Input data

{AB → 0.02, AE → 0.05, BC → 0.03, CD → 0.06, La → 0.02, xE → -0.05,
yE → 0., xD → 0.02, phi[t] → 0.523599, phi′[t] → 6.28319, phi″[t] → 0}

Joint B

Position of joint B

xB[t] = AB Cos[phi[t]]

yB[t] = AB Sin[phi[t]]

xB = 0.0173205 m

yB = 0.01 m

Linear velocity of joint B

xB'[t] = -AB Sin[phi[t]] phi′[t]

yB'[t] = AB Cos[phi[t]] phi′[t]

xB' = -0.0628319 m/s

yB' = 0.108828 m/s

Linear acceleration of joint B

xB''[t] = -AB Cos[phi[t]] phi′[t]^2 - AB Sin[phi[t]] phi″[t]

yB''[t] = -AB Sin[phi[t]] phi′[t]^2 + AB Cos[phi[t]] phi″[t]

xB'' = -0.683786 m/s^2

yB'' = -0.394784 m/s^2

Another way of calculating vB and aB:

omega1 = {0, 0, 6.28319} rad/s

alpha1 = {0, 0, 0} rad/s^2

rB = {0.0173205, 0.01, 0} m

vB= omega1 x rB = {-0.0628319, 0.108828, 0.} m/s

aB= alpha1 x rB - omega1^2 rB = {-0.683786, -0.394784, 0.} m/s^2

Link 2

Equation for phi2[t]

$\text{Tan[phi2[t]]} = \frac{-\text{yE} + \text{yB[t]}}{-\text{xE} + \text{xB[t]}}$ or

Sin[phi2[t]] (-xE + xB[t]) - Cos[phi2[t]] (-yE + yB[t]) == 0

Solution for phi2[t]=phi3[t]

phi2= 0.147465 rad = 8.44911 deg

Equation for omega2[t]=phi2'[t]

```
Cos[phi2[t]] (-xE + xB[t]) phi2′[t] +
  Sin[phi2[t]] (-yE + yB[t]) phi2′[t] + Sin[phi2[t]] xB′[t] - Cos[phi2[t]] yB′[t] == 0
```

```
-0.116879 + 0.0680592 phi2′[t] == 0
```

Solution for omega2[t]=phi2'[t]

```
         -Sin[phi2[t]] xB′[t] + Cos[phi2[t]] yB′[t]
------------------------------------------------------------
Cos[phi2[t]] (-xE + xB[t]) + Sin[phi2[t]] (-yE + yB[t])
```

phi2'=1.71731 rad/s

Equation for alpha2[t]=phi2''[t]

```
-Sin[phi2[t]] (-xE + xB[t]) phi2′[t]^2 +
  Cos[phi2[t]] (-yE + yB[t]) phi2′[t]^2 + 2 Cos[phi2[t]] phi2′[t] xB′[t] +
  2 Sin[phi2[t]] phi2′[t] yB′[t] + Cos[phi2[t]] (-xE + xB[t]) phi2″[t] +
  Sin[phi2[t]] (-yE + yB[t]) phi2″[t] + Sin[phi2[t]] xB″[t] - Cos[phi2[t]] yB″[t] == 0
```

```
0.131489 + 0.0680592 phi2″[t] == 0
```

Solution for alpha2[t]=phi2''[t]

```
(Sin[phi2[t]] (-xE + xB[t]) phi2′[t]^2 -
   Cos[phi2[t]] (-yE + yB[t]) phi2′[t]^2 - 2 Cos[phi2[t]] phi2′[t] xB′[t] -
   2 Sin[phi2[t]] phi2′[t] yB′[t] - Sin[phi2[t]] xB″[t] + Cos[phi2[t]] yB″[t]) /
 (Cos[phi2[t]] (-xE + xB[t]) + Sin[phi2[t]] (-yE + yB[t]))
```

phi2''=-1.93198 rad/s^2

Joint C

Position of joint C

Equations for C (xC[t], yC[t])

```
-BC^2 + (-xB[t] + xC[t])^2 + (-yB[t] + yC[t])^2 == 0
```

```
(-xE + xC[t]) (-yE + yB[t]) - (-xE + xB[t]) (-yE + yC[t]) == 0
```

```
-0.0009 + (-0.0173205 + xC[t])^2 + (-0.01 + yC[t])^2 == 0
```

```
0.01 (0.05 + xC[t]) - 0.0673205 (0. + yC[t]) == 0
```

Solution for C (xC[t], yC[t])

xC = 0.0469949 m

yC = 0.0144079 m

Linear velocity of joint C

Equations for vC (xC'[t], yC'[t])

```
2 (xB[t] - xC[t]) (xB′[t] - xC′[t]) + 2 (yB[t] - yC[t]) (yB′[t] - yC′[t]) == 0
```

```
(yE - yC[t]) xB′[t] + (-yE + yB[t]) xC′[t] + (-xE + xC[t]) yB′[t] + (xE - xB[t]) yC′[t] == 0
```

$0.00276958 + 0.0593488\, xC'[t] + 0.00881586\, yC'[t] == 0$

$0.011461 + 0.01\, xC'[t] == 0.0673205\, yC'[t]$

Solution for vC (xC'[t], yC'[t])

xC' = -0.0704016 m/s

yC' = 0.159788 m/s

Linear acceleration of joint C

Equations for aC (xC''[t], yC''[t])

$$2\left((xB'[t] - xC'[t])^2 + (yB'[t] - yC'[t])^2 + (xB[t] - xC[t])\,(xB''[t] - xC''[t]) + (yB[t] - yC[t])\,(yB''[t] - yC''[t])\right) == 0$$

$$2\, xC'[t]\, yB'[t] + (yE - yC[t])\, xB''[t] + (-yE + yB[t])\, xC''[t] + (-xE + xC[t])\, yB''[t] + (xE - xB[t])\, yC''[t] == 2\, xB'[t]\, yC'[t]$$

$0.0493707 + 0.0593488\, xC''[t] + 0.00881586\, yC''[t] == 0$

$-0.0437634 + 0.01\, xC''[t] - 0.0673205\, yC''[t] == -0.0200796$

Solution for aC (xC''[t], yC''[t])

xC'' = -0.762785 m/s^2

yC'' = -0.465114 m/s^2

Another way of calculating vC and aC:

omega2 = {0, 0, 1.71731} rad/s

alpha2 = {0, 0, -1.93198} rad/s^2

rC = {0.0469949, 0.0144079, 0} m

vC= vB + omega2 x BC = {-0.0704016, 0.159788, 0.} m/s

aC= aB + alpha2 x BC - omega2^2 BC = {-0.762785, -0.465114, 0.} m/s^2

Linear velocity and acceleration of point E2 on link 2

rE = {-0.05, 0., 0} m

vE2= vB + omega2 x BE = {-0.0456587, -0.00678229, 0.} m/s

aE2= aB + alpha2 x BE - omega2^2 BE = {-0.504567, -0.235231, 0.} m/s^2

Linear velocity and acceleration of point E3 on link 3: vE3=aE3=0

Relative velocity of point E2 with respect to E3, vE23

vE2 = vE3 + vE23 => vE23 = vE2 - vE3

vE23= {-0.0456587, -0.00678229, 0.} m/s

Coriolis acceleration of point E2 with respect to E3, aE23cor

aE23cor = 2 omega2 x vE23 = {0.0232946, -0.156821, 0.} m/s^2

Relative acceleration of point E2 with respect to E3, aE23

aE2 = aE3 + aE23 + aE23cor => aE23 = aE2 - aE3 - aE23cor

aE23= {-0.527862, -0.0784103, 0.} m/s^2

Joint D

Position of joint D

Equation for D (xD, yD[t])

$-CD^2 + (-xD + xC[t])^2 + (yC[t] - yD[t])^2 == 0$

$-0.00287127 + (0.0144079 - yD[t])^2 == 0$

Solution for D (xD, yD[t])

xD = 0.02 m

yD = -0.0391763 m

Linear velocity of joint D

Equation for vD (0, yD'[t])

$2 (-xD + xC[t]) xC'[t] + 2 (yC[t] - yD[t]) (yC'[t] - yD'[t]) == 0$

$-0.00380097 + 0.107169 (0.159788 - yD'[t]) == 0$

Solution for vD (0, yD'[t])

yD' = 0.124321 m/s

Equation for aD (0, yD''[t])

$2\, xC'[t]^2 + 2 (yC'[t] - yD'[t])^2 + 2 (-xD + xC[t]) xC''[t] + 2 (yC[t] - yD[t]) (yC''[t] - yD''[t]) == 0$

$-0.028754 + 0.107169 (-0.465114 - yD''[t]) == 0$

Linear acceleration of joint D

Solution for aD (0, yD''[t])

yD'' = -0.73342 m/s^2

Link 4

Equation for phi4[t]

$phi4[t]=ArcTan\left[\frac{yC[t] - yD[t]}{-xD + xC[t]}\right]$ or

$Sin[phi4[t]] (-xD + xC[t]) == Cos[phi4[t]] (yC[t] - yD[t])$

Solution for phi4[t]

phi4= 1.10413 rad = 63.2618 deg

Equation for phi4'[t]

```
Cos[phi4[t]] (-xD + xC[t]) phi4'[t] + Sin[phi4[t]] xC'[t] ==
 -Sin[phi4[t]] (yC[t] - yD[t]) phi4'[t] + Cos[phi4[t]] (yC'[t] - yD'[t])

-0.0628737 + 0.0121454 phi4'[t] == 0.0159572 - 0.0478546 phi4'[t]
```

Solution for phi4'[t]

$$\frac{-\text{Sin}[\text{phi4}[t]]\ \text{xC}'[t] + \text{Cos}[\text{phi4}[t]]\ (\text{yC}'[t] - \text{yD}'[t])}{\text{Cos}[\text{phi4}[t]]\ (-\text{xD} + \text{xC}[t]) + \text{Sin}[\text{phi4}[t]]\ (\text{yC}[t] - \text{yD}[t])}$$

phi4'=1.31385 rad/s

Equation for phi4''[t]

```
-Sin[phi4[t]] (-xD + xC[t]) phi4'[t]^2 + 2 Cos[phi4[t]] phi4'[t] xC'[t] +
  Cos[phi4[t]] (-xD + xC[t]) phi4''[t] + Sin[phi4[t]] xC''[t] ==
 -Cos[phi4[t]] (yC[t] - yD[t]) phi4'[t]^2 - 2 Sin[phi4[t]] phi4'[t] (yC'[t] - yD'[t]) -
  Sin[phi4[t]] (yC[t] - yD[t]) phi4''[t] + Cos[phi4[t]] (yC''[t] - yD''[t])

-0.806069 + 0.0121454 phi4''[t] == -0.00413256 - 0.0478546 phi4''[t]
```

Solution for phi4''[t]

```
(Sin[phi4[t]] (-xD + xC[t]) phi4'[t]^2 - Cos[phi4[t]] (yC[t] - yD[t]) phi4'[t]^2 -
   2 Cos[phi4[t]] phi4'[t] xC'[t] - 2 Sin[phi4[t]] phi4'[t] (yC'[t] - yD'[t]) -
   Sin[phi4[t]] xC''[t] + Cos[phi4[t]] (yC''[t] - yD''[t])) /
 (Cos[phi4[t]] (-xD + xC[t]) + Sin[phi4[t]] (yC[t] - yD[t]))
```

phi4''=13.3656 rad/s^2

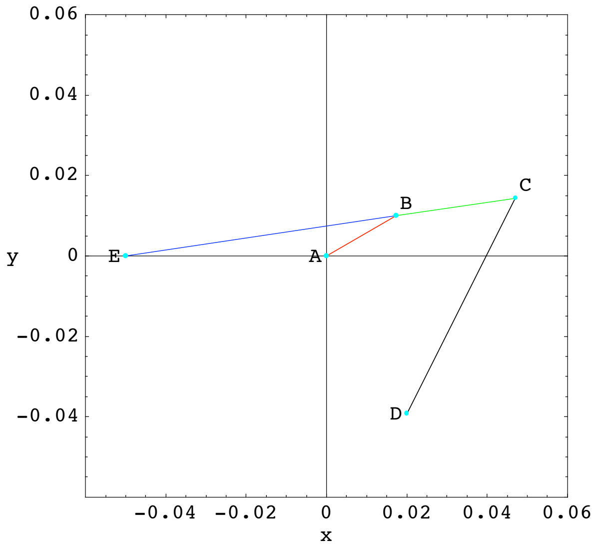
omega4 = {0, 0, 1.31385} rad/s

alpha4 = {0, 0, 13.3656} rad/s^2

rD = {0.02, -0.0391763, 0} m

vD=vD4=vD5= vC + omega4 x CD = {0, 0.124321, 0} m/s

aD=aD4=aD5= aC + alpha4 x CD - omega4^2 CD = {0, -0.73342, 0} m/s^2


0.06
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