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(* Problem 4.8 *)

(* VELOCITY and ACCELERATION ANALYSIS - input angle phi *)

Apply[Clear,Names["Global`*"]];
Off[General::spell];
Off[General::spell1];

(* Input data *)
AB = .120 ;
AC = .060 ;
BD = .240 ;
DE = .330 ;
EF = .190 ;
La = .300 ;
Lb = .070 ;

n = 100 ; (* rpm *)
omega = n*N[Pi]/30 ; (* rad/s *)

initdata = {ϕ[t]->N[Pi]150/180, ϕ'[t]->omega, ϕ''[t]->0};


(* Position of joint A *)
xA = yA = 0;

(* Position of joint C *)
xC = -AC ;
yC = 0 ;

(* Position of joint F *)
xF = La ;
yF = Lb ;

(* Position of joint B *)
XB = AB Cos[ϕ[t]] ;
YB = AB Sin[ϕ[t]] ;
xBs=XB/.initdata;
yBs=YB/.initdata;
Print["xB[t] = ", XB," = ", xBs, " m" ];
Print["yB[t] = ", YB," = ", yBs, " m" ];

(* Linear velocity of joint B *)
vBx = D[XB,t];
vBy = D[YB,t];
vBxs = vBx/.initdata;
vBys = vBy/.initdata;
Print["vBx = ", vBx ," = ", vBxs, " m/s" ];
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Print["vBy = ", vBy ," = ", vBys, " m/s" ];

(* Linear acceleration of joint B *)
aBx = D[vBx,t];
aBy = D[vBy,t];
aBxs = aBx/.initdata;
aBys = aBy/.initdata;
Print["aBx = ", aBx ," = ", aBxs, " m/s^2" ];
Print["aBy = ", aBy ," = ", aBys, " m/s^2" ];

ruleB={xB[t]->xBs,yB[t]->yBs,
       xB'[t]->vBxs,yB'[t]->vBys,
       xB''[t]→aBxs,yB''[t]→aBys};

(* Angular velocity and acceleration of the link BD (link 2)*)
phi2 = ArcTan[ (yB[t]-yC)/(xB[t]-xC) ] ;
omega2 = D[ phi2 , t ] ;
alpha2 = D[ omega2, t ] ;
Print["phi2 = phi3 = ", phi2/.ruleB , " rad =
",(phi2/.ruleB)*180/N[Pi]," deg" ];
Print["omega2 = omega3 = ", omega2/.ruleB , " rad/s" ];
Print["alpha2 = alpha3 = ", alpha2/.ruleB , " rad/s^2" ];

rB={xBs,yBs,0};
rC={xC,yC,0};
rBC=rC-rB;
Print["rB = ", rB, " m" ];
Print["rC = ", rC, " m" ];
Print["rBC = rC - rB = ", rBC, " m" ];
vB={vBxs,vBys,0};
aB={aBxs,aBys,0};
Print["vB = ", vB , " m/s" ];
Print["aB = ", aB , " m/s" ];
ω2={0,0,omega2/.ruleB};
α2={0,0,alpha2/.ruleB};
Print["ω2 = ", ω2, " rad/s" ];
Print["α2 = ", α2, " rad/s^2" ];
vC2=vB+Cross[ω2,rBC];
Print["vC2 = vB + ω2 x rBC = ",vC2, " m/s" ];
aC2=aB+Cross[α2,rBC]-ω2.ω2 rBC;
Print["aC2 = aB + α2 x rBC - ω2^2 rBC = ", aC2, " m/s^2" ];
vC32=-vC2;
Print["vC32 = vC3 - vC2 = ",vC32, " m/s" ];
aC32cor=2 Cross[ω2,vC32];
Print["aC32cor = 2 ω2 x vC32 = ",aC32cor, " m/s^2" ];
aC32=-aC2-aC32cor;
Print["aC32 = aC3 - aC2 - aC32cor = ",aC32, " m/s^2" ];

(* Position of joint D *)
eqnD1 = ( xD[t]- xB[t] )^2 + ( yD[t] - yB[t] )^2 - BD^2 == 0 ;
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eqnD2 = ( yD[t] - yB[t] ) / ( xD[t]- xB[t] ) == ( yB[t] - yC ) /
( xB[t] - xC );
solutionD = Solve [ { eqnD1/.ruleB, eqnD2/.ruleB } , { xD[t] ,
yD[t] } ];
(* Two solutions for D *)
xD1 = xD[t] /. solutionD[[1]];
yD1 = yD[t] /. solutionD[[1]];
xD2 = xD[t] /. solutionD[[2]];
yD2 = yD[t] /. solutionD[[2]];
If [ yD1 <= 0 , xDs = xD1 ; yDs = yD1 , xDs = xD2 ; yDs=yD2 ] ;
Print["xD = ",xDs," m"];
Print["yD = ",yDs," m"];

(* Velocity of joint D *)
PD={xD[t]→xDs,yD[t]→yDs};
eqnD1V = D[eqnD1,t] ;
eqnD2V = D[eqnD2,t] ;
solutionDV =
    Solve [ { (eqnD1V/.ruleB/.PD ),( eqnD2V/.ruleB/.PD )} , {
xD'[t] ,
        yD'[t] } ];
vDxs = xD'[t] /. solutionDV[[1]];
vDys = yD'[t] /. solutionDV[[1]];
Print["vDx = ",vDxs," m/s"];
Print["vDy = ",vDys," m/s"];

(* Acceleration of joint D *)
PVD={xD[t]→xDs,yD[t]→yDs,xD'[t]→vDxs,yD'[t]→vDys};
eqnD1A = D[eqnD1V,t] ;
eqnD2A = D[eqnD2V,t] ;
solutionDA =
    Solve [ { (eqnD1A/.ruleB/.PVD ),( eqnD2A/.ruleB/.PVD )} , {
xD''[t] ,
        yD''[t] } ];
aDxs = xD''[t] /. solutionDA[[1]];
aDys = yD''[t] /. solutionDA[[1]];
Print["aDx = ",aDxs," m/s^2"];
Print["aDy = ",aDys," m/s^2"];
ruleD={xD[t]→xDs,yD[t]→yDs,xD'[t]→vDxs,yD'[t]→vDys,
      xD''[t]→aDxs,yD''[t]→aDys};

rD={xDs,yDs,0};
rBD=rD-rB;
Print["rD = ", rD, " m" ];
Print["rBD = rD - rB = ", rBD, " m" ];
vD=vB+Cross[ω2,rBD];
Print["vD = vB + ω2 x rBD = ",vD, " m/s" ];
aD=aB+Cross[α2,rBD]-ω2.ω2 rBD;
Print["aD = aB + α2 x rBD - ω2^2 rBD = ", aD, " m/s^2" ];
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(* Position of joint E *)
eqnE1=(xE[t]-xD[t])^2+(yE[t]-yD[t])^2-DE^2==0;
eqnE2=(xE[t]-xF)^2+(yE[t]-yF)^2-EF^2==0;
solutionE=Solve[{eqnE1/.ruleD,eqnE2/.ruleD},{xE[t],yE[t]}];
(*Two solutions for E*)
xE1=xE[t]/.solutionE[[1]];
yE1=yE[t]/.solutionE[[1]];
xE2=xE[t]/.solutionE[[2]];
yE2=yE[t]/.solutionE[[2]];
(*Select the correct position for E*)
If[yE1 ≤ 0,xEs=xE1;yEs=yE1,xEs=xE2;yEs=yE2];
Print["xE = ",xEs," m"];
Print["yE = ",yEs," m"];

(* Velocity of joint E *)
PE={xE[t]→xEs,yE[t]→yEs};
eqnE1V = D[eqnE1,t] ;
eqnE2V = D[eqnE2,t] ;
solutionEV =
    Solve [ { (eqnE1V/.ruleD/.PE ),( eqnE2V/.ruleD/.PE )} , {
xE'[t] ,
        yE'[t] } ];
vExs = xE'[t] /. solutionEV[[1]];
vEys = yE'[t] /. solutionEV[[1]];
Print["vEx = ",vExs," m/s"];
Print["vEy = ",vEys," m/s"];

(* Acceleration of joint E *)
PVE={xE[t]→xEs,yE[t]→yEs,xE'[t]→vExs,yE'[t]→vEys};
eqnE1A = D[eqnE1V,t] ;
eqnE2A = D[eqnE2V,t] ;
solutionEA =
    Solve [ { (eqnE1A/.ruleD/.PVE ),( eqnE2A/.ruleD/.PVE )} , {
xE''[t] ,
        yE''[t] } ];
aExs = xE''[t] /. solutionEA[[1]];
aEys = yE''[t] /. solutionEA[[1]];
Print["aEx = ",aExs," m/s^2"];
Print["aEy = ",aEys," m/s^2"];
ruleE={xE[t]→xEs,yE[t]→yEs,xE'[t]→vExs,yE'[t]→vEys,
      xE''[t]→aExs,yE''[t]→aEys};


(* Angular velocity and acceleration of the link DE (link 4)*)
phi4 = ArcTan[ (yD[t]-yE[t])/(xD[t]-xE[t]) ] ;
omega4 = D[ phi4 , t ] ;
alpha4 = D[ omega4, t ] ;
Print["phi4 = ", phi4/.ruleD/.ruleE , " rad =
",(phi4/.ruleD/.ruleE)*180/N[Pi]," deg" ];
Print["omega4 = ", omega4/.ruleD/.ruleE , " rad/s" ];
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Print["alpha4 = ", alpha4/.ruleD/.ruleE , " rad/s^2" ];

(* Angular velocity and acceleration of the link EF (link 5)*)
phi5 = ArcTan[ (yE[t]-yF)/(xE[t]-xF) ] ;
omega5 = D[ phi5 , t ] ;
alpha5 = D[ omega5, t ] ;
Print["phi5 = ", phi5/.ruleE , " rad =
",(phi5/.ruleE)*180/N[Pi]," deg" ];
Print["omega5 = ", omega5/.ruleE , " rad/s" ];
Print["alpha5 = ", alpha5/.ruleE , " rad/s^2" ];
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xB[t] = 0.12 Cos[ϕ[t]] = -0.103923 m

yB[t] = 0.12 Sin[ϕ[t]] = 0.06 m

vBx = -0.12 Sin[ϕ[t]] ϕ′[t] = -0.628319 m/s

vBy = 0.12 Cos[ϕ[t]] ϕ′[t] = -1.08828 m/s

aBx = -0.12 Cos[ϕ[t]] ϕ′[t]^2 - 0.12 Sin[ϕ[t]] ϕ′′[t] = 11.3964 m/s^2

aBy = -0.12 Sin[ϕ[t]] ϕ′[t]^2 + 0.12 Cos[ϕ[t]] ϕ′′[t] = -6.57974 m/s^2

phi2 = phi3 = -0.938882 rad = -53.794 deg

omega2 = omega3 = 15.4632 rad/s

alpha2 = alpha3 = 139.461 rad/s^2

rB = {-0.103923, 0.06, 0} m

rC = {-0.06, 0, 0} m

rBC = rC - rB = {0.043923, -0.06, 0} m

vB = {-0.628319, -1.08828, 0} m/s

aB = {11.3964, -6.57974, 0} m/s

ω2 = {0, 0, 15.4632} rad/s

α2 = {0, 0, 139.461} rad/s^2

vC2 = vB + ω2 x rBC = {0.299474, -0.409089, 0.} m/s

aC2 = aB + α2 x rBC - ω2^2 rBC = {9.26164, 13.8925, 0.} m/s^2

vC32 = vC3 - vC2 = {-0.299474, 0.409089, 0.} m/s

aC32cor = 2 ω2 x vC32 = {-12.6516, -9.26164, 0.} m/s^2

aC32 = aC3 - aC2 - aC32cor = {3.39, -4.63082, 0.} m/s^2

xD = 0.0378427 m

yD = -0.133656 m

vDx = 2.36622 m/s

vDy = 1.10387 m/s
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aDx = 4.50619 m/s^2

aDy = 59.4962 m/s^2

rD = {0.0378427, -0.133656, 0} m

rBD = rD - rB = {0.141766, -0.193656, 0} m

vD = vB + ω2 x rBD = {2.36622, 1.10387, 0.} m/s

aD = aB + α2 x rBD - ω2^2 rBD = {4.50619, 59.4962, 0.} m/s^2

xE = 0.366833 m

yE = -0.107858 m

vEx = 2.38257 m/s

vEy = 0.895288 m/s

aEx = 6.00543 m/s^2

aEy = 38.6801 m/s^2

phi4 = 0.078255 rad = 4.48368 deg

omega4 = -0.634015 rad/s

alpha4 = -63.2412 rad/s^2

phi5 = -1.21135 rad = -69.4055 deg

omega5 = 13.3959 rad/s

alpha5 = 101.197 rad/s^2
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