''[t]) = 0 (2)

moment eqation: IC alpha = sum M wrt C = -rC x FO

projection on z:

$\frac{1}{12}$ L (6 FOy Cos[theta[t]] - 6 FOx Sin[theta[t]] + L m theta''[t]) = 0 (3)

from Eq.(1) => FOx = $\frac{1}{2}$ (-L m Cos[theta[t]] theta'[t]^2 - L m Sin[theta[t]] theta''[t])

from Eq.(2) => FOy = $\frac{1}{2}$ (-2 g m - L m Sin[theta[t]] theta'[t]^2 + L m Cos[theta[t]] theta''[t])

from Eqs.(1)(2)(3) => theta''[t] = $\frac{3\,g\,\mathrm{Cos}[\mathrm{theta}[t]]}{2\,L}$

Initial Conditions

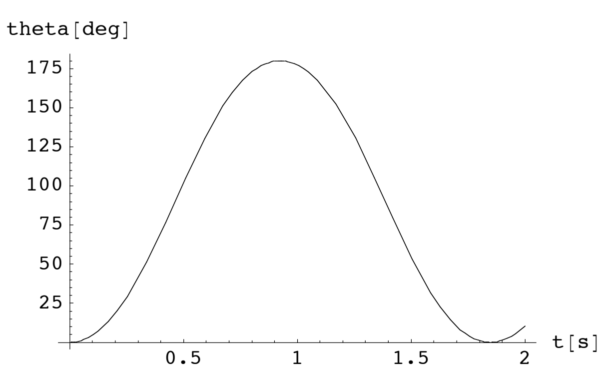
at t=0: theta[0]=0, theta'[0]=omega[0]=0

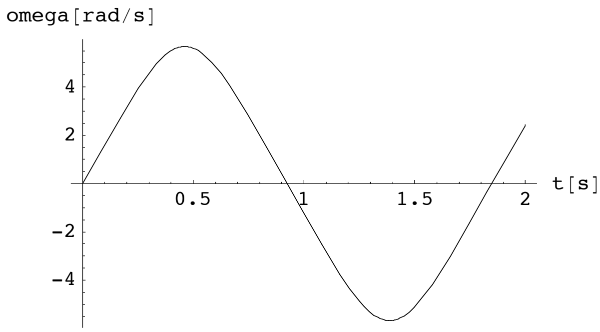
numerical data: m=12/32.2,L=3,g=32.2

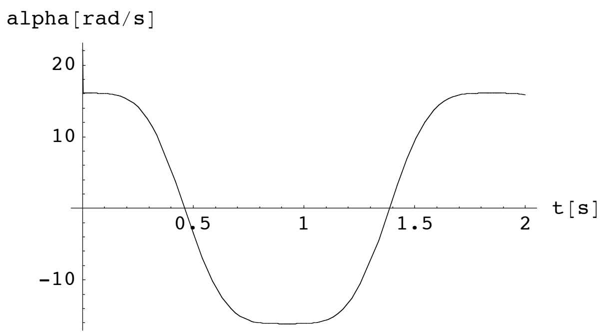
theta''[0]=alpha[0]= $\frac{3\,g}{2\,L}$=16.1 rad/s^2

V=F0y[0] = -g m + $\frac{1}{2}$ L m theta''[0]=-$\frac{g\,m}{4}$=-3. lb

H=F0x[0] = 0 lb





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Solve::ifun : Inverse functions are being used by Solve, so some
   solutions may not be found; use Reduce for complete solution information. More…
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$$\left\{\left\{\text{theta}[t] \to 2\,\text{JacobiAmplitude}\left[-\frac{1}{2}\sqrt{(-2\,C - C[1])\,(-t^2 - 2\,t\,C[2] - C[2]^2)},\ \frac{4\,C}{2\,C + C[1]}\right]\right\},\right.$$
$$\left.\left\{\text{theta}[t] \to 2\,\text{JacobiAmplitude}\left[\frac{1}{2}\sqrt{(-2\,C - C[1])\,(-t^2 - 2\,t\,C[2] - C[2]^2)},\ \frac{4\,C}{2\,C + C[1]}\right]\right\}\right\}$$