Contents

2 Fundamentals

2.1 Degrees of Freedom and Motion

The *number of degrees of freedom* (DOF) of a system is equal to the number of independent parameters (measurements) that are needed to uniquely define its position in space at any instant of time. The number of DOF is defined with respect to a reference frame.

Figure 2.1 shows a rigid body (RB) lying in a plane. The rigid body is assumed to be incapable of deformation and the distance between two particles on the rigid body is constant at any time. If this rigid body always remains in the plane, three parameters (three DOF) are required to completely define its position: two linear coordinates (x, y) to define the position of any one point on the rigid body, and one angular coordinate θ to define the angle of the body with respect to the axes. The minimum number of measurements needed to define its position are shown in the figure as x, y, and θ. A rigid body in a plane then has three degrees of freedom. Note that the particular parameters chosen to define its position are not unique. Any alternative set of three parameters could be used. There is an infinity of sets of parameters possible, but in this case there must always be three parameters per set, such as two lengths and an angle, to define the position because a rigid body in plane motion has three DOF.

Six parameters are needed to define the position of a free rigid body in a three-dimensional (3-D) space. One possible set of parameters which could be used are three lengths, (x, y, z), plus three angles $(\theta_x, \theta_y, \theta_z)$. Any free rigid body in three-dimensional space has six degrees of freedom.

A rigid body free to move in a reference frame will, in the general case, have complex motion, which is simultaneously a combination of rotation and translation. For simplicity, only the two-dimensional (2-D) or planar case will be presented. For planar motion the following terms will be defined, Fig. 2.2:

- pure rotation in which the body possesses one point (center of rotation) which has no motion with respect to a "fixed" reference frame [Fig. 2.2(a)]. All other points on the body describe arcs about that center.
- pure translation in which all points on the body describe parallel paths [Fig. 2.2(b)].
- complex motion which exhibits a simultaneous combination of rotation and translation [Fig. 2.2(c)]. With general plane motion, points on the body

will travel nonparallel paths, and there will be, at every instant, a center of rotation, which will continuously change location.

Translation and rotation represent independent motions of the body. Each can exist without the other. For a 2-D coordinate system, as shown in Fig. 2.1, the x and y terms represent the translation components of motion, and the θ term represents the rotation component.

2.2 Links and Joints

Linkages are basic elements of all mechanisms. Linkages are made up of links and joints. A *link*, sometimes known as an *element* or a *member*, is an (assumed) rigid body which possesses nodes. *Nodes* are defined as points at which links can be attached. A link connected to its neighboring elements by s nodes is an element of *degree* s. A link of degree 1 is also called unary [Fig. 2.3(a)], of degree 2, binary [Fig. 2.3(b)], and of degree 3, ternary [Fig. 2.3(c)], etc.

A *joint* is a connection between two or more links (at their nodes). A joint allows some relative motion between the connected links. Joints are also called *kinematic pairs*.

The number of independent coordinates that uniquely determine the relative position of two constrained links is termed *degree of freedom* of a given joint. Alternatively the term *joint class* is introduced. A kinematic pair is of the jth class if it diminishes the relative motion of linked bodies by j degrees of freedom; i.e. j scalar constraint conditions correspond to the given kinematic pair. It follows that such a joint has $(6 - j)$ independent coordinates. The number of degrees of freedom is the fundamental characteristic quantity of joints. One of the links of a system is usually considered to be the reference link, and the position of other RBs is determined in relation to this reference body. If the reference link is stationary, the term *frame* or *ground* is used.

The coordinates in the definition of degree of freedom can be linear or angular. Also the coordinates used can be absolute (measured with regard to the frame) or relative. Figures 2.4-2.9 show examples of joints commonly found in mechanisms. Figures 2.4(a) and 2.4(b) show two forms of a planar, one degree of freedom joint, namely a rotating pin joint and a translating slider joint. These are both typically referred to as *full joints* and are of the 5th class. The pin joint allows one rotational (R) DOF, and the slider joint allows one translational (T) DOF between the joined links. These are both special cases of another common, one degree of freedom joint, the screw and nut [Fig. 2.5(a)]. Motion of either the nut or the screw relative to the other results in helical motion. If the helix angle is made zero [Fig. 2.5(b)], the nut rotates without advancing and it becomes a pin joint. If the helix angle is made 90°, the nut will translate along the axis of the screw, and it becomes a slider joint.

Figure 2.6 shows examples of two degrees of freedom joints, which simultaneously allow two independent, relative motions, namely translation (T) and rotation (R), between the joined links. A two degrees of freedom joint is usually referred to as a *half joint* and is of the 4th class. A half joint is sometimes also called a roll-slide joint because it allows both rotation (rolling) and translation (sliding).

A joystick, ball-and-socket joint, or sphere joint [Fig. 2.7(a)], is an example of a three degrees of freedom joint (3rd class) which allows three independent angular motions between the two links that are joined. This ball joint would typically be used in a three-dimensional mechanism, one example being the ball joints used in automotive suspension systems. A plane joint [Fig. 2.7(b)], is also an example of a three degrees of freedom joint, which allows two translations and one rotation.

Note that to visualize the degree of freedom of a joint in a mechanism, it is helpful to "mentally disconnect" the two links that create the joint from the rest of the mechanism. It is easier to see how many degrees of freedoms the two joined links have with respect to one another. Figure 2.8 shows an example of a 2nd class joint (cylinder on plane) and Figure 2.9 represents a 1st class joint (sphere on plane).

The type of contact between the elements can be point (P), curve (C), or surface (S). The term *lower joint* was coined by Reuleaux to describe joints with surface contact. He used the term *higher joint* to describe joints with point or curve contact. The main practical advantage of lower joints over higher joints is their ability to better trap lubricant between their enveloping surfaces. This is especially true for the rotating pin joint.

A *closed joint* is a joint that is kept together or closed by its geometry. A pin in a hole or a slider in a two-sided slot are forms of closed joints. A *force closed joint*, such as a pin in a half-bearing or a slider on a surface, requires some external force to keep it together or closed. This force could be supplied by gravity, by a spring, or by some external means. In linkages, closed joints are usually preferred, and are easy to accomplish. For cam-follower systems force closure is often preferred.

The *order of a joint* is defined as the number of links joined minus one. The simplest joint combination of two links has order one and it is a single joint [Fig. 2.10(a)]. As additional links are placed on the same joint, the order is increased on a one for one basis [Fig. 2.10(b)]. Joint order has significance in the proper determination of overall degrees of freedom for an assembly.

Bodies linked by joints form a *kinematic chain*. Simple kinematic chains are shown in Fig. 2.11.

A *contour* or *loop* is a configuration described by a polygon consisting of links connected by joints [Fig. 2.11(a)].

The presence of loops in a mechanical structure can be used to define the following types of chains:

• *closed kinematic chains* have one or more loops so that each link and each joint is contained in at least one of the loops [Fig. 2.11(a)]. A closed kinematic chain has no open attachment point.

• *open kinematic chains* contain no closed loops [Fig. 2.11(b)]. A common example of an open kinematic chain is an industrial robot.

• *mixed kinematic chains* are a combination of closed and open kinematic chains.

Another classification is also useful:

• *simple chains* contain only binary elements

• *complex chains* contain at least one element of degree 3 or higher.

A *mechanism* is defined as a kinematic chain in which at least one link has been "grounded" or attached to the frame [Figs. 2.11(a) and 2.12]. Using Reuleaux's definition, a *machine* is a collection of mechanisms arranged to transmit forces and do work. He viewed all energy, or force-transmitting devices as machines that utilize mechanisms as their building blocks to provide the necessary motion constraints.

The following terms can be defined (Fig. 2.12):

• a *crank* is a link that makes a complete revolution about a fixed grounded pivot

• a *rocker* is a link that has oscillatory (back and forth) rotation and is fixed to a grounded pivot

• a *coupler* or connecting rod is a link that has complex motion and is not fixed to ground.

Ground is defined as any link or links that are fixed (nonmoving) with respect to the reference frame. Note that the reference frame may in fact itself be in motion.

2.3 Family and Degrees of Freedom

The concept of *number of degrees of freedom* is fundamental to the analysis of mechanisms. It is usually necessary to be able to determine quickly the number of DOF of any collection of links and joints that may be used to solve a problem.

The number of degrees of freedom or the *mobility* of a system can be defined as:

• the number of inputs which need to be provided in order to create a predictable system output, or

• the number of independent coordinates required to define the position of the system.

The *family* f of a mechanism is the number of degrees of freedom that are eliminated from all the links of the system.

Every free body in space has six degrees of freedom. A system of family f consisting of n movable links has $(6-f)\,n$ degrees of freedom. Each joint of class j diminishes the freedom of motion of the system by $j-f$ degrees of freedom. Designating the number of joints of class k as c_k, it follows that the number of degrees of freedom of the particular system is

$$M = (6-f)\,n - \sum_{j=f+1}^{5} (j-f)\,c_j. \tag{2.1}$$

This is referred to in the literature on mechanisms as the Dobrovolski formula.

A *driver* link is that part of a mechanism that causes motion. An example is a crank. The number of driver links is equal to the number of DOF of the mechanism. A *driven* link or *follower* is that part of a mechanism whose motion is affected by the motion of the driver.

Mechanisms of family f=1

The family of a mechanism can be computed with the help of a mobility table (Table 2.1). Consider the mechanism, shown in Fig. 2.13, that can be used to measure the weight of postal envelopes. The translation along the i axis is denoted by T_i, and the rotation about the i axis is denoted by R_i, where $i = x, y, z$. Every link in the mechanism is analyzed in terms of its translation and rotation about the reference frame xyz. For example the link 0 (ground) has no translations, T_i=No, and no rotations, R_i=No. The link 1 has a rotation motion about the z axis, R_z=Yes. The link 2 has a planar motion (xy is the plane of motion) with a translation along the x axis, T_x=Yes, a translation along the y axis, T_y=Yes, and a rotation about the z

axis, R_z=Yes. The link 3 has a translation along y, T_y=Yes. The link 4 has a planar motion (yz the plane of motion) with a translation along y, T_y=Yes, a translation along z, T_z=Yes, and a rotation about x, R_x=Yes. The link 5 has a rotation about the x axis, R_x=Yes. The results of this analysis are presented with the help of a mobility table (Table 2.1).

Link	T_x	T_y	T_z	R_x	R_y	R_z
0	No	No	No	No	No	No
1	No	No	No	No	No	Yes
2	Yes	Yes	No	No	No	Yes
3	No	Yes	No	No	No	No
4	No	Yes	Yes	Yes	No	No
5	No	No	No	Yes	No	No
					No	

for all links R_y=No $\Longrightarrow$ f=1

Table 2.1. Mobility table for the mechanism shown in Fig. 2.13.

From the mobility table it can be seen that link i, $i = 0, 1, 2, 3, 4, 5$, has no rotation about the y axis, i.e., there is no rotation about the y axis for any of the links of the mechanism (R_y=No). The family of the mechanism is f=1 because there is one DOF, rotation about y, which is eliminated from all the links.

There are six joints of class 5 (rotational joints) in the system at A, B, C, D, E, and F.

The number of DOF for the mechanism in Fig. 2.13, which is of f=1 family is given by

$$M = 5\,n - \sum_{j=2}^{5} (j-1)\,c_j = 5\,n - 4\,c_5 - 3\,c_4 - 2\,c_3 - c_2 = 5\,(5) \;-\; 4\,(6) \;= 1.$$

The mechanism has one DOF (one driver link).

Mechanisms of family f=2

A mobility table for a mechanism of family f=2 (Fig. 2.14), is given in Table 2.2.

Link	T_x	T_y	T_z	R_x	R_y	R_z
0	No	No	No	No	No	No
1	No	No	No	No	No	Yes
2	Yes	Yes	No	Yes	No	Yes
3	Yes	Yes	No	No	No	Yes
4	No	No	No	No	No	Yes
			No		No	

for all links T_z=No & R_y=No $\Longrightarrow$ f=2

Table 2.2. Mobility table for the mechanism shown in Fig. 2.14.

The number of DOF for the f=2 family mechanism is given by

$$M = 4\,n - \sum_{j=3}^{5} (j-2)\,c_j = 4\,n - 3\,c_5 - 2\,c_4 - c_3.$$

The mechanism in Fig. 2.14 has four moving links ($n = 4$), four rotational joints (A, B, D, E) and one screw and nut joint (C); i.e., there are five joints of class 5 ($c_5 = 5$). The number of DOF for this mechanism is

$$M = 4\,n - 3\,c_5 - 2\,c_4 - c_3 = 4\,(4)\; - \;3\,(5)\; = 1.$$

Mechanisms of family f=3

The number of DOF for mechanisms of family f=3 is given by

$$M = 3\,n - \sum_{j=4}^{5} (j-3)\,c_j = 3\,n - 2\,c_5 - c_4.$$

For the mechanism in Fig. 2.11(a) the mobility table is given in Table 2.3.

Link	T_x	T_y	T_z	R_x	R_y	R_z
0	No	No	No	No	No	No
1	No	No	No	No	No	Yes
2	Yes	Yes	No	No	No	Yes
3	No	No	No	No	No	Yes
			No	No	No	

for all links T_z=No & R_x=No & R_y=No $\Longrightarrow$ f=3

Table 2.3. Mobility table for the mechanism shown in Fig. 2.11(a).

The mechanism in Fig. 2.11(a) has three moving links ($n = 3$) and four rotational joints at A, B, C, and D, ($c_5 = 4$). The number of DOF for this mechanism is given by

$$M = 3\,n - 2\,c_5 - c_4 = 3\,(3)\ -\ 2\,(4)\ = 1.$$

The mobility table for the mechanism shown in Fig. 2.12 is given in Table 2.4.

Link	T_x	T_y	T_z	R_x	R_y	R_z
0	No	No	No	No	No	No
1	No	No	No	No	No	Yes
2	Yes	Yes	No	No	No	Yes
3	No	No	No	No	No	Yes
4	Yes	Yes	No	No	No	Yes
5	No	No	No	No	No	Yes
			No	No	No	

for all links T_z=No & R_x=No & R_y=No $\Longrightarrow$ f=3

Table 2.4. Mobility table for the mechanism shown in Fig. 2.12.

There are seven joints of class 5 ($c_5 = 7$) in the system:
• at A there is one rotational joint between link 0 and link 1;
• at B there is one rotational joint between link 1 and link 2;
• at B there is one translational joint between link 2 and link 3;
• at C there is one rotational joint between link 0 and link 3;
• at D there is one rotational joint between link 3 and link 4;
• at D there is one translational joint between link 4 and link 5;
• at A there is one rotational joint between link 5 and link 0.
The number of moving links is five ($n = 5$). The number of DOF for this mechanism is given by

$$M = 3\,n - 2\,c_5 - c_4 = 3\,(5)\ -\ 2\,(7)\ = 1,$$

and this mechanism has one driver link.

Mechanisms of family f=4

The number of DOF for mechanisms of family f=4 is given by

$$M = 2\,n - \sum_{j=5}^{5} (j-4)\,c_j = 2\,n - c_5.$$

For the mechanism shown in Fig. 2.15 the mobility table is given in Table 2.5.

Link	T_x	T_y	T_z	R_x	R_y	R_z
0	No	No	No	No	No	No
1	No	Yes	No	No	No	No
2	Yes	No	No	No	No	No
			No	No	No	No

for all links T_z=No & R_x=No & R_y=No & R_z=No $\Longrightarrow$ f=4

Table 2.5. Mobility table for the mechanism shown in Fig. 2.15.

There are three translational joints of class 5 ($c_5 = 3$) in the system:
- at B there is one translational joint between link 0 and link 1;
- at C there is one translational joint between link 1 and link 2;
- at D there is one translational joint between link 2 and link 0.

The number of DOF for this mechanism with two moving links ($n = 2$) is given by

$$M = 2\,n - c_5 = 2\,(2)\; - (3)\; = 1.$$

Mechanisms of family f=5

The number of DOF for mechanisms of family f=5 is equal with the number of moving links:

$$M = n.$$

The driver link with rotational motion [Fig. 2.16(a)] and the driver link with translational motion [Fig. 2.16(b)] are in the f=5 category.

2.4 Planar Mechanisms

For the special case of planar mechanisms (f=3) the Eq. (2.1) has the form,

$$M = 3\,n - 2c_5 - c_4, \tag{2.2}$$

where n is the number of moving links, c_5 is the number of full joints (one degree of freedom), and c_4 is the number of half joints (two degrees of freedom).

There is a special significance to kinematic chains which do not change their degrees of freedom after being connected to an arbitrary system. Kinematic chains defined in this way are called *system groups* or *fundamental kinematic chains*. Connecting them to or disconnecting them from a given system enables given systems to be modified or structurally new systems to be created while maintaining the original degrees of freedom. The term system group has been introduced for the classification of planar mechanisms used by Assur and further investigated by Artobolevski. Limiting to planar systems from Eq. (2.2), it can be obtained

$$3\,n - 2\,c_5 = 0, \tag{2.3}$$

according to which the number of system group links n is always even. In Eq. (2.3) there are no two degrees of freedom joints because a half joint, c_4, can be substituted with two full joints and an extra link (see Section 2.15).

2.5 Dyads

The simplest fundamental kinematic chain is the binary group with two links (n=2) and three full joints ($c_5 = 3$). The binary group is also called a *dyad*. The sets of links shown in Fig. 2.17 are dyads and one can distinguish the following classical types:

- rotation rotation rotation (dyad RRR) or dyad of type one $D10$ [Fig. 2.17(a)];
- rotation rotation translation (dyad RRT) or dyad of type two $D20$ [Fig. 2.17(b)];
- rotation translation rotation (dyad RTR) or dyad of type three $D30$ [Fig. 2.17(c)];
- translation rotation translation (dyad TRT) or dyad of type four $D40$ [Fig. 2.17(d)];
- translation translation rotation (dyad TTR) or dyad of type five $D50$ [Fig. 2.17(e)].

The advantage of the group classification of a system lies in its simplicity. The solution of the whole system can then be obtained by composing partial solutions. Different versions of dyads exist for each classical dyad [40, 41, 42].

For the classical dyad RRT or $D20$ there are three more different versions, $D21$, $D22$, $D23$, as shown in Fig. 2.18. For the classical dyad RTR or $D30$ there is one different version, $D31$, as shown in Fig. 2.19. Figure 2.20 shows three different versions, $D41$, $D42$, $D43$, of the dyad TRT or $D40$. Figure 2.21 shows seven different versions, $D51$, $D52$, ..., $D57$, of the dyad TTR or $D50$. In this way 19 dyads, Dij, can be obtained where i represents the type and j represents the version.

2.6 Mechanisms with One Dyad

One can connect a dyad to a driver link to create a mechanism with one degree of freedom. The driver link 1 (link AB) can have rotational (R) or translational motion (T). The driver link is connected to a first dyad comprised of the links 2 and 3, and with three joints at B, C, and D. The driver link 1 and the last link 3 are connected to the ground 0.

The closed chain R-D42 represents a mechanism with a driver link 1, with rotational motion (R) and one dyad D42 [Fig. 2.22(a)]. Figure 2.22(b) shows a mechanism R-D20 where the dyad D20 has the length l_3=0.

Figure 2.23 shows a mechanism T-D54. The mechanism has one contour with one rotational joint at A and three translational joints at A, B, and C. The angles α and β are constant angles. From the relations

$$\alpha = \phi + \beta = \text{constant} \quad \text{and} \quad \beta = \text{constant},$$

it results the angle ϕ = constant. With ϕ = constant the link 2 has a translational motion in plane. The mechanism has the family f=4 and it is a *degenerate mechanism*. In general, the planar mechanisms (Fig. 2.23) have the family f=3 with two translations and one rotation. The rotational joint at A is superfluous. For a closed chain to function as a family f=3 mechanism there must be at least two rotational joints for each contour.

2.7 Mechanisms with Two Dyads

There are also mechanisms with one driver link and two dyads. The second dyad is comprised of the links 4 and 5 and three joints at B', C', and D'.

Figure 2.24 represents the ways the second dyad can be connected to the initial mechanism with one driver and one dyad. For simplification only rotational joints are considered for the the following mechanism examples, R $-$ D10 $-$ D10.

Figure 2.24(a) shows the first link of the second dyad, link 4, connected to the driver link 1, and the second link of the second dyad, link 5, connected to ground 0. The symbolization of the dyad connection is 1+0.

Figure 2.24(b) shows the first link of the second dyad, link 4, connected to the driver link 1, and the second link of the second dyad, link 5, connected to link 2 of the first dyad. The symbolization of the dyad connection is 1+2.

Figure 2.24(c) shows the first link of the second dyad, link 4, connected to link 2 of the first dyad, and the second link of the second dyad, link 5, connected to ground 0. The symbolization of the dyad connection is 2+0.

Figure 2.24(d) shows the first link of the second dyad, link 4, connected to link 2 of the first dyad, and the second link of the second dyad, link 5, connected to link 3 of the first dyad. The symbolization of the dyad connection is 2+3.

Figure 2.24(e) shows the first link of the second dyad, link 4, connected to link 3 of the first dyad, and the second link of the second dyad, link 5, connected to ground 0. The symbolization of the dyad connection is 3+0.

Figure 2.24(f) shows the first link of the second dyad, link 4, connected to the driver link 1, and the second link of the second dyad, link 5, connected to link 3 of the first dyad. The symbolization of the dyad connection is 1+3.

Figure 2.25 represents mechanisms with two dyads with rotational and translational joints and their symbolization. Figure 2.25(a) shows a rotational driver link, R, connected to a first dyad, $D21$. The first link 4 of the second dyad $D30$ is connected to the driver link 1 at B', and the second link 5 of the second dyad $D30$ is connected to link 3 at D'. The symbolization of the mechanism is $\mathrm{R} - D21 - D30 - 1 + 3$.

Figure 2.25(b) shows a rotational driver link, R, connected to a first dyad, $D43$. The first link 4 of the second dyad $D50$ is connected to link 2 at B', and the second link 5 of the second dyad $D30$ is connected to ground 0 at D'. The symbolization of the mechanism is $\mathrm{R} - D43 - D50 - 2 + 0$.

Figure 2.25(c) shows a mechanism $\mathrm{R} - D31 - D20 - 3 + 0$. The driver link 1, with rotational motion is connected to the first dyad $D31$. The first link 4 of the second dyad $D20$ is connected to link 3, and the second link 5 is connected to the ground 0.

Figure 2.26 shows a mechanism $\mathrm{T} - D21 - D50 - 2 + 0$. There are two contours: $0 - 1 - 2 - 3 - 0$ and $0 - 1 - 2 - 4 - 5 - 0$. The first contour, $0 - 1 - 2 - 3 - 0$, has translational joints at A and B and rotational joints at C and D. The family of this contour is $f_I = 3$. The second contour, $0 - 1 - 2 - 4 - 5 - 0$, has translational joints at A, B, C', and D' and one rotational joint at B'.

The angle ϕ = constant and the angle λ_1 = constant. Then the angle $\alpha = \phi - \lambda_1$ = constant.
The angle λ_2 = constant. Then the angle $\gamma = \alpha + \lambda_2$ = constant.
The angle θ = constant and the angle λ_3 = constant. Then the angle $\delta = \theta + \lambda_3$ = constant.

With γ = constant and δ = constant, the links 2 and 4 have a translational motion in plane. The second contour has the family $f_{II} = 4$ and the mechanism is a degenerate mechanism. The closed contours that do not have a minimum of two rotational joints are contours of family $f = 4$.

To calculate the number of degrees of freedom for kinematic chains with different families the following formula is introduced [1]:

$$M = (6 - f_a)\, n - \sum_{j=f+1}^{5} (j - f_a)\, c_j, \tag{2.4}$$

where f_a is the *apparent family*.

For the mechanism in Fig. 2.26 with two contours, the apparent family is

$$f_a = \frac{f_I + f_{II}}{2} = \frac{3+4}{2} = \frac{7}{2}\ ,$$

and the number of degrees of freedom of the degenerate mechanism is

$$M = (6 - f_a)\, n - (5 - f_a)\, c_5 = \left(6 - \frac{7}{2}\right) 4 - \left(5 - \frac{7}{2}\right) 6 = 1. \tag{2.5}$$

There are $n = 4$ moving links (link 2 and link 4 form one moving link) and $c_5 = 6$.

2.8 Mechanisms with Three Dyads

There are also mechanisms with one driver link and three dyads. The second dyad is comprised of the links 4 and 5 and three joints at B', C', and D'. The third dyad has the links 6 and 7 and three joints at B'', C'', and D''.

Figure 2.27 represents some ways of connection for the the third dyad. For simplification, only rotational joints are considered, i.e., the mechanism $\mathrm{R} - D10 - D10 - D10$ is presented.

Figure 2.27(a) shows the first link of the second dyad, link 4, connected to the driver link 1, and the second link of the second dyad, link 5, connected to link 2. The connection symbolization for the second dyad is 1+2. The first link of the third dyad, link 6, is connected to link 4 and the second link of the third dyad, link 7, is connected to link 3. The connection symbolization for the third dyad is 4+3. The connection symbolization for the mechanism is 1+2-4+3.

Figure 2.27(b) represents the first link of the second dyad, link 4, connected to the driver link 1, and the second link of the second dyad, link 5, connected to link 2. The connection symbolization for the second dyad is 1+2. The first link of the third dyad, link 6, is connected to link 4 and the second link of the third dyad, link 7, is connected to ground 0. The connection symbolization for the third dyad is 4+0. The connection symbolization for the mechanism is 1+2-4+0.

Figure 2.27(c) shows the first link of the second dyad, link 4, connected to the driver link 2, and the second link of the second dyad, link 5, connected to ground 0. The connection symbolization for the second dyad is 2+0. The first link of the third dyad, link 6, is connected to the link 2 and the second link of the third dyad, link 7, is connected to link 4. The connection symbolization for the third dyad is 2+4. The connection symbolization for the mechanism is 2+0-2+4.

Figure 2.27(d) shows the first link of the second dyad, link 4, connected to the driver link 2, and the second link of the second dyad, link 5, connected to ground 0. The connection symbolization for the second dyad is 2+0. The first link of the third dyad, link 6, is connected to link 2 and the second link of the third dyad, link 7, is connected to link 5. The connection symbolization for the third dyad is 2+5. The connection symbolization for the mechanism is 2+0-2+5.

Mechanisms with three dyads with rotational and translational joints and their symbolization are shown in Fig. 2.28. Figure 2.28(a) shows a rotational driver link, R, connected to a first dyad, $D42$. The first link 4 of the second dyad $D30$ is connected to the driver link 1, and the second link 5 of the second dyad $D30$ is connected to link 2. The first link 6 of the third dyad $D21$ is connected to link 4, and the second link 7 of the second dyad $D21$ is connected to ground 0. The symbolization of the mechanism is $\mathrm{R} - D42 - D30 - D21 - 1 + 2 - 4 + 0$.

Figure 2.28(b) presents a mechanism $\mathrm{T} - D22 - D30 - D20 - 1 + 2 - 4 + 5$. The slider driver link, T, is connected to a first dyad, $D22$. The first link 4 of the second dyad $D30$ is connected to the driver link 1, and the second link 5 is connected to link 2. The first link 6 of the third dyad $D20$ is connected to link 4, and the second link 7 is connected to link 5.

2.9 Independent Contours

A contour is a configuration described by a polygon consisting of links connected by joints. A contour with at least one link that is not included in any other contour of the chain is called *independent contour*. The number of independent contours, N, of a kinematic chain can be computed as

$$N = c - n, \tag{2.6}$$

where c is the number of joints, and n is the number of moving links.

Planar kinematic chains are presented in Fig. 2.29. The kinematic chain shown in Fig. 2.29(a) has two moving links, 1 and 2 ($n = 2$), three joints ($c = 3$), and one independent contour ($N = c - n = 3 - 2 = 1$). This kinematic chain is a dyad. In Fig. 2.29(b), a new kinematic chain is obtained by connecting the free joint of the link 1 to the ground (link 0). In this case, the number of independent contours is also $N = c - n = 3 - 2 = 1$. The kinematic chain shown in Fig. 2.29(c) has three moving links, 1, 2, and 3 ($n = 3$), four joints ($c = 4$), and one independent contour ($N = c - n = 4 - 3 = 1$). A closed chain with three moving links, 1, 2, and 3 ($n = 3$), and one fixed link 0, connected by four joints ($c = 4$) is shown in Fig. 2.29(d). This is a four-bar mechanism. In order to find the number of independent contours, only the moving links are considered. Thus, there is one independent contour ($N = c - n = 4 - 3 = 1$). The kinematic chain presented in Fig. 2.29(e) has four moving links, 1, 2, 3, and 4 ($n = 4$), and six joints ($c = 6$). There are three contours: 1-2-3, 1-2-4, and 3-2-4. Only two contours are independent contours ($N = 6 - 4 = 2$).

Spatial kinematic chains are depicted in Fig. 2.30. The kinematic chain shown in Fig. 2.30(a) has five links, 1, 2, 3, 4, and 5 ($n = 5$), six joints ($c = 6$), and one independent contour ($N = c - n = 6 - 5 = 1$). For the spatial chain shown in Fig. 2.30(b), there are six links, 1, 2, 3, 4, 5, and 6 ($n = 6$), eight joints ($c = 8$), and three contours, 1-2-3-4-5, 1-2-3-6, and 5-4-3-6. In this case, two of the contours are independent contours ($N = c - n = 8 - 6 = 2$).

2.10 Spatial System Groups

The system groups for spatial mechanisms can be determined by analogy to the system groups for the planar mechanisms. The system groups have the degree of freedom $M = 0$. All possible system groups can be determined for each family of chains [40, 41, 43].

For the family $f = 0$, for system groups, from Eqs. (2.1) and (2.6) the mobility is

$$M = 6n - 5c_5 - 4c_4 - 3c_3 - 2c_2 - c_1 = 0, \tag{2.7}$$

and the number of moving links is

$$n = c - N. \tag{2.8}$$

From Eqs. (2.7) and (2.8) the number of joints of class 5 is

$$c_5 = 6N - 5c_1 - 4c_2 - 3c_3 - 2c_4, \tag{2.9}$$

and the number of moving links is

$$n = -N + c_1 + c_2 + c_3 + c_4 + c_5. \tag{2.10}$$

For the family $f = 1$, $c_1 = 0$ it results:

$$c_5 = 5N - 4c_2 - 3c_3 - 2c_4,\ n = -N + c_2 + c_3 + c_4 + c_5. \tag{2.11}$$

For the family $f = 2$, $c_1 = 0,\ c_2 = 0$ it results:

$$c_5 = 4N - 3c_3 - 2c_4,\ n = -N + c_3 + c_4 + c_5. \tag{2.12}$$

For the family $f = 3$, $c_1 = 0,\ c_2 = 0,\ c_3 = 0$ it results:

$$c_5 = 3N - 2c_4,\ n = -N + c_4 + c_5. \tag{2.13}$$

For the family $f = 4$, $c_1 = 0,\ c_2 = 0,\ c_3 = 0,\ c_4 = 0$ it results:

$$c_5 = 2N,\ n = -N + c_5. \tag{2.14}$$

Using the above conditions, all the possible solutions for spatial system groups can be determined. The number of joints, $c_1,\ c_2,\ ,c_3$, and c_4, are cycled from 0 to w, where w is a positive integer, for system groups with one or more independent contours ($N \geq 1$). The number of joints, c_5, and the number of moving links, n, are computed for each system group. An acceptable solution has to verify the conditions $n > 0$ and $c_5 > 0$. In Table 2.6, the number of possible solutions is presented for some values of w between 0 and 40 and for kinematic chains with one contour ($N = 1$), two contours ($N = 2$), and three contours ($N = 3$). For $N = 1$ and $w \geq 3$, there are 23 possible solutions. For $N = 2$, there are 85 solutions for $w \geq 6$, and for $N = 3$ there are 220 solutions for $w \geq 9$.

w	0	1	2	3	4	5	6	7	8	9	10	20	30	40
$N = 1$	5	18	22	23	23	23	23	23	23	23	23	23	23	23
$N = 2$	5	30	62	76	82	84	85	85	85	85	85	85	85	85
$N = 3$	5	31	100	158	190	205	214	218	218	220	220	220	220	220

Table 2.6. The number of configurations of system groups with one, two and three independent contours ($N = 1$, 2, and 3).

2.11 Spatial System Groups with One Independent Contour

The combinations of spatial system groups with one independent contour ($N = 1$) are presented in Table 2.7. The number of joints, c_1, c_2, c_3, and c_4, are cycled from 0 to 3, and the number of joints, c_5, and the number of moving links, n, are computed. System groups from Table 2.7 are exemplified next for each of the families $f = 0,\ 1,\ 2,\ 3$, and 4.

Index	f	c_1	c_2	c_3	c_4	c_5	n
1	0	0	0	0	0	6	5
2	0	0	0	0	1	4	4
3	0	0	0	0	2	2	3
4	0	0	0	0	3	0	2
5	0	0	0	1	0	3	3
6	0	0	0	1	1	1	2
7	0	0	0	2	0	0	1
8	0	0	1	0	0	2	2
9	0	0	1	0	1	0	1
10	0	1	0	0	0	1	1
11	1	0	0	0	0	5	4
12	1	0	0	0	1	3	3
13	1	0	0	0	2	1	2
14	1	0	0	1	0	2	2
15	1	0	0	1	1	0	1
16	1	0	1	0	0	1	1
17	2	0	0	0	0	4	3
18	2	0	0	0	1	2	2
19	2	0	0	0	2	0	1
20	2	0	0	1	0	1	1
21	3	0	0	0	0	3	2
22	3	0	0	0	1	1	1
23	4	0	0	0	0	2	1

Table 2.7 The configurations of system groups with one independent contour ($N = 1$)

For the family $f = 0$, four system groups are illustrated in Fig. 2.31. The values c_5 and n are computed from Eqs. (2.9) and (2.10), respectively. A spatial system group with no joints of class 1, 2, 3, and 4 ($c_1 = c_2 = c_3 = c_4 = 0$) is shown in Fig. 2.31(a). The system group has six joints of class 5 ($c_5 = 6(1) = 6$), and five moving links ($n = -1 + 6 = 5$). A system group with one joint of class 4 ($c_4 = 1$) and no joints of class 1, 2, and 3 ($c_1 = c_2 = c_3 = 0$) is shown in Fig. 2.31(b). The system group has four joints of class 5 ($c_5 = 6(1) - 2(1) = 4$), and four moving links ($n = -1 + 1 + 4 = 4$). A system group with two joints of class 4 ($c_4 = 2$) and no joints of class

1, 2, and 3 ($c_1 = c_2 = c_3 = 0$) is shown in Fig. 2.31(c). The system group has two joints of class 5 ($c_5 = 6(1) - 2(2) = 2$), and three moving links ($n = -1 + 2 + 2 = 3$). A system group with one joint of class 3 ($c_3 = 1$) and no joints of class 1, 2, and 4 ($c_1 = c_2 = c_4 = 0$) is shown in Fig. 2.31(d). The system group has three joints of class 5 ($c_5 = 6(1) - 3(1) = 3$), and three moving links ($n = -1 + 1 + 3 = 3$).

The spatial mechanism presented in Fig. 2.32 is built from the system group shown in Fig. 2.31(b). The mechanism has one degree of freedom ($M = 6n - 5c_5 - 4c_4 - 3c_3 - 2c_2 - c_1 = 6(5) - 5(5) - 4(1) = 1$). The driver link is link 5.

For the family $f = 1$, three systems groups are depicted in Fig. 2.33. The values c_5 and n are computed from Eq. (2.11). The missing translations and rotations with respect to the axis of the reference frame $xOyz$ are specified further on for each system group. For a cartesian reference frame $xOyz$ the rotations about the axis are represented by R and the translations along the axis are represented by T.

A spatial system group with no joints of class 1, 2, 3, and 4 ($c_1 = c_2 = c_3 = c_4 = 0$) is shown in Fig. 2.33(a). The system group has five joints of class 5 ($c_5 = 5(1) = 5$), and four moving links ($n = -1 + 5 = 4$). There are no rotations R_x (rotation about x-axis) for the links of the system group. A system group with no joints of class 1, 2, and 3 ($c_1 = c_2 = c_3 = 0$) and one joint of class 4 ($c_4 = 1$) is shown in Fig. 2.33(b). The system group has three joints of class 5 ($c_5 = 5(1) - 2(1) = 3$), and three moving links ($n = -1 + 1 + 3 = 3$). There are no translations T_z (translation along z-axis) for the links. A system group with one joint of class 3 ($c_3 = 1$) and no joints of class 1, 2, and 4 ($c_1 = c_2 = c_4 = 0$) is shown in Fig. 2.33(c). The system group has two joints of class 5 ($c_5 = 5(1) - 3(1) = 2$), and two moving links ($n = -1 + 3 = 2$). There are no translations T_y for the links.

For the family $f = 2$, four system groups are presented in Fig. 2.34. The values c_5 and n are computed from Eq. (2.12). Two spatial system groups with no joints of class 1, 2, 3, and 4 ($c_1 = c_2 = c_3 = c_4 = 0$) are shown in Figs. 2.34(a) and 2.34(b). The system groups have four joints of class 5 ($c_5 = 4(1) = 4$), and three moving links ($n = -1 + 4 = 3$). For the system group in Fig. 2.34(a), there are no translations T_x and no rotations R_y for the links. For the system group in Fig. 2.34(b), there are no translations T_y and no rotations R_x for the links. A system group with no joints of class 1, 2, and 3 ($c_1 = c_2 = c_3 = 0$) and one joint of class 4 ($c_4 = 1$) is shown in Fig. 2.34(c). The system group has two joints of class 5 ($c_5 = 4(1) - 2(1) = 2$), and two moving links ($n = -1 + 1 + 2 = 2$). There are no translations T_z and no rotations R_y for the links. A system group with one joint of class 3 ($c_3 = 1$) and no joints of class 1, 2, and 4 ($c_1 = c_2 = c_4 = 0$) is shown in Fig. 2.34(d). The system group has one joint of class 5 ($c_5 = 4(1) - 3(1) = 1$), and one moving link ($n = -1 + 1 + 1 = 1$). There are no translations T_x and T_z for the links.

The spatial mechanism presented in Fig. 2.35 is derived from the system group shown in Fig. 2.33(b). The mechanism has one degree of freedom ($M = 4n - 3c_5 - 2c_4 - c_3 = 4(4) - 3(5) = 1$). The link 4 is the driver link.

For the family $f = 3$, three system groups are presented in Fig. 2.36. The values c_5 and n are computed from Eq. (2.13). Three system groups with no joints of class 1, 2, 3, and 4 ($c_1 = c_2 = c_3 = c_4 = 0$) are shown in Fig. 2.36. The system groups have three joints of class 5 ($c_5 = 3(1) = 3$), and two moving links ($n = -1 + 3 = 2$). There are no translation T_x and no rotations R_y and R_z for the system group in Fig. 2.36(a). For the system group in Fig. 2.36(b) there are no translation T_x and no rotations R_x and R_z. There are no translation T_z and no rotations R_x and R_y for the system group shown in Fig. 2.36(c).

For the family $f = 4$, two planar system groups with no joints of class 1, 2, 3, and 4 ($c_1 = c_2 = c_3 = c_4 = 0$) are shown in Fig. 2.37. The values c_5 and n are computed from Eq. (2.14) for each system group; there are two joints of class 5 ($c_5 = 2(1) = 2$), and one moving link ($n = -1 + 2 = 1$). Also, there are two planar translations for the links and thus the family of the system is $f = 6 - 2 = 4$.

2.12 Spatial System Groups with Two Independent Contours

There are also spatial system groups with two independent contours ($N = 2$). The number of joints, c_1, c_2, c_3, and c_4, are cycled, and the number of joints, c_5, and the number of moving links, n, are computed. Examples of system groups with $N = 2$ are described next for each of the families $f = 1, 2, 3$, and 4.

For the family $f = 1$, a system group is depicted in Fig. 2.38. The system group has no joints of class 1, 2, 3, and 4 ($c_1 = c_2 = c_3 = c_4 = 0$). There are ten joints of class 5 ($c_5 = 5(2) = 10$), and eight moving links ($n = -2 + 10 = 8$). There is no translation T_x for the links.

For the family $f = 2$, two system groups are illustrated in Fig. 2.39. A system group with no joints of class 1, 2 and 3 ($c_1 = c_2 = c_3 = 0$) and one joint of class 4 ($c_4 = 1$) is shown in Fig. 2.39(a). The system group has six joints of class 5 ($c_5 = 4(2) - 2(1) = 6$), and five moving links ($n = -2+1+6 = 5$). There are no translation T_x and no rotation R_x for the links. A system group with no joints of class 1 and 2 ($c_1 = c_2 = 0$), one joint of class 3 ($c_3 = 1$), and one joint of class 4 ($c_4 = 1$) is shown in Fig. 2.39(b). The system group has three joints of class 5 ($c_5 = 4(2) - 3(1) - 2(1) = 3$), and three moving links ($n = -2 + 1 + 1 + 3 = 3$). There are no translations T_x and T_y for the links.

For the family $f = 3$, three system groups are presented in Fig. 2.40. A system group with no joints of class 1, 2, 3, and 4 ($c_1 = c_2 = c_3 = c_4 = 0$) is shown in Fig. 2.40(a). The system group has six joints of class 5 ($c_5 = 3(2) = 6$), and four moving links ($n = -2+6 = 4$). There are no translations T_x, T_y, and T_z for the links. A spatial system group and a planar system group with no joints of class 1, 2, and 3 ($c_1 = c_2 = c_3 = 0$) and one joint of class 4 ($c_4 = 1$) are shown in Figs. 2.40(b) and 2.40(c), respectively. The system groups have four joints of class 5 ($c_5 = 3(2) - 2(1) = 4$), and three moving links ($n = -2 + 1 + 4 = 3$). There are no translations T_y, T_z and no rotations R_z for the spatial system in Fig. 2.40(b).

For the family $f = 4$, a planar system group with no joints of class 1, 2, 3, and 4 ($c_1 = c_2 = c_3 = c_4 = 0$) is shown in Fig. 2.41. The system group has four joints of class 5 ($c_5 = 2(2) = 4$), and two moving links ($n = -2+4 = 2$).

The spatial mechanism shown in Fig. 2.42 contains a system group of the family $f = 0$ that has $c_1 = c_2 = 0$, $c_3 = 1$, $c_4 = 2$, $c_5 = 6(2)-3(1)-2(2) = 5$, and $n = -2 + 1 + 2 + 5 = 6$. The mechanism has two degrees of freedom $M = 6n - 5c_5 - 4c_4 - 3c_3 - 2c_2 - c_1 = 6(8) - 5(7) - 4(2) - 3(1) = 2$. The links 7 and 8 are driver links.

2.13 Spatial System Groups with Three Independent Contours

There are also spatial system groups with three independent contours ($N = 3$). The number of joints, c_1, c_2, c_3, and c_4, are cycled and the number of joints, c_5, and the number of moving links, n, are computed. System groups with $N = 3$ are exemplified for each of the families $f = 2$, 3, and 4.

For the family $f = 2$, a spatial system group with no joints of class 1 and 2 ($c_1 = c_2 = 0$), one joint of class 3 ($c_3 = 1$), and one joint of class 4 ($c_4 = 1$) is shown in Fig. 2.43. The system group has seven joints of class 5 ($c_5 = 4(3) - 3(1) - 2(1) = 7$), and six moving links ($n = -3 + 1 + 1 + 7 = 6$). There are no translations T_x and T_z for the links.

For the family $f = 3$, a planar system group with no joints of class 1, 2, and 3 ($c_1 = c_2 = c_3 = 0$) and one joint of class 4 ($c_4 = 1$) is depicted in Fig. 2.44. The system group has seven joints of class 5 ($c_5 = 3(3) - 2(1) = 7$), and five moving links ($n = -3 + 1 + 7 = 5$).

For the family $f = 4$, a planar system group with no joints of class 1, 2, 3, and 4 ($c_1 = c_2 = c_3 = c_4 = 0$) is shown in Fig. 2.45. The system group has six joints of class 5 ($c_5 = 2(3) = 6$), and three moving links ($n = -3 + 6 = 3$).

The spatial mechanism presented in Fig. 2.46 contains a system group of the family $f = 0$ that has $c_1 = c_2 = 0$, $c_3 = 3$, $c_4 = 4$, $c_5 = 6(3) - 3(3) - 2(4) = 1$, and $n = -3 + 3 + 4 + 1 = 5$. The mechanism has three degrees of freedom $M = 6n - 5c_5 - 4c_4 - 3c_3 - 2c_2 - c_1 = 6(8) - 5(4) - 4(4) - 3(3) = 3$. The links 6, 7, and 8 are driver links.

The method [40, 41] presented is based essentially on system group formation using the number of independent contours and joints as inputs. The number of joints of different classes are cycled for different families and several structures of spatial system groups with one, two, or more independent contours are obtained. For a given family, different configurations of system groups with the same number of independent contours can be obtained. Spatial mechanisms can be structured based on spatial system groups.

2.14 Decomposition of Kinematic Chains

A planar mechanism is shown in Fig. 2.47(a). This kinematic chain can be decomposed into system groups and driver links. The mobility of the mechanism will be determined first. The number of DOF for this mechanism is given by $M = 3\,n - 2\,c_5 - c_4 = 3\,n - 2\,c_5$. The mechanism has five moving links ($n = 3$). To find the number of c_5 a *connectivity table* will be used [Fig. 2.47(b)]. The links are represented with bars (binary links) or triangles (ternary links). The one degree of freedom joints (rotational joint or translation joint) are represented with a cross circle. The first column has the number of the current link, the second column shows the links connected to the current link, and the last column contains the graphical representation. The link 1 is connected to ground 0 at A and to link 2 at B [Fig. 2.47(b)]. Next, link 2 is connected to link 1 at B, link 3 at C, and link 4 at B. Link

2 is a ternary link because it is connected to three links. At B there is a multiple joint, two rotational joints, one joint between link 1 and link 2, and one joint between link 2 and link 4. Link 3 is connected to ground 0 at C and to link 2 at C. At C there is a joint between link 3 and link 0 and a joint between link 3 and link 2. Link 4 is connected to link 2 at B and to link 5 at D. The last link, 5, is connected to link 4 at D and to ground 0 at D. In this way the table in Fig. 2.47(b) is obtained. The *structural diagram* is obtained using the graphical representation of the table connecting all the links Fig. 2.47(c). The c_5 joints (with cross circles), all the links, and the way the links are connected are represented on the structural diagram. The number of one degree of freedom joints is given by the number of cross circles. From Fig. 2.47(c) it results $c_5 = 7$. The number of DOF for the mechanism is $M = 3\,(5) - 2\,(7) = 1$. If $M = 1$, there is just one driver link. One can choose link 1 as the driver link of the mechanism. Once the driver link is taken away from the mechanism the remaining kinematic chain (links 2, 3, 4, 5) has the mobility equal to zero. The dyad is the simplest system group and has two links and three joints. On the structural diagram one can notice that links 2 and 3 represent a dyad and links 4 and 5 represent another dyad. The mechanism has been decomposed into a driver link (link 1) and two dyads (links 2 and 3, and links 4 and 5).

The connectivity table and the structural diagram are not unique for this mechanism. The new connectivity table can be obtained in Fig. 2.47(d). Link 1 is connected to ground 0 at A and to link 4 at B. Link 2 is connected to link 3 at C and to link 4 at B. Link 3 is connected to link 2 at C and to ground 0 at C. Link 4 is connected to link 1 at B, to link 2 at B, and to link 5 at D. This time link 4 is the ternary link. Link 5 is connected to link 4 at D and to ground 0 at D. The structural diagram is shown in Fig. 2.47(e). Using this structural diagram the mechanism can be decomposed into a driver link (link 1) and two dyads (links 2 and 3, and links 4 and 5).

If the driver link is link 1, the mechanism has the same structure no matter what structural diagram [Fig. 2.47(c) or Fig. 2.47(e)] is used.

Next, the driver link with rotational motion (R) and the dyads are represented as shown in Fig. 2.48(a). The first dyad (BCC) has the length between 2 and 3 equal to zero, $l_{CC} = 0$. The second dyad (BDD) has the length between 5 and 0 equal to zero, $l_{DD} = 0$. Figure 2.48(b) shows the dyads with the lengths l_{CC} and l_{DD} different than zero. Using Fig. 2.48(b), the first dyad (BCC) has a rotational joint at B (R), a rotational joint at C (R), and a translational joint at C (T). The first dyad (BCC) is a rotation

rotation translation dyad (dyad RRT). Using Fig. 2.48(b), the second dyad (BDD) has a rotational joint at B (R), a translational joint at D (T), and a rotational joint at D (R). The second dyad (BDD) is a rotation translation rotation dyad (dyad RTR). The mechanism is an R-RRT-RTR mechanism.

The mechanism in Fig. 2.49(a) is formed by a driver 1 with rotational motion R [Fig. 2.49(b)], a dyad RTR [Fig. 2.49(c)], and a dyad RTR [Fig. 2.49(d)]. The mechanism in Fig. 2.49(a) is an R-RTR-RTR mechanism. The connectivity table is shown in Fig. 2.50(a) and the structural diagram is represented in Fig. 2.50(b).

2.15 Linkage Transformation

For planar mechanisms the half joints can be substituted, and in this way mechanisms with just full joints are obtained. The transformed mechanism has to be equivalent with the initial mechanism from a kinematical point of view. The number of degrees of freedom of the transformed mechanism has to be equal to the number of degrees of freedom of the initial mechanism. The relative motion of the links of the transformed mechanism has to be the same as the relative motion of the links of the initial mechanism.

A half joint constrains the possibility of motion of the connected links in motion. A constraint equation can be written and the number of degrees of freedom of a half joint is $M = -1$. To have the same number of degrees of freedom for a kinematic chain with n moving links and c_5 full joints, the following equation is obtained:

$$M = 3n - 2c_5 = -1. \tag{2.15}$$

The relation between the number of full joints and the number of moving links is obtained From Eq. (2.15)

$$c_5 = \frac{3n+1}{2}. \tag{2.16}$$

A half joint can be substituted with one link (n=1) and two full joints (c_5=2).

Figure 2.51(a) shows a cam and follower mechanism. There is a half joint at the contact point C between the links 1 and 2. One can substitute the half joint at C with one link, link 3, and two full at C and D as shown in Fig. 2.51(b). To have the same relative motion, the length of link 3 has to be equal to the radius of curvature ρ of the cam at the contact point C.

In this way the half joint at the contact point can be substituted for two full joints, C and D, and an extra link 3, between links 1 and 2. The mechanism still has one degree of freedom, and the cam and follower system (0, 1, and 2) is in fact a four-bar mechanism (0, 1, 2, and 3) in another disguise.

The half joint at the contact point of two gears in motion can be substituted for two full joints, A and B, and an extra link 3, between gears 1 and 2 (Fig. 2.52). The mechanism still has one degree of freedom, and the two gear system (0, 1, and 2) [Fig. 2.52(a)] is in fact a four-bar mechanism (0, 1, 2, and 3) in another disguise [Fig. 2.52(b)]. The following relations can be written

$$O_1O_2 = \frac{m}{2}\left(N_1 + N_2\right),$$

$$O_1A = r_1 \cos\phi,$$

$$O_2B = r_2 \cos\phi,$$

$$AB = AP + PB = \frac{mN_1}{2}\sin\phi + \frac{mN_2}{2}\sin\phi = \frac{m}{2}\left(N_1 + N_2\right)\sin\phi,$$

where m is the module, N is the number of teeth, r is the pitch radius, and ϕ is the pressure angle. Because m, N_1, N_2, and ϕ are constants, the links of the four-bar mechanism [Fig. 2.52(b)] are constant as well.

2.16 Problems

2.1 Determine the number of degrees of freedom of the planar elipsograph mechanism in Fig. 2.53.

2.2 Find the mobility of the planar mechanism represented in Fig. 2.54.

2.3 Determine the family and the number of degrees of freedom for the mechanism depicted in Fig. 2.55.

2.4 Roller 2 of the mechanism in Fig. 2.56 undergoes an independent rotation about its axis which does not influence the motion of link 3. The purpose of element 2 is to substitute the sliding friction with a rolling friction. From a kinematical point of view, roller 2 is a passive element. Find the number of degrees of freedom of the mechanism.

2.5 Find the family and the number of degrees of freedom of the mechanism in Fig. 2.57.

2.6 Determine the number of degrees of freedom and draw the structural diagram for the mechanism in Fig. 2.58.

2.7 Find the family, the number of degrees of freedom, represent the structural diagram, and find the dyads for the mechanism shown in Fig. 2.59.

2.8 Determine the family and the number of degrees of freedom for the mechanism in Fig. 2.60.

2.9 Find the family and the number of degrees of freedom for the mechanism shown in Fig. 2.61.

2.10 Determine the number of degrees of freedom for the cam mechanism in Fig. 2.62.

2.11 Find the number of degrees of freedom for the planetary gear train in Fig. 2.63.

2.12 Determine the number of degrees of freedom for the Geneva mechanism in Fig. 2.64.

2.13 Find the number of degrees of freedom for the planetary gear train in Fig. 2.65.

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Figure captions

Figure 2.1. Rigid body in planar motion with three DOF: translation along the x axis, translation along the y axis, and rotation, θ, about the z.

Figure 2.2. Rigid body in motion: (a) pure rotation, (b) pure translation, and (c) general motion.

Figure 2.3. Types of links: (a) unary, (b) binary, and (c) ternary elements

Figure 2.4. One degree of freedom joint, full joint (5th class): (a) pin joint, and (b) slider joint.

Figure 2.5. (a) Screw and nut joint; (b) helical motion

Figure 2.6. Two degrees of freedom joint, half joint (4th class): (a) general joint, (b) cylinder joint, (c) roll and slide disk, and (d) cam-follower joint.

Figure 2.7. Three degrees of freedom joint (3rd class): (a) ball and socket joint, and (b) plane joint.

Figure 2.8. Four degrees of freedom joint (2nd class) cylinder on a plane.

Figure 2.9. Five degrees of freedom joint (1st class) sphere on a plane.

Figure 2.10. Order of a joint: (a) joint of order one, and (b) joint of order two (multiple joints).

Figure 2.11. Kinematic chains: (a) closed kinematic chain, and (b) open kinematic chain.

Figure 2.12. Complex mechanism with five moving links.

Figure 2.13. Spatial mechanism of family f=1.

Figure 2.14. Spatial mechanism of family f=2.

Figure 2.15. Spatial mechanism of family f=4.

Figure 2.16. Spatial mechanism of family f=5: (a) driver link with rotational motion, and (b) driver link with translational motion.

Figure 2.17. Types of dyads: (a) RRR, (b) RRT, (c) RTR, (d) TRT, and (e) TTR.

Figure 2.18. RRT dyads.

Figure 2.19. RTR dyads.

Figure 2.20. TRT dyads.

Figure 2.21. TTR dyads.

Figure 2.22. Planar mechanisms: (a) $\mathrm{R} - D42$ and (b) $\mathrm{R} - D20$.

Figure 2.23. Planar T-$D54$ mechanism with f=4.

Figure 2.24. Mechanism with two dyads: $\mathrm{R} - D10 - D10$.

Figure 2.25. Mechanisms with two dyads: (a) $\mathrm{R} - D21 - D30 - 1 + 3$, (b) $\mathrm{R} - D43 - D50 - 2 + 0$, and (c) $\mathrm{R} - D31 - D20 - 3 + 0$.

Figure 2.26. $\mathrm{T} - D21 - D50 - 2 + 0$ mechanism.