Problem Set 7

Problem 7.1 Cable Force to Lift a Ramp

The ramp of a ship has a weight of 200 lb and a center of gravity at G. Determine the cable force in CD needed to just start lifting the ramp (i.e., so that the reaction at B is zero). Also, determine the horizontal and vertical components of force at the hinge A.

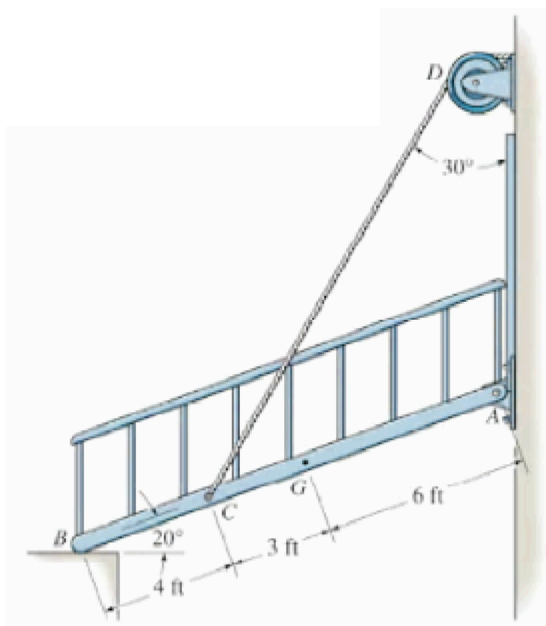

Figure P7.1: Problem 7.1

Problem 7.2 Force Reactions at Frictionless Connections

Determine the reactions on the bent rod which is supported by a smooth surface at B and a frictionless collar at A.

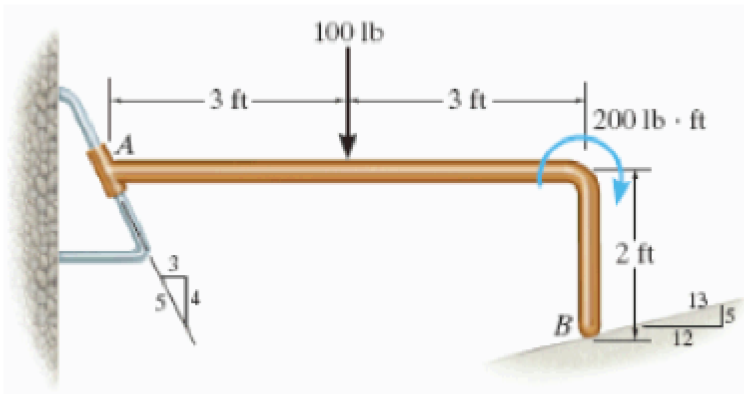

Figure P7.2: Problem 7.2

Problem 7.3

The links 1 and 2 shown in Fig. P7.3 are each connected to the ground at A and C, and to each other at B using frictionless pins. The length of link 1 is $AB = l$. The angle between the links is $\angle ABC = \theta$. A force of magnitude P is applied at the point D ($AD = 2l/3$) of the link 1. The force makes an angle θ with the horizontal. Find the force exerted by the lower link 2 on the upper link 1. Numerical application: a) $l = 1$ m, $\theta = 30°$, and $P = 1000$ N; b) $l = 2$ ft, $\theta = 45°$, and $P = 500$ lb.

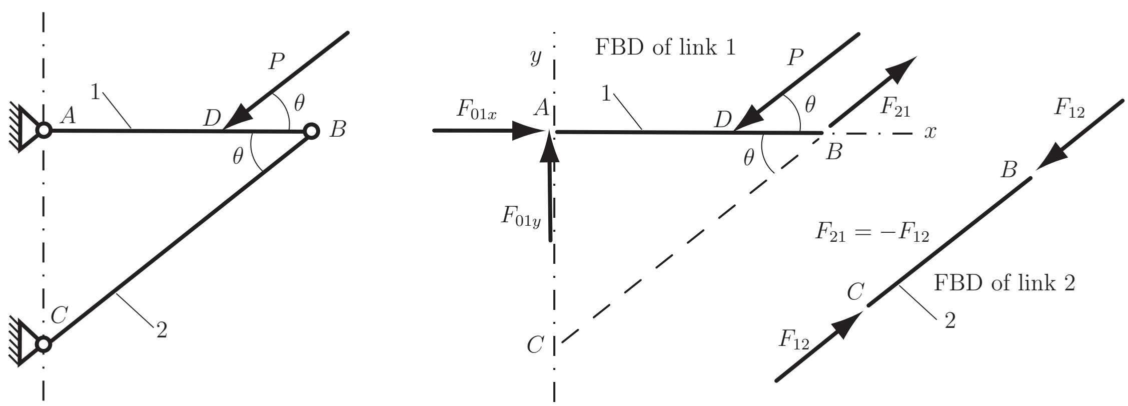

Figure P7.3: Problem 7.3

Solution

1. Mechanical System: link 1.
2. Free-Body Diagram: FBD of link 1 (see figure).
3. Equations: $\sum F_x = 0$ & $\sum F_y = 0$ & $\sum M_A = 0$.

```
% the weights of the two links are negligible
% the lower link 2 is a two force member:  any force exerted on or by the link 2
% must be parallel to the line between its two connection points
% the direction of the force applied by  link 2 onto the link 1 is parallel to BC
rB = [l, 0, 0]; % position vector of B
rD = [2*l/3, 0, 0]; % position vector of D
% the force of the ground 0 on link 1 at joint A is
F01 = [F01x, F01y, 0]; % F01x and F01y are unknowns
% the force of the link 2 on link 1 at B is
F21 = [FB*cos(theta), FB*sin(theta), 0]; % FB unknown
FD =  [-P*cos(theta), -P*sin(theta), 0]; % the force at D
```

```
% sum F = 0
F = F01 + F21 + FD;
% x-axis: F01x + FB*cos(theta) - P*cos(theta) = 0, (1)
% y-axis: F01y + FB*sin(theta) - P*sin(theta) = 0, (2)
% sum of the moments of the forces about the origin A
M = cross(rD, FD) + cross(rB, F21);
% z-axis: FB*l*sin(theta) - (2*P*l*sin(theta))/3 = 0, (3)
% =>
% reaction force at A: F01x = (P*cos(theta))/3 and F01y = (P*sin(theta))/3
% force of link 2 on link 1 is FB = |F21| = (2*P)/3
% note that FB>0 => link 2 is in compression
% a) |F21| = 666.667 (N)     F21 = [577.350 333.333 0] (N)
% b) |F21| = 333.333 (lb)    F21 = [235.702 235.702 0] (lb)
```

Problem 7.4

The dimensions of the shaft shown Fig. P7.4 are $a = 50$ mm and $l = 120$ mm. The force on the disk with the radius $r_1 = 50$ mm is $F_1 = 4000$ N and the force on the disk with the radius $r_2 = 100$ mm is $F_2 = 2000$ N. Determine the bearing loads at A and B.

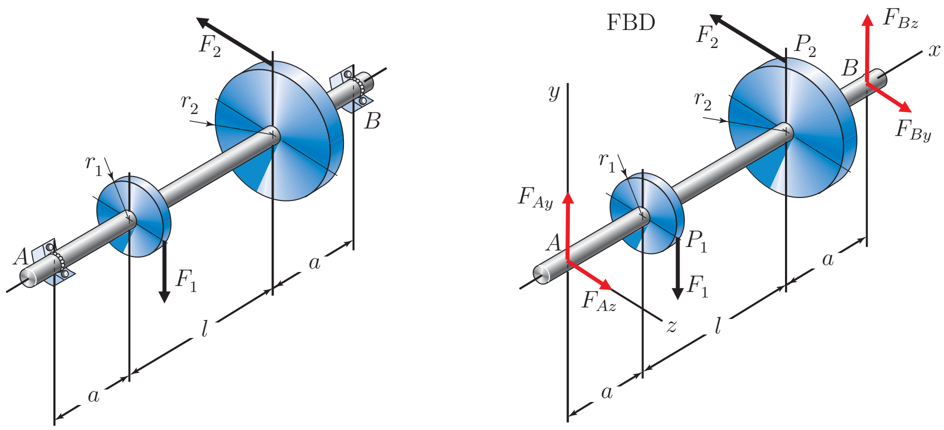

Figure P7.4: Problem 7.4

Solution

1. Mechanical System: shaft and disks
2. Free-Body Diagram: FBD of link 1 (see figure).
3. Equations: $\sum F_x = 0$ & $\sum F_y = 0$ & $\sum M_A = 0$.

```
% reaction force of the bearing on the shaft at A
FA = [0, FAy, FAz];
% reaction force of the bearing on the shaft at B
FB = [0, FBy, FBz];
% position vector of B
rB = [a+l+a, 0, 0];
% vector force F1
F1v = [0, -F1, 0];
% vector force F2
F2v = [0, 0, -F2];
% position vector of P1 application point of F1
rP1 = [a, 0, r1];
% position vector of P2 application point of F2
```

```
rP2 = [a+l, r2, 0];
% sum of the moments about A
MA = cross(rP1, F1v)+cross(rP2, F2v)+cross(rB, FB);
% MAy = 340 - (11*FBz)/50 = 0;
% MAz = (11*FBy)/50 - 200 = 0;
% FBy =  909.0909 (N)
% FBz =  1545.5 (N)

% sum F = 0
FAn= -(F1v+F2v+FBn)
% FA = [0 3090.9 454.5] (N)
```

Problem 7.5

Determine the horizontal and vertical components of reaction at the pin A and the tension developed in cable BC used to support the steel frame.

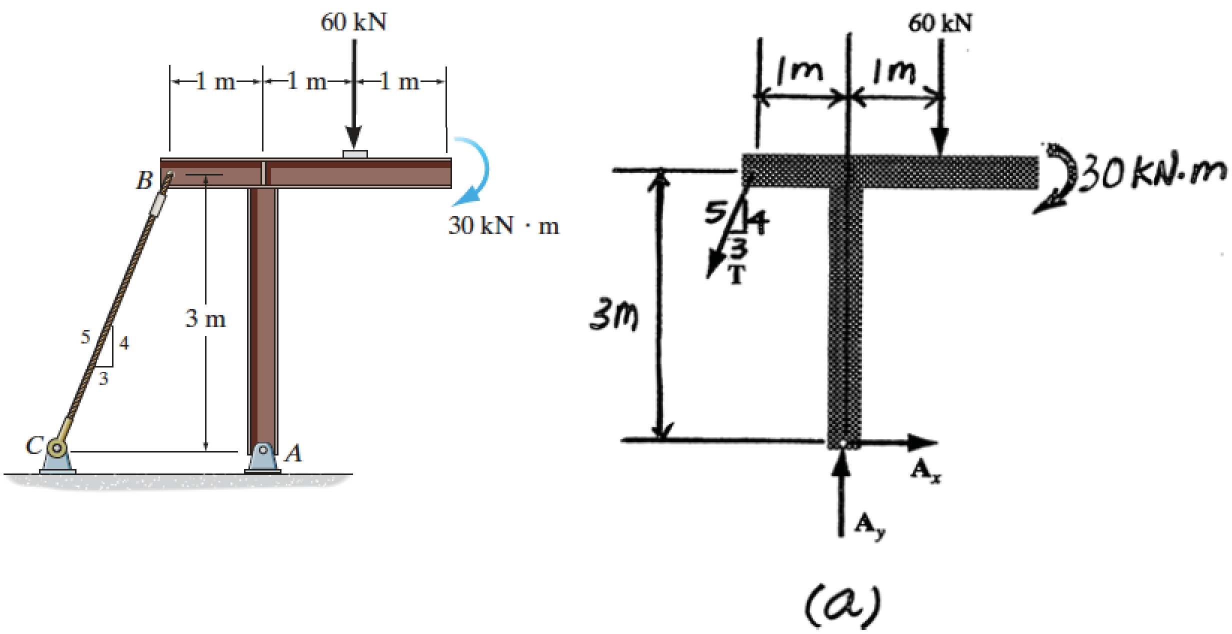

Figure P7.5: Problem 7.5

Solution

1. Mechanical System: steel frame, cable, pins A, B, and C.
2. Free-Body Diagram: FBD of the steel frame, see figure (a).
3. Equations of equilibrium: $\sum F_x = 0$ & $\sum F_y = 0$ & $\sum M_P = 0,\ \forall P$.

```
% equations of equilibrium
% tension T of cable BC can be obtained by writing
% moment equation of equilibrium about point A
% sum M_A = 0
syms T
M_A = T*(3/5)*3+T*(4/5)*1-60*1-30;
T_BC = double(solve(M_A))
% T_BC = 34.6154 (kN)

% force equations of equilibrium along x and y-axes
```

```
% sum Fx = 0
Ax = T_BC*(3/5)
% Ax =  20.7692 (kN)
% sum Fy = 0
Ay = 60+T_BC*(4/5)
% Ay =  87.6923 (kN)
```

4. Results: $T_{BC} = 34.6154$ kN, $A_x = 20.7692$ kN, $A_y = 87.6923$ kN.

Problem 7.6

Determine the horizontal and vertical components of force at the pin A and the reaction at the rocker B of the curved beam.

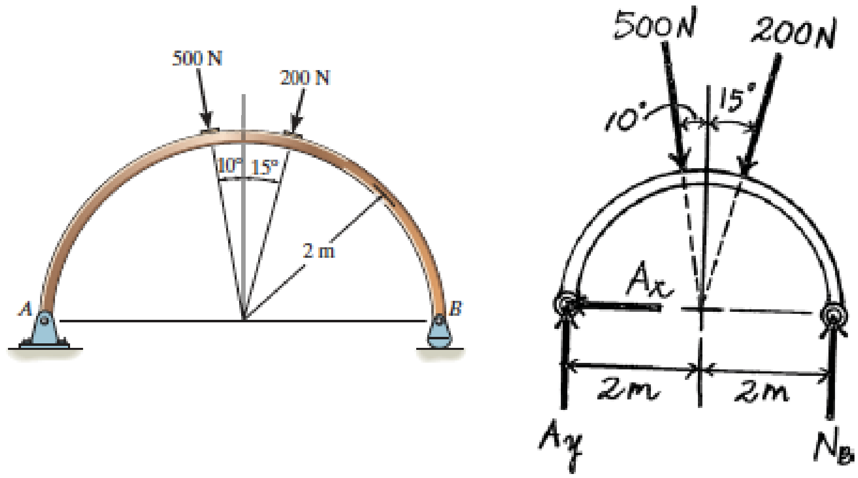

Figure P7.6: Problem 7.6

Solution

1. Mechanical System: pin A, rocker B, curved beam.
2. Free-Body Diagram: FBD of curved beam (see figure).
3. Equations of equilibrium: $\sum F_x = 0$ & $\sum F_y = 0$ & $\sum M_P = 0$, $\forall P$.

```
% moment equation of equilibrium about point A
% sum M_A = 0
NB = (200*(cosd(15))*2+500*(cosd(10))*2)/4
% NB = 342.7945 (N)
% force equation of equilibrium along y-axis
% sum Fy = 0
Ay = 500*cosd(10)+200*cosd(15)-NB
% Ay = 342.7945 (N)
% force equation of equilibrium along x-axis
% sum Fx = 0
Ax = 500*sind(10)-200*sind(15)
% Ax = 35.0603 (N)
```

4. Results: $N_B = 342.7945$ N, $A_x = 35.0603$ N, and $A_y = 342.7945$ N.

Problem 7.7

If the load has a weight of 200 lb, determine the x, y, z components of reaction at the ball-and-socket joint A and the tension in each of the wires.

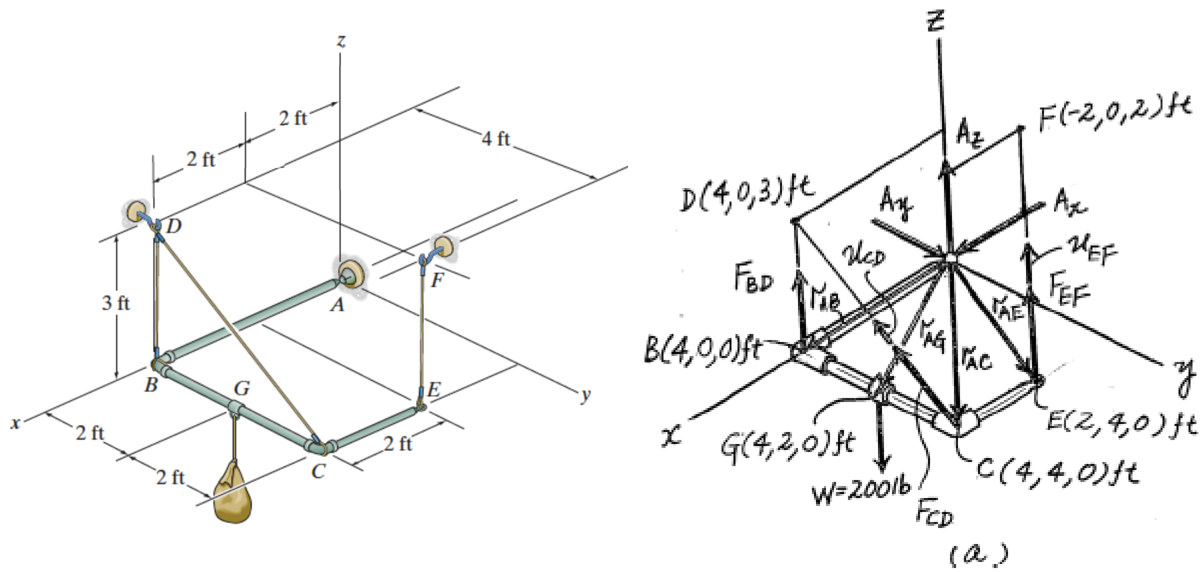

Figure P7.7: Problem 7.7

Solution

1. Mechanical System: cords BD, CD, EF, link $ABCE$, ball-and-socket A, hooks D, F.

2. Free-Body Diagram: FBD of link $ABCE$, see figure (a).

3. Equations of equilibrium: $\sum F_x = 0$ & $\sum F_y = 0$ & $\sum M_P = 0$, $\forall P$.

```
F_A = [Ax, Ay, Az]; W = [0, 0, -200]; F_BD = [0, 0, FBD];
uCD = [4-4, 0-4, 3-0]/sqrt((4-4)^2+(0-4)^2+(3-0)^2);
F_CD = FCD*uCD;
F_EF = [0, 0, FEF];
% vectorial force equation of equilibrium
F = F_A+F_BD+F_CD+F_EF+W;
% force equation of equilibrium along x-axis
Fx = F(1)  % Ax = 0    (1)
% force equation of equilibrium along y-axis
Fy = F(2) % Ay - (4*FCD)/5 = 0    (2)
```

```
% force equation of equilibrium along z-axis
Fz = F(3) % Az + FBD + (3*FCD)/5 + FEF - 200 = 0  (3)
rAB = [4, 0, 0]; rAG = [4, 2, 0]; rAC = [4, 4, 0]; rAE = [2, 4, 0] % (ft)
% moment equation of equilibrium about point A:  sum M_A = 0
MA = cross(rAB,F_BD)+cross(rAC,F_CD)+cross(rAE,F_EF)+cross(rAG,W);
% moment equation of equilibrium about point A along x-axis
MAx = MA(1) % (12*FCD)/5 + 4*FEF - 400 = 0  (4)
% moment equation of equilibrium about point A along y-axis
MAy = MA(2) % 800 - (12*FCD)/5 - 2*FEF - 4*FBD = 0  (5)
% moment equation of equilibrium about point A along z-axis
MAz = MA(3) % -(16*FCD)/5 = 0   (6)
% Eqs.(1)-(6) =>
sol = solve(Fx,Fy,Fz,MAx,MAy,MAz);
Axs = sol.Ax; Ays = sol.Ay; Azs = sol.Az;
FCDs = sol.FCD; FEFs = sol.FEF; FBDs = sol.FBD;
```

4. Results: $A_x = A_y = 0,\ A_z = -50,\ F_{CD} = 0,\ F_{EF} = 100,\ F_{BD} = 150\ \text{(lb)}$. The negative sign indicates that A_z acts in the opposite sense to that on the FBD.