Contents

6 Dynamic Force Analysis

For a kinematic chain it is important to know how forces and moments are transmitted from the input to the output, so that the links can be properly designated. The friction effects are assumed to be negligible in the force analysis presented here.

6.1 Equation of Motion for the Mass Center

Consider a system of N particles. A particle is an object whose shape and geometrical dimensions are not significant to the investigation of its motion. An arbitrary collection of matter with total mass m can be divided into N particles, the ith particle having mass, m_i (Fig. 6.1):

$$m = \sum_{i=1}^{N} m_i.$$

A rigid body can be considered as a collection of particles in which the number of particles approaches infinity and in which the distance between any two points remains constant. As N approaches infinity, each particle is treated as a differential mass element, $m_i \to dm$, and the summation is replaced by integration over the body:

$$m = \int_{\text{body}} dm.$$

The position of the mass center of a collection of particles is defined by

$$\mathbf{r}_C = \frac{1}{m}\sum_{i=1}^{N} m_i \mathbf{r}_i, \tag{6.1}$$

where $\mathbf{r}_i$ is the position vector from the origin O to the ith particle. As $N \to \infty$, the summation is replaced by integration over the body:

$$\mathbf{r}_C = \frac{1}{m}\int_{\text{body}} \mathbf{r}\, dm, \tag{6.2}$$

where $\mathbf{r}$ is the vector from the origin O to differential element dm.

The time derivative of Eq. (6.1) gives

$$\sum_{i=1}^{N} m_i \frac{d^2\mathbf{r}_i}{dt^2} = m\frac{d^2\mathbf{r}_C}{dt^2} = m\mathbf{a}_C, \tag{6.3}$$

where $\mathbf{a}_C$ is the acceleration of the mass center. The acceleration of the mass center can be related to the external forces acting on the system. This relationship is obtained by applying Newton's laws to each of the individual particles in the system. Any such particle is acted on by two types of forces. One type is exerted by other particles that are also part of the system. Such forces are called internal forces (internal to the system). Additionally, a particle can be acted on by a force that is exerted by a particle or object not included in the system. Such a force is known as an external force (external to the system). Let $\mathbf{f}_{ij}$ be the internal force exerted on the jth particle by the ith particle. Newton's third law (action and reaction) states that the jth particle exerts a force on the ith particle of equal magnitude, and opposite direction, and collinear with the force exerted by the ith particle on the jth particle (Fig. 6.1):

$$\mathbf{f}_{ji} = -\mathbf{f}_{ij}, \quad j \neq i.$$

Newton's second law for the ith particle must include all of the internal forces exerted by all of the other particles in the system on the ith particle, plus the sum of any external forces exerted by particles, objects outside of the system on the ith particle:

$$\sum_{j} \mathbf{f}_{ji} + \mathbf{F}_i^{ext} = m_i \frac{d^2\mathbf{r}_i}{dt^2}, \quad j \neq i, \tag{6.4}$$

where $\mathbf{F}_i^{ext}$ is the external force on the ith particle. Equation (6.4) is written for each particle in the collection of particles. Summing the resulting equations over all of the particles from $i = 1$ to N the following relation is obtained:

$$\sum_{i}\sum_{j} \mathbf{f}_{ji} + \sum_{i} \mathbf{F}_i^{ext} = m\mathbf{a}_C, \quad j \neq i. \tag{6.5}$$

The sum of the internal forces includes pairs of equal and opposite forces. The sum of any such pair must be zero. The sum of all of the internal forces on the collection of particles is zero (Newton's third law):

$$\sum_{i}\sum_{j} \mathbf{f}_{ji} = \mathbf{0}, \quad j \neq i.$$

The term $\sum_i \mathbf{F}_i^{ext}$ is the sum of the external forces on the collection of particles

$$\sum_i \mathbf{F}_i^{ext} = \mathbf{F}.$$

One can conclude that the sum of the external forces acting on a closed system equals the product of the mass and the acceleration of the mass center:

$$m\,\mathbf{a}_C = \mathbf{F}. \tag{6.6}$$

Considering Fig. 6.2 for a rigid body and introducing the distance $\mathbf{q}$ in Eq. (6.2) gives

$$\mathbf{r}_C = \frac{1}{m}\int_{\text{body}} \mathbf{r}\,dm = \frac{1}{m}\int_{\text{body}} (\mathbf{r}_C + \mathbf{q})\,dm = \mathbf{r}_C + \frac{1}{m}\int_{\text{body}} \mathbf{q}\,dm. \tag{6.7}$$

It results

$$\frac{1}{m}\int_{\text{body}} \mathbf{q}\,dm = \mathbf{0}, \tag{6.8}$$

that is the weighed average of the displacement vector about the mass center is zero. The equation of motion for the differential element dm is

$$\mathbf{a}\,dm = d\mathbf{F},$$

where $d\mathbf{F}$ is the total force acting on the differential element. For the entire body:

$$\int_{\text{body}} \mathbf{a}\,dm = \int_{\text{body}} d\mathbf{F} = \mathbf{F}, \tag{6.9}$$

where $\mathbf{F}$ is the resultant of all forces. This resultant contains contributions only from the external forces, as the internal forces cancel each other. Introducing Eq. (6.7) into Eq. (6.9), the Newton's second law for a rigid body is obtained:

$$m\,\mathbf{a}_C = \mathbf{F}$$

The derivation of the equations of motion is valid for the general motion of a rigid body. These equations are equally applicable to planar and three-dimensional motions.

Resolving the sum of the external forces into cartesian rectangular components

$$\mathbf{F} = F_x\,\boldsymbol{\imath} + F_y\,\boldsymbol{\jmath} + F_z\,\mathbf{k},$$

and the position vector of the mass center

$$\mathbf{r}_C = x_C(t)\,\boldsymbol{\imath} + y_C(t)\,\boldsymbol{\jmath} + z_C(t)\,\mathbf{k},$$

Newton's second law for the rigid body is

$$m\ddot{\mathbf{r}}_C = \mathbf{F}, \tag{6.10}$$

or

$$m\ddot{x}_C = F_x,\;\; m\ddot{y}_C = F_y,\;\; m\ddot{z}_C = F_z. \tag{6.11}$$

6.2 Angular Momentum Principle for a System of Particles

An arbitrary system with the mass m can be divided into N particles P_1, $P_2, ..., P_N$. The position vector of the ith particle relative to an origin point O is $\mathbf{r}_i = \mathbf{r}_{OP_i}$ and the mass of the ith particle is m_i (Fig. 6.3). The position of the mass center C of the system is $\mathbf{r}_C = \sum_{i=1}^{N} m_i\mathbf{r}_i/m$. The position of the the particle P_i of the system relative to O is

$$\mathbf{r}_i = \mathbf{r}_C + \mathbf{r}_{CP_i}. \tag{6.12}$$

Multiplying Eq. (6.12) by m_i, summing from 1 to N, the following relation is obtained:

$$\sum_{i=1}^{N} m_i\mathbf{r}_{CP_i} = \mathbf{0}. \tag{6.13}$$

The total angular momentum of the system about its mass center C, is the sum of the angular momenta of the particles about C:

$$\mathbf{H}_C = \sum_{i=1}^{N} \mathbf{r}_{CP_i} \times m_i\mathbf{v}_i, \tag{6.14}$$

where $\mathbf{v}_i = \dfrac{d\mathbf{r}_i}{dt}$ is the velocity of the particle P_i.
The total angular momentum of the system about O is the sum of the angular momenta of the particles

$$\mathbf{H}_O = \sum_{i=1}^{N} \mathbf{r}_i \times m_i \mathbf{v}_i = \sum_{i=1}^{N} (\mathbf{r}_C + \mathbf{r}_{CP_i}) \times m_i \mathbf{v}_i = \mathbf{r}_C \times m\mathbf{v}_C + \mathbf{H}_C, \quad (6.15)$$

or the total angular momentum about O is the sum of the angular momentum about O due to the velocity $\mathbf{v}_C$ of the mass center of the system and the total angular momentum about the mass center (Fig. 6.4).

Newton's second law for the ith particle is

$$\sum_j \mathbf{f}_{ji} + \mathbf{F}_i^{ext} = m_i \frac{d\mathbf{v}_i}{dt}, \quad j \neq i,$$

and the cross product with the position vector $\mathbf{r}_i$, and sum from $i = 1$ to N gives

$$\sum_i \sum_j \mathbf{r}_i \times \mathbf{f}_{ji} + \sum_i \mathbf{r}_i \times \mathbf{F}_i^{ext} = \sum_i \mathbf{r}_i \times \frac{d}{dt}(m_i \mathbf{v}_i), \quad j \neq i. \quad (6.16)$$

The first term on the left side of Eq. (6.16) is the sum of the moments about O due to internal forces, and

$$\mathbf{r}_i \times \mathbf{f}_{ji} + \mathbf{r}_i \times \mathbf{f}_{ij} = \mathbf{r}_i \times (\mathbf{f}_{ji} + \mathbf{f}_{ij}) = \mathbf{0}, \quad j \neq i.$$

The term vanishes because the internal forces between each pair of particles are equal, opposite, and directed along the straight line between the two particles (Fig. 6.1). The second term on the left side of Eq. (6.16),

$$\sum_i \mathbf{r}_i \times \mathbf{F}_i^{ext} = \sum \mathbf{M}_O,$$

represents the sum of the moments about O due to external forces and couples. The term on the right side of Eq. (6.16) is

$$\sum_i \mathbf{r}_i \times \frac{d}{dt}(m_i \mathbf{v}_i) = \sum_i \left[\frac{d}{dt}(\mathbf{r}_i \times m_i \mathbf{v}_i) - \mathbf{v}_i \times m_i \mathbf{v}_i\right] = \frac{d\mathbf{H}_O}{dt}, \quad (6.17)$$

which represents the rate of change of the total angular momentum of the system about the point O.
Equation (6.16) is rewritten as

$$\frac{d\mathbf{H}_O}{dt} = \sum \mathbf{M}_O. \tag{6.18}$$

The rate of change of the angular momentum about O equals the sum of the moments about O due to external forces and couples.
Using Eqs. (6.15) and (6.18), the following result is obtained:

$$\sum \mathbf{M}_O = \frac{d}{dt}(\mathbf{r}_C \times m\mathbf{v}_C + \mathbf{H}_C) = \mathbf{r}_C \times m\mathbf{a}_C + \frac{d\mathbf{H}_C}{dt}, \tag{6.19}$$

where $\mathbf{a}_C$ is the acceleration of the mass center.
With the relation

$$\sum \mathbf{M}_O = \sum \mathbf{M}_C + \mathbf{r}_C \times \mathbf{F} = \sum \mathbf{M}_C + \mathbf{r}_C \times m\mathbf{a}_C,$$

Eq. (6.19) becomes

$$\frac{d\mathbf{H}_C}{dt} = \sum \mathbf{M}_C. \tag{6.20}$$

The rate of change of the angular momentum about the mass center equals the sum of the moments about the mass center.

6.3 Equations of Motion for General Plane Motion

An arbitrary rigid body with the mass m can be divided into N particles P_i, $i = 1, 2, ..., N$. The position vector of the P_i particle is $\mathbf{r}_i = \mathbf{OP}_i$ and the mass of the particle is m_i. Figure 6.5(a) represents the rigid body moving with general planar motion in the (X, Y) plane. The origin of the cartesian reference frame is O. The mass center C of the rigid body is located in the plane of the motion, $C \in (X, Y)$.

Let $d_O = OZ$ be the axis through the fixed origin point O that is perpendicular to the plane of motion of the rigid body (X, Y), $d_O \perp (X, Y)$. Let $d_C = Czz$ be the parallel axis through the mass center C, $d_C || d_O$. The rigid body has a general planar motion and the angular velocity vector is $\boldsymbol{\omega} = \omega\mathbf{k}$. The unit vector of the $d_C = Czz$ axis is $\mathbf{k}$.

The velocity of the P_i particle relative to the mass center is

$$\frac{d\mathbf{R}_i}{dt} = \omega\mathbf{k} \times \mathbf{R}_i,$$

where $\mathbf{R}_i = \mathbf{r}_{CP_i}$. The sum of the moments about O due to external forces and couples is

$$\sum \mathbf{M}_O = \frac{d\mathbf{H}_O}{dt} = \frac{d}{dt}[(\mathbf{r}_C \times m\mathbf{v}_C) + \mathbf{H}_C], \tag{6.21}$$

where

$$\mathbf{H}_C = \sum_i [\mathbf{R}_i \times m_i(\omega\mathbf{k} \times \mathbf{R}_i)],$$

is the angular momentum about d_C. The magnitude of the angular momentum about d_C is

$$\begin{aligned}
&H_C = \mathbf{H}_C \cdot \mathbf{k} = \sum_i [\mathbf{R}_i \times m_i(\omega\mathbf{k} \times \mathbf{R}_i)] \cdot \mathbf{k} = \\
&\sum_i m_i \left[(\mathbf{R}_i \times \mathbf{k}) \times \mathbf{R}_i)\right] \cdot \mathbf{k}\,\omega = \sum_i m_i \left[(\mathbf{R}_i \times \mathbf{k}) \cdot (\mathbf{R}_i \times \mathbf{k})\right] \omega = \\
&\sum_i m_i \left|\mathbf{R}_i \times \mathbf{k}\right|^2 \omega = \sum_i m_i r_i^2 \omega,
\end{aligned} \tag{6.22}$$

where the term $|\mathbf{k} \times \mathbf{R}_i| = r_i$ is the perpendicular distance from d_C to the P_i particle. The identity

$$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}).$$

has been used.

The summation $\sum_i m_i r_i^2$ is replaced by integration over the body $\int r^2\, dm$ and is defined as mass moment of inertia I_{Czz} of the body about the z-axis through C:

$$I_{Czz} = \sum_i m_i r_i^2.$$

The mass moment of inertia I_{Czz} is a constant property of the body and is a measure of the rotational inertia or resistance to change in angular velocity due to the radial distribution of the rigid body mass around z-axis through C.

Equation (6.22) defines the angular momentum of the rigid body about d_C (z-axis through C):

$$H_C = I_{Czz}\,\omega \text{ or } \mathbf{H}_C = I_{Czz}\,\omega\,\mathbf{k} = I_{Czz}\,\boldsymbol{\omega}.$$

Substituting this expression into Eq. (6.21) gives

$$\sum \mathbf{M}_O = \frac{d}{dt}[(\mathbf{r}_C \times m\mathbf{v}_C) + I_{Czz}\boldsymbol{\omega}] = (\mathbf{r}_C \times m\mathbf{a}_C) + I_{Czz}\boldsymbol{\alpha}. \qquad (6.23)$$

The rotational equation of motion for the rigid body is

$$I_{Czz}\,\boldsymbol{\alpha} = \sum \mathbf{M}_C \quad \text{or} \quad I_{Czz}\,\alpha\,\mathbf{k} = \sum M_C\,\mathbf{k}. \qquad (6.24)$$

For general planar motion the angular acceleration is

$$\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \ddot{\theta}\mathbf{k}, \qquad (6.25)$$

where the angle θ describes the position, or orientation, of the rigid body about a fixed axis.

If the rigid body is a plate moving in the plane of motion (X, Y), the mass moment of inertia of the rigid body about z-axis through C becomes the polar mass moment of inertia of the rigid body about C, $I_{Czz} = I_C$. For this case the Eq. (6.24) gives

$$I_C\,\boldsymbol{\alpha} = \sum \mathbf{M}_C. \qquad (6.26)$$

A special application of Eq. (6.26) is for rotation about a fixed point. Consider the special case when the rigid body rotates about the fixed point O as shown in Fig. 6.5(b). It follows that the acceleration of the mass center is expressed as

$$\mathbf{a}_C = \boldsymbol{\alpha} \times \mathbf{r}_C - \omega^2\mathbf{r}_C. \qquad (6.27)$$

The relation between the sum of the moments of the external forces about the fixed point O and the product $I_{Czz}\,\boldsymbol{\alpha}$ is given by Eq. (6.23):

$$\sum \mathbf{M}_O = \mathbf{r}_C \times m\mathbf{a}_C + I_{Czz}\boldsymbol{\alpha}. \qquad (6.28)$$

Equations (6.27) and (6.28) give

$$\begin{aligned}\sum \mathbf{M}_O &= \mathbf{r}_C \times m\,(\boldsymbol{\alpha} \times \mathbf{r}_C - \omega^2\mathbf{r}_C) + I_{Czz}\boldsymbol{\alpha} = \\ & m\,\mathbf{r}_C \times (\boldsymbol{\alpha} \times \mathbf{r}_C) + I_{Czz}\boldsymbol{\alpha} = \\ & m\,[(\mathbf{r}_C \cdot \mathbf{r}_C)\boldsymbol{\alpha} - (\mathbf{r}_C \cdot \boldsymbol{\alpha})\mathbf{r}_C] + I_{Czz}\boldsymbol{\alpha} = \\ & m\,\mathbf{r}_C^2\boldsymbol{\alpha} + I_{Czz}\boldsymbol{\alpha} = (m\,r_C^2 + I_{Czz})\boldsymbol{\alpha}. \end{aligned} \qquad (6.29)$$

According to parallel-axis theorem

$$I_{Ozz} = I_{Czz} + m\, r_C^2,$$

where I_{Ozz} denotes the mass moment of inertia of the rigid body about z-axis through O. For the special case of rotation about a fixed point O one can use the formula

$$I_{Ozz}\boldsymbol{\alpha} = \sum \mathbf{M}_O. \tag{6.30}$$

The general equations of motion for a rigid body in plane motion are (Fig. 6.6):

$$\mathbf{F} = m\,\mathbf{a}_C \quad \text{or} \quad \mathbf{F} = m\ddot{\mathbf{r}}_C, \tag{6.31}$$

$$\sum \mathbf{M}_C = I_{Czz}\,\boldsymbol{\alpha}, \tag{6.32}$$

or using the cartesian components:

$$m\ddot{x}_C = \sum F_x, \;\; m\ddot{y}_C = \sum F_y, \;\; I_{Czz}\ddot{\theta} = \sum M_C. \tag{6.33}$$

Equations (6.31) and (6.32) are interpreted in two ways:

1. The forces and moments are known and the equations are solved for the motion of the rigid body (direct dynamics).
2. The motion of the RB is known and the equations are solved for the force and moments (inverse dynamics).

The dynamic force analysis in this chapter is based on the known motion of the mechanism.

6.4 D'Alembert's principle

Newton's second law can be writen as

$$\mathbf{F} + (-m\mathbf{a}_C) = \mathbf{0}, \;\text{ or }\; \mathbf{F} + \mathbf{F}_{in} = \mathbf{0}, \tag{6.34}$$

where the term $\mathbf{F}_{in} = -m\mathbf{a}_C$ is the *inertia force*. Newton's second law can be regarded as an "equilibrium" equation.

Equation (6.23) relates the total moment about a fixed point O to the acceleration of the mass center and the angular acceleration

$$\sum \mathbf{M}_O = (\mathbf{r}_C \times m\mathbf{a}_C) + I_{Czz}\boldsymbol{\alpha},$$

or

$$\sum \mathbf{M}_O + [\mathbf{r}_C \times (-m\mathbf{a}_C)] + (-I_{Czz}\boldsymbol{\alpha}) = \mathbf{0}. \tag{6.35}$$

The term $\mathbf{M}_{in} = -I_{Czz}\boldsymbol{\alpha}$ is the *inertia moment.* The sum of the moments about any point, including the moment due to the inertial force $-m\mathbf{a}$ acting at mass center and the inertial moment, equals zero.

The equations of motion for a rigid body are analogous to the equations for static equilibrium:
The sum of the forces equals zero and the sum of the moments about any point equals zero when the inertial forces and moments are taken into account. This is called *D'Alembert's principle.*

The dynamic force analysis is expressed in a form similar to static force analysis:

$$\sum \mathbf{R} = \sum \mathbf{F} + \mathbf{F}_{in} = \mathbf{0}, \tag{6.36}$$
$$\sum \mathbf{T}_C = \sum \mathbf{M}_C + \mathbf{M}_{in} = \mathbf{0}, \tag{6.37}$$

where $\sum \mathbf{F}$ is the vector sum of all external forces (resultant of external force), and $\sum \mathbf{M}_C$ is the sum of all external moments about the center of mass C (resultant external moment).
For a rigid body in plane motion in the xy plane,

$$\mathbf{a}_C = \ddot{x}_C\,\mathbf{\imath} + \ddot{y}_C\,\mathbf{\jmath}, \quad \boldsymbol{\alpha} = \alpha\,\mathbf{k},$$

with all external forces in that plane, Eqs. (6.36) and (6.37) become

$$\sum R_x = \sum F_x + F_{in\,x} = \sum F_x + (-m\,\ddot{x}_C) = 0, \tag{6.38}$$
$$\sum R_y = \sum F_y + F_{in\,y} = \sum F_y + (-m\,\ddot{y}_C) = 0, \tag{6.39}$$
$$\sum T_C = \sum M_C + M_{in} = \sum M_C + (-I_C\,\alpha) = 0. \tag{6.40}$$

With d'Alembert's principle the moment summation can be about any arbitrary point P:

$$\sum \mathbf{T}_P = \sum \mathbf{M}_P + \mathbf{M}_{in} + \mathbf{r}_{PC} \times \mathbf{F}_{in} = \mathbf{0}, \tag{6.41}$$

where
- $\sum \mathbf{M}_P$ is the sum of all external moments about P,
- $\mathbf{M}_{in}$ is the inertia moment,

- $\mathbf{F}_{in}$ is the inertia force, and
- $\mathbf{r}_{PC}$ is a vector from P to C.

The dynamic analysis problem is reduced to a static force and moment balance problem where the inertia forces and moments are treated in the same way as external forces and moments.

6.5 Free-Body Diagrams

A free-body diagram is a drawing of a part of a complete system, isolated in order to determine the forces acting on that rigid body.

The following force convention is defined: $\mathbf{F}_{ij}$ represents the force exerted by link i on link j.

Figure 6.7 shows various free-body diagrams that are considered in the analysis of a crank slider mechanism Fig. 6.7(a).

In Fig. 6.7(b), the free body consists of the three moving links isolated from the frame 0. The forces acting on the system include a driving moment $\mathbf{M}$, external driven force $\mathbf{F}$, and the forces transmitted from the frame at joint A, $\mathbf{F}_{01}$, and at joint C, $\mathbf{F}_{03}$. Figure 6.7(c) is a free-body diagram of the two links 1 and 2. Figure 6.7(d) is a free-body diagram of a single link.

The force analysis can be accomplished by examining individual links or a subsystem of links. In this way the joint forces between links as well as the required input force or moment for a given output load are computed.

6.6 Joint Forces Analysis Using Individual Links

Figure 6.8(a) is a schematic diagram of a crank slider mechanism comprised of a crank 1, a connecting rod 2, and a slider 3. The center of mass of link 1 is C_1, the center of mass of link 2 is C_2, and the center of mass of slider 3 is C. The mass of the crank is m_1, the mass of the connecting rod is m_2, and the mass of the slider is m_3. The moment of inertia of link i is $I_{Ci},\ i = 1, 2, 3$. The gravitational force is $\mathbf{G}_i = -m_i\, g\, \mathbf{j},\ i = 1, 2, 3$, where g=9.81 m/s^2 is the acceleration of gravity.

For a given value of the crank angle ϕ and a known driven force $\mathbf{F}_{ext}$ the joint reactions and the drive moment $\mathbf{M}$ on the crank are computed using free-body diagrams of the individual links.

Figures 6.8(b), (c), and (d) show free-body diagrams of the crank 1, the connecting rod 2, and the slider 3. For each moving link the dynamic equilibrium equations are applied.

For the slider 3 the vector sum of the all the forces (external forces $\mathbf{F}_{ext}$, gravitational force $\mathbf{G}_3$, inertia forces $\mathbf{F}_{in\,3}$, joint forces $\mathbf{F}_{23}$, $\mathbf{F}_{03}$) is zero [Fig. 6.8(d)]

$$\sum \mathbf{F}^{(3)} = \mathbf{F}_{23} + \mathbf{F}_{in\,3} + \mathbf{G}_3 + \mathbf{F}_{ext} + \mathbf{F}_{03} = \mathbf{0}.$$

Projecting this force onto x and y axes gives

$$\sum \mathbf{F}^{(3)} \cdot \imath = F_{23x} + (-m_3\,\ddot{x}_C) + F_{ext} = 0, \tag{6.42}$$
$$\sum \mathbf{F}^{(3)} \cdot \jmath = F_{23y} - m_3\,g + F_{03y} = 0. \tag{6.43}$$

For the connecting rod 2 [Fig. 6.8(c)], two vectorial equations can be written

$$\sum \mathbf{F}^{(2)} = \mathbf{F}_{32} + \mathbf{F}_{in\,2} + \mathbf{G}_2 + \mathbf{F}_{12} = \mathbf{0},$$
$$\sum \mathbf{M}_B^{(2)} = (\mathbf{r}_C - \mathbf{r}_B) \times \mathbf{F}_{32} + (\mathbf{r}_{C2} - \mathbf{r}_B) \times (\mathbf{F}_{in\,2} + \mathbf{G}_2) + \mathbf{M}_{in\,2} = \mathbf{0},$$

or

$$\sum \mathbf{F}^{(2)} \cdot \imath = F_{32x} + (-m_2\,\ddot{x}_{C2}) + F_{12x} = 0, \tag{6.44}$$
$$\sum \mathbf{F}^{(2)} \cdot \jmath = F_{32y} + (-m_2\,\ddot{y}_{C2}) - m_2\,g + F_{12y} = 0, \tag{6.45}$$
$$\begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_C - x_B & y_C - y_B & 0 \\ F_{32x} & F_{32y} & 0 \end{vmatrix} + \begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_{C2} - x_B & y_{C2} - y_B & 0 \\ -m_2\,\ddot{x}_{C2} & -m_2\,\ddot{y}_{C2} - m_2\,g & 0 \end{vmatrix}$$
$$-I_{C2}\,\alpha_2\,\mathbf{k} = \mathbf{0}. \tag{6.46}$$

For the crank 1 [Fig. 6.8(b)], there are two vectorial equations:

$$\sum \mathbf{F}^{(1)} = \mathbf{F}_{21} + \mathbf{F}_{in\,1} + \mathbf{G}_1 + \mathbf{F}_{01} = \mathbf{0},$$
$$\sum \mathbf{M}_A^{(1)} = \mathbf{r}_B \times \mathbf{F}_{21} + \mathbf{r}_{C1} \times (\mathbf{F}_{in\,1} + \mathbf{G}_1) + \mathbf{M}_{in\,1} + \mathbf{M} = \mathbf{0},$$

or

$$\sum \mathbf{F}^{(1)} \cdot \imath = F_{21x} + (-m_1\,\ddot{x}_{C1}) + F_{01x} = 0, \tag{6.47}$$
$$\sum \mathbf{F}^{(1)} \cdot \jmath = F_{21y} + (-m_1\,\ddot{y}_{C1}) - m_1\,g + F_{01y} = 0, \tag{6.48}$$
$$\begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_B & y_B & 0 \\ F_{21x} & F_{21y} & 0 \end{vmatrix} + \begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_{C1} & y_{C1} & 0 \\ -m_1\,\ddot{x}_{C1} & -m_1\,\ddot{y}_{C1} - m_1\,g & 0 \end{vmatrix}$$
$$-I_{C1}\,\alpha_1\,\mathbf{k} + M\,\mathbf{k} = \mathbf{0}, \tag{6.49}$$

where $M = |\mathbf{M}|$ is the magnitude of the input moment on the crank.
The eight scalar unknowns F_{03y}, $F_{23x} = -F_{32x}$, $F_{23y} = -F_{32y}$, $F_{12x} = -F_{21x}$, $F_{12y} = -F_{21y}$, F_{01x}, F_{01y}, and M are computed from the set of eight equations (6.42), (6.43), (6.44), (6.45), (6.46), (6.47), (6.48) and (6.49).

6.7 Joint Force Analysis Using Contour Method

An analytical method to compute joint forces that can be applied for both planar and spatial mechanisms will be presented. The method is based on the decoupling of a closed kinematic chain and writing the dynamic equilibrium equations. The kinematic links are loaded with external forces and inertia forces and moments.

A general monocontour closed kinematic chain is considered in Fig. 6.9. The joint force between the links $i-1$ and i (joint A_i) will be determined. When these two links $i-1$ and i are separated [Fig. 6.9(b)] the joint forces $\mathbf{F}_{i-1,i}$ and $\mathbf{F}_{i,i-1}$ are introduced and

$$\mathbf{F}_{i-1,i} + \mathbf{F}_{i,i-1} = \mathbf{0}. \tag{6.50}$$

Table 6.1 shows the joint forces for several joints. The following notations have been used: $\mathbf{M}_\Delta$ is the moment with respect to the axis Δ, and F_Δ is the projection of the force vector $\mathbf{F}$ onto the axis Δ.

It is helpful to "mentally disconnect" the two links $(i-1)$ and i, which create joint A_i, from the rest of the mechanism. The joint at A_i will be replaced by the joint forces $\mathbf{F}_{i-1,i}$ and $\mathbf{F}_{i,i-1}$. The closed kinematic chain has been transformed into two open kinematic chains, and two paths I and II are associated. The two paths start from A_i.

For the path I (counterclockwise), starting at A_i and following I the first joint encountered is A_{i-1}. For the link $i-1$ left behind, dynamic equilibrium equations are written according to the type of the joint at A_{i-1}. Following the same path I, the next joint encountered is A_{i-2}. For the subsystem ($i-1$ and $i-2$) equilibrium conditions corresponding to the type of joint at A_{i-2} can be specified, and so on. A similar analysis is performed for the path II of the open kinematic chain. The number of equilibrium equations written is equal to the number of unknown scalars introduced by joint A_i (joint forces at this joint). For a joint, the number of equilibrium conditions is equal to the number of relative mobilities of the joint.

The five moving link ($j = 1, 2, 3, 4, 5$) mechanism shown in Fig. 6.10(a) has the center of mass locations designated by $C_j(x_{Cj},\, y_{Cj}, 0)$. The following

analysis will consider the relationships of the inertia forces $\mathbf{F}_{in\,j}$, the inertia moments $\mathbf{M}_{in\,j}$, the gravitational force $\mathbf{G}_j$, the driven force, $\mathbf{F}_{ext}$, to the joint reactions $\mathbf{F}_{ij}$ and the drive moment $\mathbf{M}$ on the crank 1.
To simplify the notation the total vector force at C_j is written as $\mathbf{F}_j = \mathbf{F}_{in\,j} + \mathbf{G}_j$ and the inertia moment of link j is written as $\mathbf{M}_j = \mathbf{M}_{in\,j}$.
The diagram representing the mechanism is depicted in Fig. 6.10(b) and has two contours 0-1-2-3-0 and 0-3-4-5-0.

Remark. The joint at C represents a ramification point for the mechanism and the diagram, and the dynamic force analysis will start with this joint. The force computation starts with the contour 0-3-4-5-0 because the driven load $\mathbf{F}_{ext}$ on link 5 is given.

I. Contour 0-3-4-5-0

Reaction $\mathbf{F}_{34}$

The rotation joint at C (or C_R, where the subscript R means rotation), between 3 and 4, is replaced with the unknown reaction [Fig. 6.11]:

$$\mathbf{F}_{34} = -\mathbf{F}_{43} = F_{34x}\,\mathbf{ı} + F_{34y}\,\mathbf{ȷ}.$$

If the path I is followed [Fig. 6.11(a)], for the rotation joint at E (E_R) a moment equation is written

$$\sum \mathbf{M}_E^{(4)} = (\mathbf{r}_C - \mathbf{r}_E) \times \mathbf{F}_{32} + (\mathbf{r}_{C4} - \mathbf{r}_E) \times \mathbf{F}_4 + \mathbf{M}_4 = \mathbf{0},$$

or

$$\begin{vmatrix} \mathbf{ı} & \mathbf{ȷ} & \mathbf{k} \\ x_C - x_E & y_C - y_E & 0 \\ F_{34x} & F_{34y} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{ı} & \mathbf{ȷ} & \mathbf{k} \\ x_{C4} - x_E & y_{C4} - y_E & 0 \\ F_{4x} & F_{4y} & 0 \end{vmatrix}$$
$$+M_4\mathbf{k} = \mathbf{0}. \tag{6.51}$$

Continuing on path I, the next joint is the translational joint at D (D_T). The projection of all the forces that act on 4 and 5 onto the sliding direction Δ (x-axis) should be zero.

$$\sum \mathbf{F}_\Delta^{(4\&5)} = \sum \mathbf{F}^{(4\&5)} \cdot \mathbf{ı} = (\mathbf{F}_{34} + \mathbf{F}_4 + \mathbf{F}_5 + \mathbf{F}_{ext}) \cdot \mathbf{ı} =$$
$$F_{34x} + F_{4x} + F_{5x} + F_{ext} = 0. \tag{6.52}$$

The system of Eqs. (6.51) and (6.52) is solved and the two unknowns F_{34x} and F_{34y} are obtained.

Reaction $\mathbf{F}_{45}$
The rotation joint at E (E_R), between 4 and 5, is replaced with the unknown reaction (Fig. 6.12):

$$\mathbf{F}_{45} = -\mathbf{F}_{54} = F_{45x}\,\mathbf{\imath} + F_{45y}\,\mathbf{\jmath}.$$

If the path I is traced [Fig. 6.12(a)], for the pin joint at C (C_R), a moment equation is written

$$\sum \mathbf{M}_C^{(4)} = (\mathbf{r}_E - \mathbf{r}_C) \times \mathbf{F}_{54} + (\mathbf{r}_{C4} - \mathbf{r}_C) \times \mathbf{F}_4 + \mathbf{M}_4 = \mathbf{0},$$

or

$$\begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_E - x_C & y_E - y_C & 0 \\ -F_{45x} & -F_{45y} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_{C4} - x_C & y_{C4} - y_C & 0 \\ F_{4x} & F_{4y} & 0 \end{vmatrix}$$
$$+ M_4 \mathbf{k} = \mathbf{0}. \tag{6.53}$$

For the path II, the slider joint at E (E_T) is encountered. The projection of all the forces that act on 5 onto the sliding direction Δ (x-axis) should be zero.

$$\sum \mathbf{F}_\Delta^{(5)} = \sum \mathbf{F}^{(5)} \cdot \mathbf{\imath} = (\mathbf{F}_{45} + \mathbf{F}_5 + \mathbf{F}_{ext}) \cdot \mathbf{\imath} =$$
$$F_{45x} + F_{5x} + F_{ext} = 0. \tag{6.54}$$

The unknown force components F_{45x} and F_{45y} are calculated from Eqs. (6.53) and (6.54).

Reaction $\mathbf{F}_{05}$
The slider joint at E (E_T), between 0 and 5, is replaced with the unknown reaction [Fig. 6.13]:

$$\mathbf{F}_{05} = F_{05y}\,\mathbf{\jmath}.$$

The reaction joint introduced by the translational joint is perpendicular on the sliding direction $\mathbf{F}_{05} \perp \Delta$. The application point P of force $\mathbf{F}_{05}$ is unknown.

If the path I is followed [Fig. 6.13(a)], for the pin joint at E (E_R), a moment equation is written for link 5:

$$\sum \mathbf{M}_E^{(5)} = (\mathbf{r}_P - \mathbf{r}_E) \times \mathbf{F}_{05} = \mathbf{0},$$

or

$$x\,F_{05y} = 0 \quad \Rightarrow x = 0. \tag{6.55}$$

The application point is at E ($P \equiv E$).

Continuing on path I, the next joint is the pin joint C (C_R).

$$\sum \mathbf{M}_C^{(4\&5)} = (\mathbf{r}_E - \mathbf{r}_C) \times (\mathbf{F}_{05} + \mathbf{F}_5 + \mathbf{F}_{ext}) + (\mathbf{r}_{C4} - \mathbf{r}_C) \times \mathbf{F}_4 + \mathbf{M}_4 = \mathbf{0},$$

or

$$\begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_E - x_C & y_E - y_C & 0 \\ F_{5x} + F_{ext} & F_{05y} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_{C4} - x_C & y_{C4} - y_C & 0 \\ F_{4x} & F_{4y} & 0 \end{vmatrix}$$
$$+M_4\mathbf{k} = \mathbf{0}. \tag{6.56}$$

The joint reaction force F_{05y} is computed from Eq. (6.56).

II. Contour 0-1-2-3-0

For this contour the joint force $\mathbf{F}_{43} = -\mathbf{F}_{34}$ at the ramification point C is considered as a known external force.

Reaction $\mathbf{F}_{03}$

The pin joint D_R, between 0 and 3, is replaced with the unknown reaction force (Fig. 6.14):

$$\mathbf{F}_{03} = F_{03x}\,\mathbf{\imath} + F_{03y}\,\mathbf{\jmath}.$$

If the path I is followed [Fig. 6.14(a)], a moment equation is written for the pin joint C_R for link 3:

$$\sum \mathbf{M}_C^{(3)} = (\mathbf{r}_D - \mathbf{r}_C) \times \mathbf{F}_{03} + (\mathbf{r}_{C3} - \mathbf{r}_C) \times \mathbf{F}_3 + \mathbf{M}_3 = \mathbf{0},$$

or

$$\begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_D - x_C & y_D - y_C & 0 \\ F_{03x} & F_{03y} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_{C3} - x_C & y_{C3} - y_C & 0 \\ F_{3x} & F_{3y} & 0 \end{vmatrix}$$
$$+M_3\mathbf{k} = \mathbf{0}. \tag{6.57}$$

Continuing on path I, the next joint is the pin joint B_R and a moment equation is written for links 3 and 2:

$$\sum \mathbf{M}_B^{(3\&2)} = (\mathbf{r}_D - \mathbf{r}_B) \times \mathbf{F}_{03} + (\mathbf{r}_{C3} - \mathbf{r}_B) \times \mathbf{F}_3 + \mathbf{M}_3 + (\mathbf{r}_C - \mathbf{r}_B) \times \mathbf{F}_{43}$$
$$+(\mathbf{r}_{C2} - \mathbf{r}_B) \times \mathbf{F}_2 + \mathbf{M}_2 = \mathbf{0},$$

or

$$\begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_D - x_B & y_D - y_B & 0 \\ F_{03x} & F_{03y} & 0 \end{vmatrix} + \begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_{C3} - x_B & y_{C3} - y_B & 0 \\ F_{3x} & F_{3y} & 0 \end{vmatrix}$$
$$+ M_3\mathbf{k} + \begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_C - x_B & y_C - y_B & 0 \\ F_{43x} & F_{43y} & 0 \end{vmatrix} +$$
$$\begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_{C2} - x_B & y_{C2} - y_B & 0 \\ F_{2x} & F_{2y} & 0 \end{vmatrix} + M_2\mathbf{k} = \mathbf{0}. \tag{6.58}$$

The two components F_{03x} and F_{03y} of the joint force are obtained from Eqs. (6.57) and (6.58).

Reaction $\mathbf{F}_{23}$

The pin joint C_R, between 2 and 3, is replaced with the unknown reaction force (Fig. 6.15)

$$\mathbf{F}_{23} = F_{23x}\,\imath + F_{23y}\,\jmath.$$

If the path I is followed, as in Fig. 6.15(a), a moment equation is written for the pin joint D_R for link 3:

$$\sum \mathbf{M}_D^{(3)} = (\mathbf{r}_C - \mathbf{r}_D) \times (\mathbf{F}_{23} + \mathbf{F}_{43}) + (\mathbf{r}_{C3} - \mathbf{r}_D) \times \mathbf{F}_3 + \mathbf{M}_3 = \mathbf{0},$$

or

$$\begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_C - x_D & y_C - y_D & 0 \\ F_{23x} + F_{43x} & F_{23y} + F_{43y} & 0 \end{vmatrix} + \begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_{C3} - x_D & y_{C3} - y_D & 0 \\ F_{3x} & F_{3y} & 0 \end{vmatrix}$$
$$+ M_3\mathbf{k} = \mathbf{0}. \tag{6.59}$$

For the path II, the first joint encountered is the pin joint B_R and a moment equation is written for link 2:

$$\sum \mathbf{M}_B^{(2)} = (\mathbf{r}_C - \mathbf{r}_B) \times (-\mathbf{F}_{23}) + (\mathbf{r}_{C2} - \mathbf{r}_B) \times \mathbf{F}_2 + \mathbf{M}_2 = \mathbf{0},$$

or

$$\begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_C - x_B & y_C - y_B & 0 \\ -F_{23x} & -F_{23y} & 0 \end{vmatrix} + \begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_{C2} - x_B & y_{C2} - y_B & 0 \\ F_{2x} & F_{2y} & 0 \end{vmatrix}$$
$$+ M_2\mathbf{k} = \mathbf{0}. \tag{6.60}$$

The two force components F_{23x} and F_{23y} of the joint force are obtained from Eqs. (6.59) and (6.60).

Reaction $\mathbf{F}_{12}$

The pin joint B_R, between 1 and 2, is replaced with the unknown reaction force (Fig. 6.16):

$$\mathbf{F}_{12} = F_{12x}\,\imath + F_{12y}\,\jmath.$$

If the path I is followed, as in Fig. 6.16(a), a moment equation is written for the pin joint C_R for link 2:

$$\sum \mathbf{M}_C^{(2)} = (\mathbf{r}_B - \mathbf{r}_C) \times \mathbf{F}_{12} + (\mathbf{r}_{C2} - \mathbf{r}_C) \times \mathbf{F}_2 + \mathbf{M}_2 = \mathbf{0},$$

or

$$\begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_B - x_C & y_B - y_C & 0 \\ F_{12x} & F_{12y} & 0 \end{vmatrix} + \begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_{C2} - x_C & y_{C2} - y_C & 0 \\ F_{2x} & F_{2y} & 0 \end{vmatrix} + M_2\mathbf{k} = \mathbf{0}. \tag{6.61}$$

Continuing on path I the next joint encountered is the pin joint D_R, and a moment equation is written for links 2 and 3:

$$\sum \mathbf{M}_D^{(2\&3)} = (\mathbf{r}_B - \mathbf{r}_D) \times \mathbf{F}_{12} + (\mathbf{r}_{C2} - \mathbf{r}_D) \times \mathbf{F}_2 + \mathbf{M}_2 + (\mathbf{r}_C - \mathbf{r}_D) \times \mathbf{F}_{43} + (\mathbf{r}_{C3} - \mathbf{r}_D) \times \mathbf{F}_3 + \mathbf{M}_3 = \mathbf{0},$$

or

$$\begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_B - x_D & y_B - y_D & 0 \\ F_{12x} & F_{12y} & 0 \end{vmatrix} + \begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_{C2} - x_D & y_{C2} - y_D & 0 \\ F_{2x} & F_{2y} & 0 \end{vmatrix} + M_2\mathbf{k} + \begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_C - x_D & y_C - y_D & 0 \\ F_{43x} & F_{43y} & 0 \end{vmatrix} + \begin{vmatrix} \imath & \jmath & \mathbf{k} \\ x_{C3} - x_D & y_{C3} - y_D & 0 \\ F_{3x} & F_{3y} & 0 \end{vmatrix} + M_3\mathbf{k} = \mathbf{0}. \tag{6.62}$$

The two components F_{12x} and F_{12y} of the joint force are computed from Eqs. (6.61) and (6.62).

Reaction $\mathbf{F}_{01}$ and driver moment $\mathbf{M}$

The pin joint A_R, between 0 and 1, is replaced with the unknown reaction force (Fig. 6.17):

$$\mathbf{F}_{01} = F_{01x}\,\mathbf{\imath} + F_{01y}\,\mathbf{\jmath}.$$

The unknown driver moment is $\mathbf{M} = M\,\mathbf{k}$. If the path I is followed [Fig. 6.17(a)], a moment equation is written for the pin joint B_R for link 1:

$$\sum \mathbf{M}_B^{(1)} = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}_{01} + (\mathbf{r}_{C1} - \mathbf{r}_B) \times \mathbf{F}_1 + \mathbf{M}_1 + \mathbf{M} = \mathbf{0},$$

or

$$\begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_A - x_B & y_A - y_B & 0 \\ F_{01x} & F_{01y} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_{C1} - x_B & y_{C1} - y_B & 0 \\ F_{1x} & F_{1y} & 0 \end{vmatrix}$$
$$+M_1\,\mathbf{k} + M\,\mathbf{k} = \mathbf{0}. \tag{6.63}$$

Continuing on path I, the next joint encountered is the pin joint C_R and a moment equation is written for links 1 and 2:

$$\sum \mathbf{M}_C^{(1\&2)} = (\mathbf{r}_A - \mathbf{r}_C) \times \mathbf{F}_{01} + (\mathbf{r}_{C1} - \mathbf{r}_C) \times \mathbf{F}_1 + \mathbf{M}_1 + \mathbf{M} +$$
$$(\mathbf{r}_{C2} - \mathbf{r}_C) \times \mathbf{F}_2 + \mathbf{M}_2 = \mathbf{0}. \tag{6.64}$$

Equation (6.64) is the vector sum of the moments about D_R of all forces and moments that act on links 1, 2, and 3.

$$\sum \mathbf{M}_D^{(1\&2\&3)} = (\mathbf{r}_A - \mathbf{r}_D) \times \mathbf{F}_{01} + (\mathbf{r}_{C1} - \mathbf{r}_D) \times \mathbf{F}_1 + \mathbf{M}_1 + \mathbf{M} +$$
$$(\mathbf{r}_{C2} - \mathbf{r}_D) \times \mathbf{F}_2 + \mathbf{M}_2 + (\mathbf{r}_C - \mathbf{r}_D) \times \mathbf{F}_{43} + (\mathbf{r}_{C3} - \mathbf{r}_D) \times \mathbf{F}_3$$
$$+\mathbf{M}_3 = \mathbf{0}. \tag{6.65}$$

The components F_{01x}, F_{01y} and M are computed from Eqs. (6.63), (6.64), and (6.65).

6.8 Joint Force Analysis Using Dyads

RRR dyad

Figure 6.18 shows an RRR dyad with two links 2 and 3, and three pin joints, B, C, and D. The unknowns are the joint reaction forces:

$$\mathbf{F}_{12} = F_{12x}\mathbf{\imath} + F_{12y}\mathbf{\jmath},$$
$$\mathbf{F}_{43} = F_{43x}\mathbf{\imath} + F_{43y}\mathbf{\jmath},$$
$$\mathbf{F}_{23} = -\mathbf{F}_{32} = F_{23x}\mathbf{\imath} + F_{23y}\mathbf{\jmath}. \tag{6.66}$$

The inertia forces and external forces $\mathbf{F}_j = F_j\mathbf{\imath} + F_j\mathbf{\jmath}$, inertia moments and external moments $\mathbf{M}_j = M_j\mathbf{k}$, ($j$=2,3) are given.

To determine $\mathbf{F}_{12}$ and $\mathbf{F}_{43}$, the following equations are written:

- sum of all forces on links 2 and 3 is zero:

$$\sum \mathbf{F}^{(2\&3)} = \mathbf{F}_{12} + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_{43} = \mathbf{0},$$

or

$$\sum \mathbf{F}^{(2\&3)} \cdot \mathbf{\imath} = F_{12x} + F_{2x} + F_{3x} + F_{43x} = 0,$$
$$\sum \mathbf{F}^{(2\&3)} \cdot \mathbf{\jmath} = F_{12y} + F_{2y} + F_{3y} + F_{43y} = 0. \quad (6.67)$$

- sum of moments of all forces and moments on link 2 about C is zero:

$$\sum \mathbf{M}_C^{(2)} = (\mathbf{r}_B - \mathbf{r}_C) \times \mathbf{F}_{12} + (\mathbf{r}_{C2} - \mathbf{r}_C) \times \mathbf{F}_2 + \mathbf{M}_2 = \mathbf{0}. \quad (6.68)$$

- sum of moments of all forces and moments on link 3 about C is zero:

$$\sum \mathbf{M}_C^{(3)} = (\mathbf{r}_D - \mathbf{r}_C) \times \mathbf{F}_{43} + (\mathbf{r}_{C3} - \mathbf{r}_C) \times \mathbf{F}_3 + \mathbf{M}_3 = \mathbf{0}. \quad (6.69)$$

The components F_{12x}, F_{12y}, F_{43x}, and F_{43y} are calculated from Eqs. (6.67), (6.68), and (6.69).
The reaction force $\mathbf{F}_{32} = -\mathbf{F}_{23}$ is computed from the sum of all forces on link 2

$$\sum \mathbf{F}^{(2)} = \mathbf{F}_{12} + \mathbf{F}_2 + \mathbf{F}_{32} = \mathbf{0},$$

or

$$\sum \mathbf{F}^{(2)} \cdot \mathbf{\imath} = F_{12x} + F_{2x} + F_{32x} = 0,$$
$$\sum \mathbf{F}^{(2)} \cdot \mathbf{\jmath} = F_{12y} + F_{2y} + F_{32y} = 0. \quad (6.70)$$

RRT dyad

Figure 6.19 shows an RRT dyad with the unknown joint reaction forces $\mathbf{F}_{12}$, $\mathbf{F}_{43}$, and $\mathbf{F}_{23} = -\mathbf{F}_{32}$. The joint reaction force $\mathbf{F}_{43}$ is perpendicular to the sliding direction $\mathbf{F}_{43} \perp \Delta$ or

$$\mathbf{F}_{43} \cdot \Delta = (F_{43x}\mathbf{\imath} + F_{43y}\mathbf{\jmath}) \cdot (\cos\theta\mathbf{\imath} + \sin\theta\mathbf{\jmath}) = 0. \quad (6.71)$$

In order to determine $\mathbf{F}_{12}$ and $\mathbf{F}_{43}$ the following equations are written:
• sum of all the forces on links 2 and 3 is zero:

$$\sum \mathbf{F}^{(2\&3)} = \mathbf{F}_{12} + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_{43} = \mathbf{0},$$

or

$$\sum \mathbf{F}^{(2\&3)} \cdot \mathbf{\imath} = F_{12x} + F_{2x} + F_{3x} + F_{43x} = 0,$$
$$\sum \mathbf{F}^{(2\&3)} \cdot \mathbf{\jmath} = F_{12y} + F_{2y} + F_{3y} + F_{43y} = 0. \quad (6.72)$$

• sum of moments of all the forces and the moments on link 2 about C is zero:

$$\sum \mathbf{M}_C^{(2)} = (\mathbf{r}_B - \mathbf{r}_C) \times \mathbf{F}_{12} + (\mathbf{r}_{C2} - \mathbf{r}_C) \times \mathbf{F}_2 + \mathbf{M}_2 = \mathbf{0}. \quad (6.73)$$

The components F_{12x}, F_{12y}, F_{43x}, and F_{43y} are calculated from Eqs. (6.71), (6.72), and (6.73).
The reaction force components F_{32x} and F_{32y} are computed from the sum of all the forces on link 2:

$$\sum \mathbf{F}^{(2)} = \mathbf{F}_{12} + \mathbf{F}_2 + \mathbf{F}_{32} = \mathbf{0},$$

or

$$\sum \mathbf{F}^{(2)} \cdot \mathbf{\imath} = F_{12x} + F_{2x} + F_{32x} = 0,$$
$$\sum \mathbf{F}^{(2)} \cdot \mathbf{\jmath} = F_{12y} + F_{2y} + F_{32y} = 0. \quad (6.74)$$

RTR dyad

The unknown joint reaction forces $\mathbf{F}_{12}$ and $\mathbf{F}_{43}$ are calculated from the relations (Fig. 6.20):
• sum of all the forces on links 2 and 3 is zero:

$$\sum \mathbf{F}^{(2\&3)} = \mathbf{F}_{12} + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_{43} = \mathbf{0},$$

or

$$\sum \mathbf{F}^{(2\&3)} \cdot \mathbf{\imath} = F_{12x} + F_{2x} + F_{3x} + F_{43x} = 0,$$
$$\sum \mathbf{F}^{(2\&3)} \cdot \mathbf{\jmath} = F_{12y} + F_{2y} + F_{3y} + F_{43y} = 0. \quad (6.75)$$

• sum of the moments of all the forces and moments on links 2 and 3 about B is zero:

$$\sum \mathbf{M}_B^{(2\&3)} = (\mathbf{r}_D - \mathbf{r}_B) \times \mathbf{F}_{43} + (\mathbf{r}_{C3} - \mathbf{r}_B) \times \mathbf{F}_3 + \mathbf{M}_3 + (\mathbf{r}_{C2} - \mathbf{r}_B) \times \mathbf{F}_2 + \mathbf{M}_2 = \mathbf{0}. \quad (6.76)$$

• sum of all the forces on link 2 projected onto the sliding direction $\Delta = \cos\theta\boldsymbol{\imath} + \sin\theta\boldsymbol{\jmath}$ is zero:

$$\sum \mathbf{F}^{(2)} \cdot \Delta = (\mathbf{F}_{12} + \mathbf{F}_2) \cdot (\cos\theta\boldsymbol{\imath} + \sin\theta\boldsymbol{\jmath}) = 0. \quad (6.77)$$

The components F_{12x}, F_{12y}, F_{43x}, and F_{43y} are calculated from Eqs. (6.75), (6.76), and (6.77).
The force components F_{32x} and F_{32y} are computed from the sum of all the forces on link 2:

$$\sum \mathbf{F}^{(2)} = \mathbf{F}_{12} + \mathbf{F}_2 + \mathbf{F}_{32} = \mathbf{0},$$

or

$$\sum \mathbf{F}^{(2)} \cdot \boldsymbol{\imath} = F_{12x} + F_{2x} + F_{32x} = 0,$$
$$\sum \mathbf{F}^{(2)} \cdot \boldsymbol{\jmath} = F_{12y} + F_{2y} + F_{32y} = 0. \quad (6.78)$$

6.9 Examples

Example 6.1. The R-RTR mechanism shown in Fig. 6.21(a) has the dimensions: $AB = 0.14$ m, $AC = 0.06$ m, and $CF = 0.2$ m. The driver link 1 makes an angle $\phi = \phi_1 = \dfrac{\pi}{3}$ rad with the horizontal axis and rotates with a constant speed of $n = n_1 = 30\,\pi$ rpm. The position vectors of the points A, B, C, and F are

$$\begin{aligned}
\mathbf{r}_A &= 0\mathbf{\imath} + 0\mathbf{\jmath} \text{ m},\\
\mathbf{r}_B &= \mathbf{r}_{C_2} = x_B\mathbf{\imath} + y_B\mathbf{\jmath} = 0.07\mathbf{\imath} + 0.121\mathbf{\jmath} \text{ m},\\
\mathbf{r}_C &= x_C\mathbf{\imath} + y_C\mathbf{\jmath} = 0\mathbf{\imath} + 0.06\mathbf{\jmath} \text{ m},\\
\mathbf{r}_F &= x_F\mathbf{\imath} + y_F\mathbf{\jmath} = 0.150\mathbf{\imath} + 0.191\mathbf{\jmath} \text{ m},
\end{aligned}$$

where the mass center of the slider 2 is at B ($B = C_2$). The position vectors of the mass centers of links 1 and 3 are

$$\begin{aligned}
\mathbf{r}_{C_1} &= x_{C_1}\mathbf{\imath} + y_{C_1}\mathbf{\jmath} = \frac{x_B}{2}\mathbf{\imath} + \frac{y_B}{2}\mathbf{\jmath} = 0.035\mathbf{\imath} + 0.06\mathbf{\jmath} \text{ m},\\
\mathbf{r}_{C_3} &= x_{C_3}\mathbf{\imath} + y_{C_3}\mathbf{\jmath} = \frac{x_C + x_F}{2}\mathbf{\imath} + \frac{y_C + y_F}{2}\mathbf{\jmath} = 0.075\mathbf{\imath} + 0.125\mathbf{\jmath} \text{ m}.
\end{aligned}$$

The total forces and moments at C_j, $j = 1, 2, 3$ are $\mathbf{F}_j = \mathbf{F}_{in\,j} + \mathbf{G}_j$ and $\mathbf{M}_j = \mathbf{M}_{in\,j}$, where $\mathbf{F}_{in\,j}$ is the inertia force, $\mathbf{M}_j$ is the inertia moment, and $\mathbf{G}_j = -m_j\, g\,\mathbf{\jmath}$ is the gravity force with gravity acceleration $g = 9.81$ m/s^2.

$$\begin{aligned}
\mathbf{F}_1 &= 0.381\mathbf{\imath} + 0.437\mathbf{\jmath} \text{ N}, & \mathbf{M}_1 &= 0\mathbf{k} \text{ N}\cdot\text{m},\\
\mathbf{F}_2 &= 0.545\mathbf{\imath} + 0.160\mathbf{\jmath} \text{ N}, & \mathbf{M}_2 &= -0.001\mathbf{k} \text{ N}\cdot\text{m},\\
\mathbf{F}_3 &= 3.302\mathbf{\imath} - 0.539\mathbf{\jmath} \text{ N}, & \mathbf{M}_3 &= -0.046\mathbf{k} \text{ N}\cdot\text{m}.
\end{aligned}$$

The external moment on link 3 is $\mathbf{M}_{3ext} = -1000\mathbf{k}$ N·m. Determine the moment $\mathbf{M}$ required for dynamic equilibrium and the joint forces for the mechanism using the free-body diagrams of the individual links.

Solution.

For each link two vectorial equations are written:

$$\sum \mathbf{F}_j + \mathbf{F}_{in\,j} = \mathbf{0} \quad \text{and} \quad \sum \mathbf{M}_{C_j} + \mathbf{M}_{in\,j} = \mathbf{0}, \tag{6.79}$$

where $\sum \mathbf{F}_j$ is the vector sum of all external forces (resultant of external force) on link j, and $\sum \mathbf{M}_{C_j}$ is the sum of all external moments on link j about the mass center C_j.

The force analysis will start with link 3 because the moment $\mathbf{M}_{3ext}$ is known.

Link 3

For the free-body diagram of link 3 shown in Fig. 6.21(b), Eq. (6.79) gives

$$\mathbf{F}_{03} + \mathbf{F}_{in\,3} + \mathbf{G}_3 + \mathbf{F}_{23} = \mathbf{0},$$
$$\mathbf{r}_{C_3C} \times \mathbf{F}_{03} + \mathbf{r}_{C_3Q} \times \mathbf{F}_{23} + \mathbf{M}_{in\,3} + \mathbf{M}_{3ext} = \mathbf{0},$$

or

$$\mathbf{F}_{03} + \mathbf{F}_3 + \mathbf{F}_{23} = \mathbf{0},$$
$$\mathbf{r}_{C_3C} \times \mathbf{F}_{03} + \mathbf{r}_{C_3Q} \times \mathbf{F}_{23} + \mathbf{M}_3 + \mathbf{M}_{3ext} = \mathbf{0}, \tag{6.80}$$

where the unknowns are

$$\mathbf{F}_{03} = F_{03x}\,\imath + F_{03y}\,\jmath, \quad \mathbf{F}_{23} = F_{23x}\,\imath + F_{23y}\,\jmath,$$

and the position vector $\mathbf{r}_Q = x_Q\imath + y_Q\jmath$ of the application point of the joint force $\mathbf{F}_{23}$.

Numerically Eq. (6.80) becomes

$$3.302 + F_{03x} + F_{23x} = 0, \tag{6.81}$$
$$-0.539 + F_{03y} + F_{23y} = 0, \tag{6.82}$$
$$-1000.05 + 0.065F_{03x} - 0.075F_{03y} + 0.125F_{23x} - 0.075F_{23y} + F_{23y}x_Q - F_{23x}y_Q = 0. \tag{6.83}$$

The application point Q of the joint force $\mathbf{F}_{23}$ is on the line BC:

$$\frac{y_B - y_C}{x_B - x_C} = \frac{y_Q - y_C}{x_Q - x_C} \quad \text{or} \quad 0.874 - \frac{y_Q - 0.06}{x_Q} = 0. \tag{6.84}$$

The joint force $\mathbf{F}_{23}$ is perpendicular to the sliding direction BC:

$$\mathbf{F}_{23} \cdot \mathbf{r}_{BC} = 0 \quad \text{or} \quad -0.07F_{23x} - 0.061F_{23y} = 0. \tag{6.85}$$

There are five scalar equations, Eqs. (6.81) throught (6.85), and six unknowns,
F_{03x}, F_{03y}, F_{23x}, F_{23y}, x_Q, y_Q. The force analysis will continue with link 2.

Link 2

Figure 6.21(c) shows the free-body diagram of link 2 and Eq. (6.79) gives

$$\begin{aligned}&\mathbf{F}_{12} + \mathbf{F}_{in\,2} + \mathbf{G}_2 + \mathbf{F}_{32} = \mathbf{0},\\ &\mathbf{r}_{BQ} \times \mathbf{F}_{32} + \mathbf{M}_{in\,2} = \mathbf{0},\end{aligned}$$

or

$$\begin{aligned}&\mathbf{F}_{12} + \mathbf{F}_2 - \mathbf{F}_{23} = \mathbf{0},\\ &\mathbf{r}_{BQ} \times (-\mathbf{F}_{23}) + \mathbf{M}_2 = \mathbf{0},\end{aligned}$$

where the new unknown is introduced (the reaction of link 1 on link 2):

$$\mathbf{F}_{12} = F_{12x}\,\mathbf{\imath} + F_{12y}\,\mathbf{\jmath}.$$

Numerically, the previous equations becomes

$$0.545 + F_{12x} - F_{23x} = 0, \tag{6.86}$$

$$0.160 + F_{12y} - F_{23y} = 0, \tag{6.87}$$

$$-0.001 - 0.121\,F_{23x} + 0.07\,F_{23y} - x_Q\,F_{23y} + y_Q\,F_{23x} = 0. \tag{6.88}$$

Now there is a system of eight scalar equations, Eqs. (6.81) through (6.88), eight unknowns, and the solution is

$$\begin{aligned}&\mathbf{F}_{03} = F_{03x}\,\mathbf{\imath} + F_{03y}\,\mathbf{\jmath} = 7078.41\,\mathbf{\imath} - 8093.7\,\mathbf{\jmath} \quad \text{N},\\ &\mathbf{F}_{23} = F_{23x}\,\mathbf{\imath} + F_{23y}\,\mathbf{\jmath} = -7081.72\,\mathbf{\imath} + 8094.24\,\mathbf{\jmath} \quad \text{N},\\ &\mathbf{F}_{12} = F_{12x}\,\mathbf{\imath} + F_{12y}\,\mathbf{\jmath} = -7082.26\,\mathbf{\imath} + 8094.08\,\mathbf{\jmath} \quad \text{N},\\ &\mathbf{r}_Q = x_Q\,\mathbf{\imath} + y_Q\,\mathbf{\jmath} = 0.069\,\mathbf{\imath} + 0.121\,\mathbf{\jmath} \quad \text{m}.\end{aligned}$$

Link 1

Figure 6.21(d) shows the free-body diagram of link 1. The sum of all the forces for the driver link 1 gives

$$\mathbf{F}_{21} + \mathbf{F}_{in\,1} + \mathbf{G}_1 + \mathbf{F}_{01} = \mathbf{0}, \quad \text{or} \quad -\mathbf{F}_{12} + \mathbf{F}_1 + \mathbf{F}_{01} = \mathbf{0}.$$

The reaction of the ground 0 on the link 1 is

$$\begin{aligned}\mathbf{F}_{01} &= \mathbf{F}_{12} - \mathbf{F}_1 = -7082.26\,\mathbf{\imath} + 8094.08\,\mathbf{\jmath} - (0.381\mathbf{\imath} + 0.437\mathbf{\jmath})\\ &= -7082.64\,\mathbf{\imath} + 8094.52\mathbf{\jmath} \;\; \text{N}.\end{aligned}$$

The sum of the moments about the mass center C_1 for link 1 gives the equilibrium moment

$$\mathbf{r}_{C_1B} \times \mathbf{F}_{21} + \mathbf{r}_{C_1A} \times \mathbf{F}_{01} + \mathbf{M} = \mathbf{0},$$

or

$$\begin{aligned}\mathbf{M} &= \mathbf{r}_{C_1B} \times \mathbf{F}_{12} - \mathbf{r}_{C_1A} \times \mathbf{F}_{01} \\ &= 712.632\,\mathbf{k} + 712.671\,\mathbf{k} = 1425.303\,\mathbf{k} \quad \mathrm{N \cdot m}.\end{aligned}$$

Example 6.2. Calculate the moment $\mathbf{M}$ required for dynamic equilibrium and the joint forces for the mechanism shown in Fig. 6.22 using the contour method. The position of the crank angle is $\phi = \frac{\pi}{4}$ rad. The dimensions are $AC = 0.10$ m, $BC = 0.30$ m, $BD = 0.90$ m, and $L_a = 0.10$ m, and the external force on slider 5 is $F_{ext} = 100$ N. The angular speed of crank 1 is $n_1 = 100$ rpm, or $\omega_1 = 100\,\frac{\pi}{30}$ rad/s. The center of mass locations of links $j = 1, 2, ..., 5$ (with the masses m_j) are designated by $C_j(x_{Cj},\, y_{Cj}, 0)$. The position vectors of the joints and the centers of mass are

$$\begin{aligned}
&\mathbf{r}_A = 0\,\imath + 0\,\jmath \;\; \mathrm{m},\\
&\mathbf{r}_{C1} = 0.212\imath + 0.212\jmath \;\; \mathrm{m},\\
&\mathbf{r}_B = \mathbf{r}_{C2} = 0.256\imath + 0.256\jmath \;\; \mathrm{m},\\
&\mathbf{r}_{C3} = 0.178\imath + 0.128\jmath \;\; \mathrm{m},\\
&\mathbf{r}_C = 0.100\imath + 0.000\jmath \;\; \mathrm{m},\\
&\mathbf{r}_{C4} = 0.699\imath + 0.178\jmath \;\; \mathrm{m},\\
&\mathbf{r}_D = \mathbf{r}_{C5} = 1.142\imath + 0.100\jmath \;\; \mathrm{m}.
\end{aligned}$$

The total forces and moments at C_j are $\mathbf{F}_j = \mathbf{F}_{in\,j} + \mathbf{G}_j$ and $\mathbf{M}_j = \mathbf{M}_{in\,j}$, where $\mathbf{F}_{in\,j}$ is the inertia force, $\mathbf{M}_j$ is the inertia moment, and $\mathbf{G}_j = -m_j\,g\,\jmath$ is the gravity force with gravity acceleration $g = 9.81$ m/s^2.

$$\begin{aligned}
&\mathbf{F}_1 = 5.514\imath + 3.189\jmath \;\; \mathrm{N},\\
&\mathbf{F}_2 = 0.781\imath + 1.843\jmath \;\; \mathrm{N},\\
&\mathbf{F}_3 = 1.202\imath + 1.660\jmath \;\; \mathrm{N},\\
&\mathbf{F}_4 = 6.466\imath + 4.896\jmath \;\; \mathrm{N},
\end{aligned}$$

$$\mathbf{F}_5 = 0.643\imath - 0.382\jmath \ \text{N},$$
$$\mathbf{M}_1 = \mathbf{M}_2 = \mathbf{M}_5 = 0\,\mathbf{k} \ \text{N}\cdot\text{m},$$
$$\mathbf{M}_3 = 0.023\mathbf{k} \ \text{N}\cdot\text{m},$$
$$\mathbf{M}_4 = -1.274\mathbf{k} \ \text{N}\cdot\text{m}.$$

Solution.

The diagram representing the mechanism is shown in Fig. 6.22(b) and has two contours, 0-1-2-3-0 and 0-3-4-5-0.

I. Contour 0-3-4-5-0

The joint at B represents a ramification point, and the dynamic force analysis will start with this joint.

Reaction $\mathbf{F}_{34}$

The rotation joint at B_R, between 3 and 4, is replaced with the unknown reaction (Fig. 6.23):

$$\mathbf{F}_{34} = -\mathbf{F}_{43} = F_{34x}\,\imath + F_{34y}\,\jmath.$$

If the path I is followed [Fig. 6.23(a)], a moment equation is written for the rotation joint D_R:

$$\sum \mathbf{M}_D^{(4)} = (\mathbf{r}_B - \mathbf{r}_D) \times \mathbf{F}_{34} + (\mathbf{r}_{C4} - \mathbf{r}_D) \times \mathbf{F}_4 + \mathbf{M}_4 = \mathbf{0}. \tag{6.89}$$

Continuing on path I the next joint is the slider joint D_T, and a force equation is written. The projection of all the forces that act on 4 and 5 onto the sliding direction x is zero:

$$\sum \mathbf{F}^{(4\&5)} \cdot \imath = (\mathbf{F}_{34} + \mathbf{F}_4 + \mathbf{F}_5 + \mathbf{F}_{ext}) \cdot \imath =$$
$$F_{34x} + F_{4x} + F_{5x} + F_{ext} = 0. \tag{6.90}$$

Solving the system of Eqs. (6.89) and (6.90):

$$F_{34x} = -107.110 \ \text{N} \ \text{and} \ F_{34y} = 14.415 \ \text{N}.$$

Reaction $\mathbf{F}_{45}$

The pin joint at D_R, between 4 and 7, is replaced with the reaction force (Fig. 6.24):

$$\mathbf{F}_{45} = -\mathbf{F}_{54} = F_{45x}\,\imath + F_{45y}\,\jmath.$$

For the path I, shown Fig. 6.24(a), a moment equation about B_R is written for link 4:

$$\sum \mathbf{M}_B^{(4)} = (\mathbf{r}_D - \mathbf{r}_B) \times \mathbf{F}_{54} + (\mathbf{r}_{C4} - \mathbf{r}_B) \times \mathbf{F}_4 + \mathbf{M}_4 = \mathbf{0}. \qquad (6.91)$$

For the path II, an equation for the forces projected onto the sliding direction of the joint D_T is written for link 5:

$$\sum \mathbf{F}^{(5)} \cdot \mathbf{\imath} = (\mathbf{F}_{45} + \mathbf{F}_5 + \mathbf{F}_{ext}) \cdot \mathbf{\imath} =$$
$$F_{45x} + F_{5x} + F_{ext} = 0. \qquad (6.92)$$

The joint force $\mathbf{F}_{45}$ is obtained from the system of Eqs. (6.91) and (6.92):

$$F_{45x} = -100.643 \text{ N and } F_{45y} = 19.310 \text{ N}.$$

Reaction $\mathbf{F}_{05}$
The reaction force $\mathbf{F}_{05}$ is perpendicular to the sliding direction of joint D_T (Fig. 6.25):

$$\mathbf{F}_{05} = F_{05y}\,\mathbf{\jmath}.$$

The application point of the unknown reaction force $\mathbf{F}_{05}$ is computed from a moment equation about D_R for link 5 (path I) [Fig. 6.25(a)]:

$$\sum \mathbf{M}_D^{(5)} = (\mathbf{r}_P - \mathbf{r}_D) \times \mathbf{F}_{05} = \mathbf{0}, \qquad (6.93)$$

or

$$x\,F_{05y} = 0 \quad \Rightarrow x = 0. \qquad (6.94)$$

The application point of the reaction force $\mathbf{F}_{05}$ is at D ($P \equiv D$). The magnitude of the reaction force F_{05y} is obtained from a moment equation about B_R for the links 5 and 4 (path I):

$$\sum \mathbf{M}_B^{(5\&4)} = (\mathbf{r}_D - \mathbf{r}_B) \times (\mathbf{F}_{05} + \mathbf{F}_5 + \mathbf{F}_{ext}) +$$
$$(\mathbf{r}_{C4} - \mathbf{r}_B) \times \mathbf{F}_4 + \mathbf{M}_4 = \mathbf{0}. \qquad (6.95)$$

Solving the above equation:

$$F_{05y} = -18.928 \text{ N}.$$

II. Contour 0-1-2-3-0
The reaction force $\mathbf{F}_{43} = 107.110\,\mathbf{\imath} - 14.415\,\mathbf{\jmath}$ N is considered as an external

force for this contour at B.

Reaction $\mathbf{F}_{23}$
The rotation joint at B_R, between 2 and 3, is replaced with the unknown reaction force (Fig. 6.26):

$$\mathbf{F}_{23} = -\mathbf{F}_{32} = F_{23x}\,\imath + F_{23y}\,\jmath.$$

If the path I is followed, as in Fig. 6.26(a), a moment equation is written for the pin joint C_R for link 3:

$$\sum \mathbf{M}_C^{(3)} = (\mathbf{r}_B - \mathbf{r}_C) \times (\mathbf{F}_{23} + \mathbf{F}_{43}) + (\mathbf{r}_{C3} - \mathbf{r}_C) \times \mathbf{F}_3 + \mathbf{M}_3 = \mathbf{0}. \quad (6.96)$$

For the path II, an equation for the forces projected in the direction Δ, the sliding direction of the joint B_T is written for link 2:

$$\sum \mathbf{F}^{(2)} \cdot \Delta = (\mathbf{F}_{32} + \mathbf{F}_2) \cdot (\cos\phi\,\imath + \sin\phi\,\jmath) = 0. \quad (6.97)$$

The joint force $\mathbf{F}_{23}$ is calculated from Eqs. (6.96) and (6.97):

$$F_{23x} = -71.155 \text{ N and } F_{23y} = 73.397 \text{ N}.$$

Reaction $\mathbf{F}_{03}$
For the joint reaction force $\mathbf{F}_{03}$ at C_R, there is only path I. For the pin joint B_R one moment equation is written for link 3 (Fig. 6.27):

$$\sum \mathbf{M}_B^{(3)} = (\mathbf{r}_C - \mathbf{r}_B) \times \mathbf{F}_{03} + (\mathbf{r}_{C3} - \mathbf{r}_B) \times \mathbf{F}_3 + \mathbf{M}_3 = \mathbf{0}. \quad (6.98)$$

A force equation is written for links 3 and 2 for the slider joint B_T:

$$\sum \mathbf{F}^{(3\&2)} \cdot \Delta = (\mathbf{F}_{03} + \mathbf{F}_3 + \mathbf{F}_{43} + \mathbf{F}_2) \cdot (\cos\phi\,\imath + \sin\phi\,\jmath) = 0. \quad (6.99)$$

The components of the unknown force are obtained by solving the system of Eqs. (6.98) and (6.99):

$$F_{03x} = -37.156 \text{ N and } F_{03y} = -60.643 \text{ N}.$$

Reaction $\mathbf{F}_{12}$
The slider joint at B_T, between 1 and 2, is replaced with the reaction force (Fig. 6.28):

$$\mathbf{F}_{12} = -\mathbf{F}_{21} = F_{12x}\,\imath + F_{12y}\,\jmath.$$

The reaction force $\mathbf{F}_{12}$ is perpendicular to the sliding direction Δ:

$$\mathbf{F}_{12} \cdot \Delta = (F_{12x}\,\imath + F_{12y}\,\jmath) \cdot (\cos\phi\imath + \sin\phi\jmath) = \\ F_{12x}\,\cos\phi + F_{12y}\,\sin\phi = 0. \qquad (6.100)$$

The point of application of force $\mathbf{F}_{12}$ is determined from the equation (path I)

$$\sum \mathbf{M}_B^{(2)} = (\mathbf{r}_Q - \mathbf{r}_B) \times \mathbf{F}_{12} = \mathbf{0}, \qquad (6.101)$$

or

$$x\,F_{12} = 0 \quad \Rightarrow x = 0, \qquad (6.102)$$

and the force $\mathbf{F}_{12}$ acts at B.
Continuing on path I, a moment equation is written for links 2 and 3 with respect to the pin joint C_R:

$$\sum \mathbf{M}_C^{(2\&3)} = (\mathbf{r}_B - \mathbf{r}_C) \times (\mathbf{F}_{12} + \mathbf{F}_2 + \mathbf{F}_{43}) + \\ (\mathbf{r}_{C3} - \mathbf{r}_C) \times \mathbf{F}_3 + \mathbf{M}_3 = \mathbf{0}. \qquad (6.103)$$

The two components of the joint force $\mathbf{F}_{12}$ are computed from Eqs. (6.100) and (6.103):

$$F_{12x} = -71.936 \text{ N and } F_{12y} = 71.936 \text{ N}.$$

Reaction $\mathbf{F}_{01}$ and equilibrium moment $\mathbf{M}$
The pin joint A_R, between 0 and 1, is replaced with the unknown reaction (Fig. 6.29):

$$\mathbf{F}_{01} = F_{01x}\,\imath + F_{01y}\,\jmath.$$

The unknown equilibrium moment is $\mathbf{M} = M\,\mathbf{k}$. If the path I is followed [Fig. 6.29(a)] for the slider joint B_T, a force equation is written for link 1:

$$\sum \mathbf{F}^{(1)} \cdot \Delta = (\mathbf{F}_{01} + \mathbf{F}_1) \cdot (\cos\phi\imath + \sin\phi\jmath) = 0. \qquad (6.104)$$

Continuing on path I the next joint encountered is the pin joint B_R, and a moment equation is written for links 1 and 2:

$$\sum \mathbf{M}_B^{(1\&2)} = -\mathbf{r}_B \times \mathbf{F}_{01} + (\mathbf{r}_{C1} - \mathbf{r}_B) \times \mathbf{F}_1 + \mathbf{M} = \mathbf{0}. \qquad (6.105)$$

Equation (6.105) is the vector sum of the moments about C_R of all forces and moments that acts on links 1, 2, and 3:

$$\sum \mathbf{M}_C^{(1\&2\&3)} = -\mathbf{r}_C \times \mathbf{F}_{01} + (\mathbf{r}_{C1} - \mathbf{r}_C) \times \mathbf{F}_1 + \mathbf{M} + \\ (\mathbf{r}_B - \mathbf{r}_C) \times (\mathbf{F}_2 + \mathbf{F}_{43}) + \mathbf{M}_3 + (\mathbf{r}_{C3} - \mathbf{r}_C) \times \mathbf{F}_3 = \mathbf{0}. \qquad (6.106)$$

From Eqs. (6.104), (6.105), and (6.106) the components F_{01x}, F_{01y} and M are computed:

$$F_{01x} = -77.451 \text{ N}, \quad F_{01y} = 68.747 \text{ N}, \quad \text{and} \quad M = 37.347 \text{ N} \cdot \text{m}.$$

Example 6.3. For the R-TRR-RRT mechanism in Example 6.2 calculate the moment $\mathbf{M}$ required for dynamic equilibrium of the mechanism and the joint forces using the dyad method.

Solution.

$B_R\, D_R\, D_T$ dyad

Figure 6.30(a) shows the last dyad $B_R\, D_R\, D_T$ with the unknown joint reactions $\mathbf{F}_{34}$, $\mathbf{F}_{05}$, and $\mathbf{F}_{45} = -\mathbf{F}_{54}$. The joint reaction $\mathbf{F}_{05}$ is perpendicular to the sliding direction $\mathbf{F}_{05} \perp \Delta = \imath$ or

$$\mathbf{F}_{05} = F_{05y}\jmath. \tag{6.107}$$

The following equations are written to determine $\mathbf{F}_{34}$ and $\mathbf{F}_{05}$:

• sum of all the forces on links 4 and 5 is zero:

$$\sum \mathbf{F}^{(4\&5)} = \mathbf{F}_{34} + \mathbf{F}_4 + \mathbf{F}_5 + \mathbf{F}_{ext} + \mathbf{F}_{05} = \mathbf{0},$$

or

$$\sum \mathbf{F}^{(2\&3)} \cdot \imath = F_{43x} + F_{4x} + F_{5x} + F_{ext} = 0,$$
$$\sum \mathbf{F}^{(2\&3)} \cdot \jmath = F_{43y} + F_{4y} + F_{5y} + F_{05y} = 0. \tag{6.108}$$

• sum of moments of all the forces and moments on link 4 about D_R is zero:

$$\sum \mathbf{M}_D^{(4)} = (\mathbf{r}_B - \mathbf{r}_D) \times \mathbf{F}_{43} + (\mathbf{r}_{C4} - \mathbf{r}_D) \times \mathbf{F}_4 + \mathbf{M}_4 = \mathbf{0}. \tag{6.109}$$

From Eqs. (6.108) and (6.109) the unknown components are calculated:

$$F_{34x} = -107.110 \text{ N}, \quad F_{34y} = 14.415 \text{ N}, \quad \text{and} \quad F_{05y} = -18.928 \text{ N}.$$

The reaction components F_{54x} and F_{54y} are computed from the sum of all the forces on link 4 [Fig. 6.30(b)]:

$$\sum \mathbf{F}^{(4)} = \mathbf{F}_{34} + \mathbf{F}_4 + \mathbf{F}_{54} = \mathbf{0},$$

or

$$\sum \mathbf{F}^{(4)} \cdot \imath = F_{34x} + F_{4x} + F_{54x} = 0,$$
$$\sum \mathbf{F}^{(4)} \cdot \jmath = F_{34y} + F_{5y} + F_{54y} = 0, \qquad (6.110)$$

and

$$F_{54x} = 100.643 \text{ N and } F_{54y} = -19.310 \text{ N}.$$

$B_T\, B_R\, C_R$ dyad

Figure 6.31(a) shows the first dyad $B_T\, B_R\, C_R$ with the unknown joint reaction forces $\mathbf{F}_{12}$, $\mathbf{F}_{03}$, and $\mathbf{F}_{23} = -\mathbf{F}_{32}$. The joint reaction force $\mathbf{F}_{12}$ is perpendicular to the sliding direction $\mathbf{F}_{12} \perp \Delta$ or

$$\mathbf{F}_{12} \cdot \Delta = (F_{12x}\imath + F_{12y}\jmath) \cdot (\cos\phi\imath + \sin\phi\jmath) = 0. \qquad (6.111)$$

The following equations are written in order to determine the forces $\mathbf{F}_{12}$ and $\mathbf{F}_{03}$:

• sum of all forces on links 2 and 3 is zero:

$$\sum \mathbf{F}^{(2\&3)} = \mathbf{F}_{12} + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_{43} + \mathbf{F}_{03} = \mathbf{0},$$

or

$$\sum \mathbf{F}^{(2\&3)} \cdot \imath = F_{12x} + F_{2x} + F_{3x} + F_{43x} + F_{03x} = 0,$$
$$\sum \mathbf{F}^{(2\&3)} \cdot \jmath = F_{12y} + F_{2y} + F_{3y} + F_{43y} + F_{03x} = 0. \qquad (6.112)$$

• sum of moments of all the forces and the moments on link 3 about B_R is zero:

$$\sum \mathbf{M}_B^{(3)} = (\mathbf{r}_C - \mathbf{r}_B) \times \mathbf{F}_{03} + (\mathbf{r}_{C3} - \mathbf{r}_B) \times \mathbf{F}_3 + \mathbf{M}_3 = \mathbf{0}. \qquad (6.113)$$

From Eqs. (6.111), (6.112), and (6.113) the following components are obtained:

$$F_{12x} = -71.936 \text{ N and } F_{12y} = 71.936 \text{ N},$$

$$F_{03x} = -37.156 \text{ N and } F_{03y} = -60.643 \text{ N}.$$

The reaction components F_{23x} and F_{23y} are computed from the sum of all the forces on link 3 [Fig. 6.31(b)]:

$$\sum \mathbf{F}^{(3)} = \mathbf{F}_{23} + \mathbf{F}_3 + \mathbf{F}_{43} + \mathbf{F}_{03} = \mathbf{0},$$

or

$$\sum \mathbf{F}^{(3)} \cdot \imath = F_{23x} + F_{3x} + F_{43x} + F_{03x} = 0,$$
$$\sum \mathbf{F}^{(2)} \cdot \jmath = F_{23y} + F_{3y} + F_{43y} + F_{03y} = 0, \tag{6.114}$$

and solving the equations

$$F_{23x} = -71.155 \text{ N and } F_{23y} = 73.397 \text{ N}.$$

Driver link

A force equation for the driver can be written to determine the joint reaction $\mathbf{F}_{01}$ (Fig. 6.32):

$$\sum \mathbf{F}^{(1)} = \mathbf{F}_{01} + \mathbf{F}_1 + \mathbf{F}_{21} = \mathbf{0},$$

or

$$\sum \mathbf{F}^{(1)} \cdot \imath = F_{01x} + F_{1x} + F_{21x} = 0,$$
$$\sum \mathbf{F}^{(1)} \cdot \jmath = F_{01y} + F_{1y} + F_{21y} = 0, \tag{6.115}$$

Solving the above equations gives

$$F_{01x} = -77.451 \text{ N and } F_{01y} = 68.747 \text{ N}.$$

Sum of the moments about A_R for link 1 gives the equilibrium moment

$$\sum \mathbf{M}_A^{(1)} = \mathbf{r}_B \times \mathbf{F}_{21} + \mathbf{r}_{C1} \times \mathbf{F}_1 + \mathbf{M} = \mathbf{0}, \tag{6.116}$$

and $M = 37.347$ N·m.

6.10 Problems

6.1 Figure 6.33 shows a uniform rod of mass m and length L. The rod is free to swing in a vertical plane. The rod is connnected to the ground by a pin joint at the distance D from one end of the rod. The rod makes an angle $\theta(t)$ with the horizontal axis. The local acceleration of gravity is g. a) Find the differential equation or equations describing the motion of the rod. b) Determine the axial and shear components of the force exerted by the pin on the rod as the rod swings by any arbitrary position. c) When the rod is released from rest in the horizontal position the initial value of the angular velocity is zero. Find the initial angular acceleration and the initial pin force components.

6.2 The four-bar mechanism shown in Fig. 3.10(a) has the dimensions: $AB = 80$ mm, $BC = 210$ mm, $CD = 120$ mm, and $AD = 190$ mm. The driver link AB rotates with a constant angular speed of 2400 rpm. The links are homogeneous rectangular prisms made of steel with the width $h = 0.010$ m and the depth $d = 0.001$ m. The external moment applied on the link CD is opposed to the motion of the link and has the value $|\mathbf{M}_{ext}| = 600$ N·m. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on link AB and the joint forces for $\phi = 120°$ using: a) free-body diagram of individual links; b) contour method; and c) dyads.

6.3 The slider crank mechanism shown in Fig. 4.10 has the dimensions $AB = 0.4$ m and $BC = 1$ m. The driver link 1 rotates with a constant angular speed of $n = 1600$ rpm. The links 1 and 2 have a rectangular shape made of steel with the width $h = 0.010$ m and the depth $d = 0.001$ m. The steel slider 3 has the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d = 0.001$ m. The external force applied on the slider 3 is opposed to the motion of the slider and has the value $|\mathbf{F}_{ext}| = 800$ N. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces for $\phi = 30°$ using: a) free-body diagram of individual links; b) contour method; and c) dyads.

6.4 The planar mechanism considered is shown in Fig. 3.19 and has the following data: $AB = 0.150$ m, $BC = 0.400$ m, $CD = 0.370$ m, $CE = 0.230$ m, $EF = CE$, $L_a = 0.300$ m, $L_b = 0.450$ m, and $L_c = CD$. The constant angular speed of the driver link 1 is 1800 rpm. The links 1, 2, 3, and 4 are homogeneous rectangular prisms with the width $h = 0.010$ m and the depth $d = 0.001$ m. The slider 5 has the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d = 0.001$ m. The external force applied on the slider 5 is opposed to the motion of the slider and has the value $|\mathbf{F}_{ext}| = 500$ N. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces for $\phi = \phi_1 = 60°$.

6.5 The R-RRR-RTT mechanism is shown in Fig. 3.20. The following data are given: $AB = 0.080$ m, $BC = 0.350$ m, $CE = 0.200$ m, $CD = 0.150$ m, $L_a = 0.200$ m, $L_b = 0.350$ m, and $L_c = 0.040$ m. The driver link 1 rotates with a constant angular speed of $n = 2200$ rpm. The links 1, 2, 3, and 5 are homogeneous rectangular prisms made of aluminum with the width $h = 0.010$ m and the depth $d = 0.001$ m. The aluminum slider 4 has the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d = 0.001$ m. The external force applied on 5 is opposed to the motion of the link and has the value $|\mathbf{F}_{ext}| = 1000$ N. The density of the material is $\rho_{Al} = 2.8$ Mg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. For $\phi = 145°$ find the equilibrium moment on the driver link 1 and the joint forces. Select suitable dimensions for the link 5.

6.6 The mechanism shown in Fig. 3.21 has the following dimensions: $AB = 100$ mm, $AD = 350$ mm, $BC = 240$ mm, $CE = 70$ mm, $EF = 300$ mm, and $a = 240$ mm. The constant angular speed of the driver link 1 is $n = 1400$ rpm. The links 1 and 4 are homogeneous rectangular prisms with the width $h = 0.010$ m and the depth $d = 0.001$ m. The link 2 has the width $h = 0.010$ m and the depth $d = 0.001$ m. The sliders 3 and 5 have the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d = 0.001$ m. The external force applied on 5 is opposed to the motion of the link and has the value $|\mathbf{F}_{ext}| = 1200$ N. The density of the material is $\rho_{Iron} = 7.2$ Mg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces for $\phi = \phi_1 = 30°$. Select a suitable dimension for link 2.

6.7 The dimensions for the mechanism shown in Fig. 3.22 are: $AB =$ 60 mm, $BD = 160$ mm, $BC = 55$ mm, $CD = 150$ mm, $DE = 100$ mm, $CF = 250$ mm, $AE = 150$ mm, and $b = 40$ mm. The constant angular speed of the driver link 1 is $n = 1400$ rpm. The links 1, 3, and 4 are homogeneous rectangular prisms with the width $h = 0.010$ m and the depth $d = 0.001$ m. The slider 5 has the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d = 0.001$ m. The plate 2 has the width $h = 0.010$ m and the depth $d = 0.001$ m. The external force applied on 5 is opposed to the motion of the link and has the value $|\mathbf{F}_{ext}| = 1500$ N. The density of the material is $\rho_{Bronze} = 8.7$ Mg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces for $\phi = \phi_1 = 60°$.

6.8 The mechanism in Fig. 3.23 has the dimensions: $AB = 110$ mm, $AC =$ 55 mm, $BD = 220$ mm, $DE = 300$ mm, $EF = 175$ mm, $L_a = 275$ mm, and $L_b = 65$ mm. The links 1, 2, 4, and 5 are homogeneous rectangular prisms with the width $h = 0.010$ m and the depth $d = 0.001$ m. The slider 3 has the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d = 0.001$ m. The constant angular speed of the driver link 1 is $n = 2400$ rpm. The external moment on 5 is opposed to the motion of the link $\mathbf{M}_{ext} = -|\mathbf{M}_{ext}|\dfrac{\boldsymbol{\omega}_5}{|\boldsymbol{\omega}_5|}$, where $|\mathbf{M}_{ext}| = 600$ N·m. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces for $\phi = \phi_1 = 150°$.

6.9 The dimensions for the mechanism shown in Fig. 3.24 are: $AB =$ 250 mm, $BC = 650$ mm, $AD = 600$ mm, $CD = 350$ mm, $DE =$ 200 mm, $EF = 600$ mm, and $L_a = 100$ mm. The constant angular speed of the driver link 1 is $n = 2500$ rpm. The links 1, 2, 3, and 4 are homogeneous rectangular prisms with the width $h = 0.010$ m and the depth $d = 0.001$ m. The slider 5 has the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d = 0.001$ m. The external force on 5 is opposed to the motion of the link $\mathbf{F}_{ext} = -|\mathbf{F}_{ext}|\dfrac{\mathbf{v}_F}{|\mathbf{v}_F|}$, where $|\mathbf{F}_{ext}| = 1600$ N. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces for $\phi = \phi_1 = 60°$.

6.10 The mechanism in Fig. 3.25 has the dimensions: $AB = 50$ mm, $AC = 160$ mm, $BD = 250$ mm, $L_a = 30$ mm, and $L_b = 60$ mm. The driver link 1 rotates with a constant angular speed of $n = 1500$ rpm. The links 1, 2, and 5 are homogeneous rectangular prisms with the width $h = 0.010$ m and the depth $d = 0.001$ m. The sliders 3 and 4 have the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d = 0.001$ m. The external moment on 5 is opposed to the motion of the link $\mathbf{M}_{ext} = -|\mathbf{M}_{ext}|\frac{\boldsymbol{\omega}_5}{|\boldsymbol{\omega}_5|}$ where $|\mathbf{M}_{ext}| = 900$ N·m. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces for $\phi = \phi_1 = 130°$. Select a suitable dimension for the link 5.

6.11 Figure 3.26 shows a mechanism with the following dimensions: $AB = 150$ mm, $BD = 500$ mm, and $L_a = 180$ mm. The constant angular speed of the driver link 1 is $n = 1600$ rpm. The links 1, 2, and 4 are homogeneous rectangular prisms with the width $h = 0.010$ m and the depth $d = 0.001$ m. The sliders 3 and 5 have the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d = 0.001$ m. The external force on 5 is opposed to the motion of the link $\mathbf{F}_{ext} = -|\mathbf{F}_{ext}|\frac{\mathbf{v}_D}{|\mathbf{v}_D|}$ where $|\mathbf{F}_{ext}| = 2000$ N. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces for $\phi = 210°$. Select a suitable dimension for the link 4.

6.12 The mechanism in Fig. 3.27 has the dimensions: $AB = 20$ mm, $AC = 50$ mm, $BD = 150$ mm, $DE = 40$ mm, $EF = 27$ mm, $L_a = 7$ mm, and $L_b = 30$ mm. The constant angular speed of the driver link 1 is $n = 1400$ rpm. The links 1, 2, 4, and 5 are homogeneous rectangular prisms with the width $h = 0.010$ m and the depth $d = 0.001$ m. The slider 3 has the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d = 0.001$ m. The external moment on 5 is opposed to the motion of the link $\mathbf{M}_{ext} = -|\mathbf{M}_{ext}|\frac{\boldsymbol{\omega}_5}{|\boldsymbol{\omega}_5|}$ where $|\mathbf{M}_{ext}| = 1500$ N·m. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces for $\phi = \phi_1 = 120°$.

6.13 Figure 3.28 shows a mechanism with the following dimensions: $AB = 250$ mm, $BC = 940$ mm, $CD = DE = 380$ mm, $EF = 700$ mm, $L_a = 930$ mm, and $L_b = L_c = 310$ mm. The driver link 1 rotates with a constant angular speed of $n = 1500$ rpm. The links 1, 2, 3, and 4 are homogeneous rectangular prisms with the width $h = 0.010$ m and the depth $d = 0.001$ m. The slider 5 has the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d = 0.001$ m. The external force on 5 is opposed to the motion of the link $\mathbf{F}_{ext} = -|\mathbf{F}_{ext}|\dfrac{\mathbf{v}_F}{|\mathbf{v}_F|}$ where $|\mathbf{F}_{ext}| = 2000$ N. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces or $\phi = \phi_1 = 120°$.

6.14 Figure 3.29 shows a mechanism with the following dimensions: $AB = 200$ mm, $BC = 900$ mm, $CE = 300$ mm, $CD = 600$ mm, $EF = 600$ mm, $L_a = 500$ mm, $L_b = 800$ mm, and $L_c = 1100$ mm. The constant angular speed of the driver link 1 is $n = 1000$ rpm. The links 1, 2, 3, and 4 are homogeneous rectangular prisms with the width $h = 0.010$ m and the depth $d = 0.001$ m. The slider 5 has the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d = 0.001$ m. The external force on 5 is opposed to the motion of the link $\mathbf{F}_{ext} = -|\mathbf{F}_{ext}|\dfrac{\mathbf{v}_F}{|\mathbf{v}_F|}$ where $|\mathbf{F}_{ext}| = 3000$ N. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces for $\phi = \phi_1 = 150°$.

6.15 Figure 3.30 shows a mechanism with the following dimensions: $AB = 200$ mm, $BC = 540$ mm, $CF = 520$ mm, $CD = 190$ mm, $DE = 600$ mm, $L_a = 700$ mm, $L_b = 400$ mm, and $L_c = 240$ mm. The constant angular speed of the driver link 1 is $n = 1200$ rpm. The links 1, 2, 3, and 4 are homogeneous rectangular prisms with the width $h = 0.010$ m and the depth $d = 0.001$ m. The slider 5 has the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d = 0.001$ m. The external force on 5 is opposed to the motion of the link $\mathbf{F}_{ext} = -|\mathbf{F}_{ext}|\dfrac{\mathbf{v}_E}{|\mathbf{v}_E|}$ where $|\mathbf{F}_{ext}| = 900$ N. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. For $\phi = 30°$ find the equilibrium moment on the driver link 1 and the joint forces.

6.16 Figure 3.31 shows a mechanism with the following dimensions: $AB = 80$ mm, $BC = 200$ mm, $AD = 90$ mm, and $BE = 220$ mm. The constant angular speed of the driver link 1 is $n = 1300$ rpm. The links 1, 2, and 4 are homogeneous rectangular prisms with the width $h = 0.010$ m and the depth $d = 0.001$ m. The sliders 3 and 5 have the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d = 0.001$ m. The external force on 3 is opposed to the motion of the link $\mathbf{F}_{ext} = -|\mathbf{F}_{ext}|\dfrac{\mathbf{v}_C}{|\mathbf{v}_C|}$ where $|\mathbf{F}_{ext}| = 1900$ N. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces for $\phi = 60°$..

6.17 The dimensions of the mechanism shown in Fig. 3.32 are: $AB = 80$ mm, $BC = 150$ mm, $BE = 300$ mm, $CE = 450$ mm, $CD = 170$ mm, $EF = 600$ mm, $L_a = 200$ mm, $L_b = 150$ mm, and $L_c = 50$ mm. The constant angular speed of the driver link 1 is $n = 1500$ rpm. The links 1, 3, and 4 are homogeneous rectangular prisms with the width $h = 0.010$ m and the depth $d = 0.001$ m. The slider 5 has the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d = 0.001$ m. The plate 2 has the width $h = 0.010$ m and the depth $d = 0.001$ m. The external force applied on 5 is opposed to the motion of the link and has the value $|\mathbf{F}_{ext}| = 2000$ N. The density of the material is $\rho_{Bronze} = 8.7$ Mg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces for $\phi = \phi_1 = 210°$.

6.18 The dimensions of the mechanism shown in Fig. 3.33 are: $AB = 140$ mm, $AC = 200$ mm, $CD = 350$ mm, $DE = 180$ mm, and $L_a = 300$ mm. The constant angular speed of the driver link 1 is $n = 900$ rpm. The links 1, 3, and 4 are homogeneous rectangular prisms with the width $h = 0.010$ m and the depth $d = 0.001$ m. The sliders 2 and 5 have the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d = 0.001$ m. The external force on 5 is opposed to the motion of the link $\mathbf{F}_{ext} = -|\mathbf{F}_{ext}|\dfrac{\mathbf{v}_E}{|\mathbf{v}_E|}$ where $|\mathbf{F}_{ext}| = 1000$ N. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces for $\phi = 60°$.

6.19 The dimensions of the mechanism shown in Fig. 3.34 are: AB = 250 mm, AC = 100 mm, CD = 280 mm, and DE = 800 mm. The constant angular speed of the driver link 1 is n = 1600 rpm. The links 1, 3, and 4 are homogeneous rectangular prisms with the width h = 0.010 m and the depth d = 0.001 m. The sliders 2 and 5 have the width w_{Slider} = 0.050 m, the height h_{Slider} = 0.020 m, and the depth d = 0.001 m. The external force on 5 is opposed to the motion of the link $\mathbf{F}_{ext} = -|\mathbf{F}_{ext}|\frac{\mathbf{v}_E}{|\mathbf{v}_E|}$ where $|\mathbf{F}_{ext}|$ = 900 N. The density of the material is ρ_{Steel} = 8000 kg/m^3 and the gravitational acceleration is g = 9.807 m/s^2. For $\phi = \phi_1 = 210°$ find the equilibrium moment on the driver link 1 and the joint forces.

6.20 The dimensions of the mechanism shown in Fig. 3.35 are: AB = 100 mm, AC = 200 mm, and CD = 350 mm. The constant angular speed of the driver link 1 is n = 900 rpm. The links 1, 3, and 5 are homogeneous rectangular prisms with the width h = 0.010 m and the depth d = 0.001 m. The sliders 2 and 4 have the width w_{Slider} = 0.050 m, the height h_{Slider} = 0.020 m, and the depth d = 0.001 m. The external force on 5 is opposed to the motion of the link $\mathbf{F}_{ext} = -|\mathbf{F}_{ext}|\frac{\mathbf{v}_G}{|\mathbf{v}_G|}$ where $|\mathbf{F}_{ext}|$ = 2500 N. The density of the material is ρ_{Steel} = 8000 kg/m^3 and the gravitational acceleration is g = 9.807 m/s^2. Find the equilibrium moment on the driver link 1 and the joint reaction forces for $\phi = \phi_1 = 45°$. Select suitable dimensions for the link 5 and the distance b.

6.21 The dimensions of the mechanism shown in Fig. 3.36 are: AB = 140 mm, AC = 60 mm, and CD = 140 mm. The constant angular speed of the driver link 1 is n = 2200 rpm. The links 1, 3, and 5 are homogeneous rectangular prisms with the width h = 0.010 m and the depth d = 0.001 m. The sliders 2 and 4 has the width w_{Slider} = 0.050 m, the height h_{Slider} = 0.020 m, and the depth d = 0.001 m. The external moment on 5 is opposed to the motion of the link $\mathbf{M}_{ext} = -|\mathbf{M}_{ext}|\frac{\boldsymbol{\omega}_5}{|\boldsymbol{\omega}_5|}$ where $|\mathbf{M}_{ext}|$ = 1500 N·m. The density of the material is ρ_{Steel} = 8000 kg/m^3 and the gravitational acceleration is g = 9.807 m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces for $\phi = \phi_1 = 60°$. Select suitable lengths for the link 3 and 5.

6.22 The dimensions of the mechanism shown in Fig. 3.37 are: $AB = 110$ mm, $AC = 260$ mm, $BD = L_a = 400$ mm, and $DE = 270$ mm. The constant angular speed of the driver link 1 is $n = n_1 = 1000$ rpm. The links 1, 2, and 4 are homogeneous rectangular prisms with the width $h = 0.010$ m and the depth $d = 0.001$ m. The sliders 3 and 5 have the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d = 0.001$ m. The external force on 5 is opposed to the motion of the link $\mathbf{F}_{ext} = -|\mathbf{F}_{ext}|\frac{\mathbf{v}_E}{|\mathbf{v}_E|}$ where $|\mathbf{F}_{ext}| = 900$ N. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces for $\phi = \phi_1 = 45°$.

6.23 The dimensions of the mechanism shown in Fig. 3.38 are: $AB = 180$ mm, $AD = 450$ mm, and $BC = 200$ mm. The constant angular speed of the driver link 1 is $n = 1600$ rpm. The links 1, 2, and 5 are homogeneous rectangular prisms with the width $h = 0.010$ m and the depth $d = 0.001$ m. The sliders 3 and 4 have the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d = 0.001$ m. The external force on 5 is opposed to the motion of the link $\mathbf{F}_{ext} = -|\mathbf{F}_{ext}|\frac{\mathbf{v}_G}{|\mathbf{v}_G|}$ where $|\mathbf{F}_{ext}| = 1500$ N. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces. Select suitable lengths for the link 5 for $\phi = \phi_1 = 135°$.

6.24 The mechanism in Fig. 3.11(a) has the dimensions: $AB = 0.20$ m, $AD = 0.40$ m, $CD = 0.70$ m, $CE = 0.30$ m, and $y_E = 0.35$ m. The constant angular speed of the driver link 1 is $n = 2600$ rpm. The links 1, 3, and 4 are homogeneous rectangular prisms with the width $h = 0.010$ m and the depth $d = 0.001$ m. The sliders 2 and 5 have the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d = 0.001$ m. The external force on 5 is opposed to the motion of the link $\mathbf{F}_{ext} = -|\mathbf{F}_{ext}|\frac{\mathbf{v}_E}{|\mathbf{v}_E|}$ where $|\mathbf{F}_{ext}| = 1500$ N. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces for $\phi = \phi_1 = 30°$.

6.25 The mechanism in Fig. 3.12 has the dimensions: $AB = 0.04$ m, $BC =$

0.07 m, $CD = 0.12$ m, $AE = 0.10$ m, and $L_a = 0.035$ m. The constant angular speed of the driver link 1 is $n = 900$ rpm. The links 1, 2, and 4 are homogeneous rectangular prisms with the width $h = 0.010$ m and the depth $d = 0.001$ m. The sliders 3 and 5 have the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d =$ 0.001 m. The external force on 5 is opposed to the motion of the link $\mathbf{F}_{ext} = -|\mathbf{F}_{ext}|\dfrac{\mathbf{v}_D}{|\mathbf{v}_D|}$ where $|\mathbf{F}_{ext}| = 1250$ N. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces for $\phi = \phi_1 = 60°$.

6.26 The mechanism in Fig. 3.15 has the dimensions: $AC = 0.080$ m, $BC = 0.150$ m, $BD = 0.400$ m, and $L_a = 0.020$ m. The constant angular speed of the driver link 1 is $n = 1500$ rpm. The links 1, 3, and 4 are homogeneous rectangular prisms with the width $h = 0.010$ m and the depth $d = 0.001$ m. The sliders 2 and 5 have the width $w_{Slider} = 0.050$ m, the height $h_{Slider} = 0.020$ m, and the depth $d =$ 0.001 m. The external force on 5 is opposed to the motion of the link $\mathbf{F}_{ext} = -|\mathbf{F}_{ext}|\dfrac{\mathbf{v}_D}{|\mathbf{v}_D|}$ where $|\mathbf{F}_{ext}| = 2000$ N. The density of the material is $\rho_{Steel} = 8000$ kg/m^3 and the gravitational acceleration is $g = 9.807$ m/s^2. Find the equilibrium moment on the driver link 1 and the joint forces for $\phi = \phi_1 = 60°$. Select a suitable length for the link 1.

References

[1] P. Antonescu, *Mechanisms*, Printech, Bucharest, 2003.

[2] P. Appell, *Traité de Mécanique Rationnelle*, Gautier-Villars, Paris, 1941.

[3] I.I. Artobolevski, *Mechanisms in Modern Engineering Design*, MIR, Moscow, 1977.

[4] M. Atanasiu, *Mecanica*, EDP, Bucharest, 1973.

[5] H. Baruh, *Analytical Dynamics*, WCB/McGraw-Hill, Boston, 1999.

[6] A. Bedford and W. Fowler, *Dynamics*, Addison Wesley, Menlo Park, 1999.

[7] A. Bedford and W. Fowler, *Statics*, Addison Wesley, Menlo Park, 1999.

[8] M.I. Buculei, *Mechanisms*, University of Craiova Press, Craiova, 1976.

[9] M.I. Buculei, D. Bagnaru, G. Nanu, D.B. Marghitu, *Analysis of Mechanisms with Bars*, Scrisul romanesc, Craiova, 1986.

[10] T. Demian et al., *Mechanisms - problems*, EDP, Bucharest, 1969.

[11] A.G. Erdman, and G.N. Sandor, *Mechanisms Design*, Prentice-Hall, Upper Saddle River, 1984.

[12] A. Ertas and J.C. Jones, *The Engineering Design Process*, John Wiley & Sons, New York, 1996.

[13] F. Freudenstein, "An Application of Boolean Algebra to the Motion of Epicyclic Drives," *Transaction of the ASME, Journal of Engineering for Industry*, pp.176-182, 1971.

[14] J.H. Ginsberg, *Advanced Engineering Dynamics*, Cambridge University Press, Cambridge, 1995.

[15] D.T. Greenwood, *Principles of Dynamics*, Prentice-Hall, Englewood Cliffs, 1998.

[16] A.S. Hall, Jr., A.R. Holowenko, and H.G. Laughlin, *Theory and problems of machine design*, McGraw-Hill, New York, 1961.

[17] R.C. Hibbeler, *Engineering Mechanics - Statics and Dynamics*, Prentice-Hall, Upper Saddle River, New Jersey, 1995.

[18] R.C. Juvinall and K.M. Marshek, *Fundamentals of Machine Component Design*, John Wiley & Sons, New York, 1983.

[19] T.R. Kane, *Analytical Elements of Mechanics*, Vol. 1, Academic Press, New York, 1959.

[20] T.R. Kane, *Analytical Elements of Mechanics*, Vol. 2, Academic Press, New York, 1961.

[21] T.R. Kane and D.A. Levinson, "The Use of Kane's Dynamical Equtions in Robotics", *MIT International Journal of Robotics Research*, No. 3, pp. 3-21, 1983.

[22] T.R. Kane, P.W. Likins, and D.A. Levinson, *Spacecraft Dynamics*, McGraw-Hill, New York, 1983.

[23] T.R. Kane and D.A. Levinson, *Dynamics*, McGraw-Hill, New York, 1985.

[24] J.T. Kimbrell, *Kinematics Analysis and Synthesis*, McGraw-Hill, New York, 1991.

[25] R. Maeder, *Programming in Mathematica*, Addison–Wesley Publishing Company, Redwood City, California, 1990.

[26] N.H. Madsen, *Statics and Dynamics*, class notes, www.eng.auburn.edu/users/nmadsen/, 2004.

[27] N.I. Manolescu, F. Kovacs, and A. Oranescu, *The Theory of Mechanisms and Machines*, EDP, Bucharest, 1972.

[28] D.B. Marghitu, *Mechanical Engineer's Handbook*, Academic Press, San Diego, California, 2001.

[29] D.B. Marghitu and M.J. Crocker, *Analytical Elements of Mechanisms*, Cambridge University Press, Cambridge, 2001.

[30] D.B. Marghitu and E.D. Stoenescu, *Kinematics and Dynamics of Machines and Machine Design*, class notes, www.eng.auburn.edu/users/marghitu/, 2004.

[31] J.L. Meriam and L.G. Kraige, *Engineering Mechanics: Dynamics*, John Wiley & Sons, New York, 1997.

[32] D.J. McGill and W.W. King, *Engineering Mechanics: Statics and an Introduction to Dynamics*, PWS Publishing Company, Boston, 1995.

[33] R.L. Mott, *Machine elements in mechanical design*, Prentice Hall, Upper Saddle River, New Jersey, 1999.

[34] D.H. Myszka, *Machines and Mechanisms*, Prentice-Hall, Upper Saddle River, New Jersey, 1999.

[35] R.L. Norton, *Machine Design*, Prentice-Hall, Upper Saddle River, New Jersey, 1996.

[36] R.L. Norton, *Design of Machinery*, McGraw-Hill, New York, 1999.

[37] W.C. Orthwein, *Machine Component Design*, West Publishing Company, St. Paul, 1990.

[38] L.A. Pars, *A treatise on analytical dynamics*, Wiley, New York, 1965.

[39] R.M. Pehan, *Dynamics of Machinery*, McGraw-Hill, New York, 1967.

[40] I. Popescu, *Mechanisms*, University of Craiova Press, Craiova, 1990.

[41] I. Popescu and C. Ungureanu, *Structural Synthesis and Kinematics of Mechanisms with Bars*, Universitaria Press, Craiova, 2000.

[42] I. Popescu and D.B. Marghitu, "Dyad Classification for Mechanisms," *World Conference on Integrated Design and Process Technology*, Austin, Texas, December 3-5, 2003.

[43] I. Popescu, E.D. Stoenescu, and D.B. Marghitu, "Analysis of Spatial Kinematic Chains Using the System Groups," *8th International Congress on Sound and Vibration*, St. Petersburg, Russia, July 5-8, 2004.

[44] M. Radoi and E. Deciu, *Mecanica*, EDP, Bucharest, 1981.

[45] F. Reuleaux, *The Kinematics of Machinery*, Dover, New York, 1963.

[46] C.A. Rubin, *The Student Edition of Working Model*, Addison–Wesley Publishing Company, Reading, Massachusetts, 1995.

[47] I.H. Shames, *Engineering Mechanics - Statics and Dynamics*, Prentice-Hall, Upper Saddle River, New Jersey, 1997.

[48] J.E. Shigley and C.R. Mischke, *Mechanical Engineering Design*, McGraw-Hill, New York, 1989.

[49] J.E. Shigley and J.J. Uicker, *Theory of Machines and Mechanisms*, McGraw-Hill, New York, 1995.

[50] R.W. Soutas-Little and D.J. Inman, *Engineering Mechanics: Statics and Dynamics*, Prentice-Hall, Upper Saddle River, New Jersey, 1999.

[51] A. Stan and M. Grumarescu, *Mechanics Problems*, EDP, Bucharest, 1973.

[52] A. Stoenescu, A. Ripianu, and M. Atanasiu, *Theoretical Mechanics Problems*, EDP, Bucharest, 1965.

[53] A. Stoenescu and G. Silas, *Theoretical Mechanics*, ET, Bucharest, 1957.

[54] E.D. Stoenescu, *Dynamics and Synthesis of Kinematic Chains with Impact and Clearance*, Ph.D. Dissertation, Mechanical Engineering, Auburn University, 2005.

[55] L. W. Tsai, *Mechanism Design: Enumeration of Kinematic Structures According to Function*, CRC Press, Boca Raton, Florida, 2001.

[56] R. Voinea, D. Voiculescu, and V. Ceausu, *Mecanica*, EDP, Bucharest, 1983.

[57] K.J. Waldron and G.L. Kinzel, *Kinematics, Dynamics, and Design of Machinery*, John Wiley&Sons, New York, 1999.

[58] C.E. Wilson and J.P. Sadler, *Kinematics and Dynamics of Machinery*, Harper Collins College Publishers, 1991.

[59] C.W. Wilson, *Computer integrated macine design*, Prentice Hall, Inc., Upper Saddle River, New Jersey, 1997.

[60] S. Wolfram, *Mathematica*, Wolfram Media/Cambridge University Press, Cambridge, 1999.

[61] * * * ,*The theory of mechanisms and machines (Teoria mehanizmov i masin)*, Vassaia scola, Minsc, 1970.

[62] * * * ,*Working Model 2D, Users Manual*, Knowledge Revolution, San Mateo, California, 1996.

Figure captions

Figure 6.1 Rigid body as a collection of particles

Figure 6.2 Rigid body with differential element dm

Figure 6.3 System of particles

Figure 6.4 Angular momentum about the mass center for a system of particles

Figure 6.5 (a) Rigid body in general plane motion; (b) rotation about a fixed point

Figure 6.6 Rigid body in plane motion

Figure 6.7 Free-body diagrams for a crank slider mechanism

Figure 6.8 (a) Crank slider mechanism; free-body diagrams: (b) crank 1, (c) connecting, and (d) slider 3

Figure 6.9 (a) Monocontour closed kinematic chain, (b) joint at A_i replaced by the joint forces $\mathbf{F}_{i-1,i}$ and $\mathbf{F}_{i,i-1}$: $\mathbf{F}_{i-1,i} + \mathbf{F}_{i,i-1} = \mathbf{0}$

Figure 6.10 (a) Mechanism, and (b) diagram representing the mechanism

Figure 6.11 Joint force $\mathbf{F}_{34}$ (a) calculation diagram, and (b) force diagram

Figure 6.12 Joint force $\mathbf{F}_{45}$ (a) calculation diagram, and (b) force diagram

Figure 6.13 Joint force $\mathbf{F}_{05}$ (a) calculation diagram, and (b) force diagram

Figure 6.14 Joint force $\mathbf{F}_{03}$ (a) calculation diagram, and (b) force diagram

Figure 6.15 Joint force $\mathbf{F}_{23}$ (a) calculation diagram, and (b) force diagram

Figure 6.16 Joint force $\mathbf{F}_{12}$ (a) calculation diagram, and (b) force diagram

Figure 6.17 Joint force $\mathbf{F}_{01}$ (a) calculation diagram, and (b) force diagram

Figure 6.18 Joint forces for RRR dyad

Figure 6.19 Joint forces for RRT dyad

Figure 6.20 Joint forces for RTR dyad

Figure 6.21 Joint forces for R-RTR mechanism (Example 6.1)

Figure 6.22 (a) Mechanism, and (b) diagram representing the mechanism with two contours

Figure 6.23 Joint force $\mathbf{F}_{34}$ (a) calculation diagram, and (b) force diagram

Figure 6.24 Joint force $\mathbf{F}_{45}$ (a) calculation diagram, and (b) force diagram

Figure 6.25 Joint force $\mathbf{F}_{05}$ (a) calculation diagram, and (b) force diagram

Figure 6.26 Joint force $\mathbf{F}_{23}$ (a) calculation diagram, and (b) force diagram

Figure 6.27 Joint force $\mathbf{F}_{03}$ (a) calculation diagram, and (b) force diagram

Figure 6.28 Joint force $\mathbf{F}_{12}$ (a) calculation diagram, and (b) force diagram

Figure 6.29 Joint force $\mathbf{F}_{01}$ (a) calculation diagram, and (b) force diagram

Figure 6.30 Joint reactions for the dyad $B_R\, D_R\, D_T$

Figure 6.31 Joint reactions for the dyad $B_T\, B_R\, C_R$

Figure 6.32 Joint reactions for the driver link