Problem Set 5

Problem 5.1 Product of Inertia of a Cross Section

Determine the product of inertia for the cross sectional area with respect to the x- and y-axes with origin located at the centroid C.

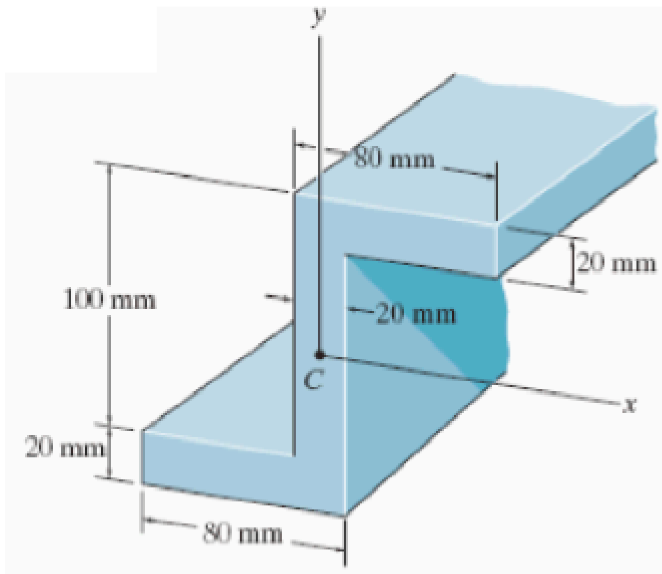

Figure 5.1: Problem 5.1

Problem 5.2 Product of Inertia Transformations

Determine the moments of inertia I_u and I_v as well as the product of inertia I_uv for the channel's cross section. Use $\theta = 45°$.

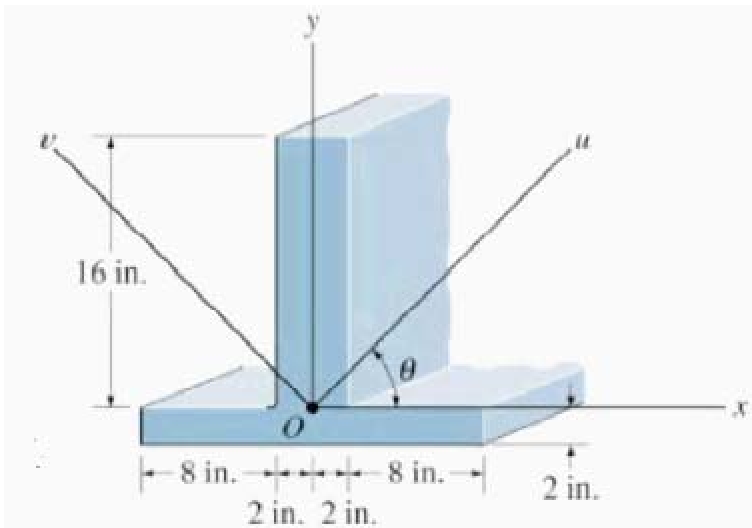

Figure P5.2: Problem 5.2

Problem 5.3

The polar moment of inertia of the area shown in Fig. P5.3, is I_{Czz} about the z-axis passing through the centroid C. If the moment of inertia about the y' axis is $I_{y'y'}$ and the moment of inertia about the x-axis is I_{xx}. Determine the area A. Numerical application: $I_{Czz} = 548 \times 10^6$ mm^4, $I_{y'y'} = 383 \times 10^6$ mm^4, $I_{xx} = 856 \times 10^6$ mm^4, and $h = 250$ mm.

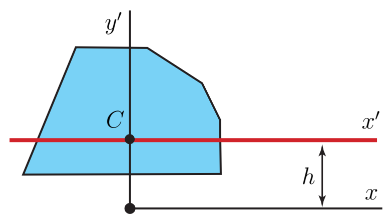

Figure P5.3: Problem 5.3

Solution

```
ICzz = 548*10^6; % mm^4
Iyyp = 383*10^6; % mm^4
Ixx  = 856*10^6; % mm^4
h = 250; % mm

Ixxp = Ixx - A*h^2;

% ICzz = Ixxp + Iyyp
% ICzz = Ixx - A*h^2 + Iyyp
% => A = (Ixx + Iyyp - ICzz)/h^2;
A = (Ixx + Iyyp - ICzz)/h^2;
% A = 1.11e+04 (mm^2)
```

Problem 5.4

Determine the product of inertia of the area with respect to the x and y axes.

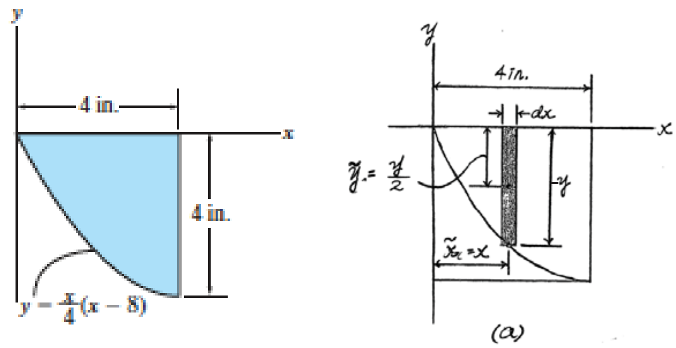

Figure P5.4: Problem 5.4

Solution

```
y = (x/4)*(x-8);

% using a thickness dx
% area of the differential element parallel to y-axis
% height of the the differential element is 0 - y = -y
% dA = - y dx = -(x/4)*(x-8) dx
% coordinates of the centroid C of the differential element
 x_ = x;
 y_ = y/2; y_ = (x/4)*(x-8)/2;
% product of inertia of the differential element
% with respect to x and y-axes is
% dIxy = dICxy + x_ * y_ * dA = 0 + x_* y_* dA
dIxy = x_ * y_ * (-y);
% dIxy = x_ * y_ * dA =  x_ * y_ * (-y ) * dx
% dIxy = [-(x^3*(x - 8)^2)/32] dx

% Ixy = int(dIxy) where 0<x<4
Ixy = int(dIxy, x, 0, 4);

% Ixy = -46.933 (in^4)
```

Problem 5.5

Determine the orientation of the principal axes, which have their origin at centroid C of the beam's cross-sectional area. Also, find the principal moments of inertia.

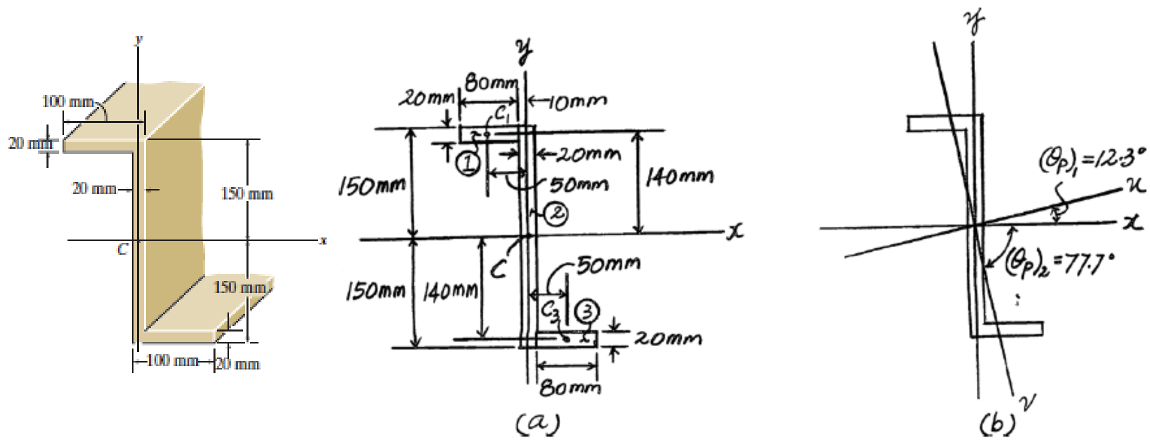

Figure P5.5: Problem 5.5

Solution

```
% moment and product of inertia with respect to x and y-axes
% the perpendicular distances measured from each subdivided
% segment to the x and y-axes are indicated in figure (a)
% applying the parallel axis theorem

Ixx = 2*((1/12)*80*20^3+80*20*140^2)+(1/12)*20*300^3;
% Ixx = 1.078e+08 (mm^4)

Iyy = 2*((1/12)*20*80^3+20*80*50^2)+(1/12)*300*20^3;
% Iyy = 9.907e+06 (mm^4)

Ixy = 80*20*(-50)*140 + 80*20*50*(-140);
% Ixy = -2.24e+07 (mm^4)

% principal moment of inertia
Imax = (Ixx+Iyy)/2 + sqrt(((Ixx-Iyy)/2)^2+Ixy^2);
Imin = (Ixx+Iyy)/2 - sqrt(((Ixx-Iyy)/2)^2+Ixy^2);
```

```
% Imax = 1.13e+08 (mm^4)
% Imin = 5.03e+06 (mm^4)

% orientation of principal axes
% tan(2*theta)) =  -Ixy/((Ixx-Iyy)/2)
theta = atand(-Ixy/((Ixx-Iyy)/2))/2;
% tan(2*theta)) = 0.458
% theta = 12.3 (deg) and theta = -77.7 (deg)
```

Problem 5.6

Determine the mass moment of inertia I_{yy} of the slender rod. The rod is made of material having a variable density $\rho = \rho_0(1 + x/l)$, where ρ_0 is constant. The cross-sectional area of the rod is A. Express the result in terms of the mass m of the rod.

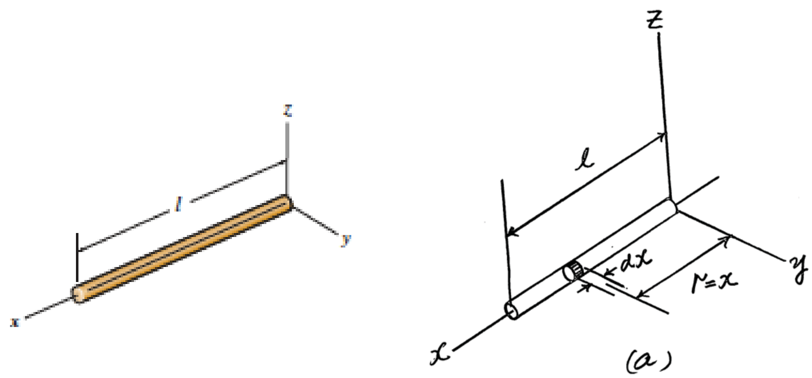

Figure P5.6: Problem 5.6

Solution

```
rho = rho0*(1+x/l);
% mass of the differential element parallel to x-axis
% dm = rho dV = rho (A dx) = rho0 A (1+x/l) dx;
% A is the cross-sectional area of the rod
% mass of the is determined by integrating dm
% m = int(dm) = int (rho0 A (1+x/l) dx) where 0<x<l
m = int(rho*A, x, 0, l);
% m = (3*A*l*rho0)/2
% mass moment of inertia of the differential element about y-axis
% dIyy = r^2 dm = x^2 dm where r = x
% dIyy = x^2*rho*A;
% mass moment of inertia can be determined by integrating dIyy
% Iyy = int(dIyy) where 0<x<l
Iyy = int(x^2*rho*A, x, 0, l);
% Iyy = (7*A*l^3*rho0)/12

% m = (3*A*l*rho0)/2 => A*l*rho0 = 2*m/3
% => Iyy = Izz = 7 m l^2/18
```

Problem 5.7

The thin plate has a mass per unit area of 10 kg/m^2. Determine its mass moment of inertia about the z axis.

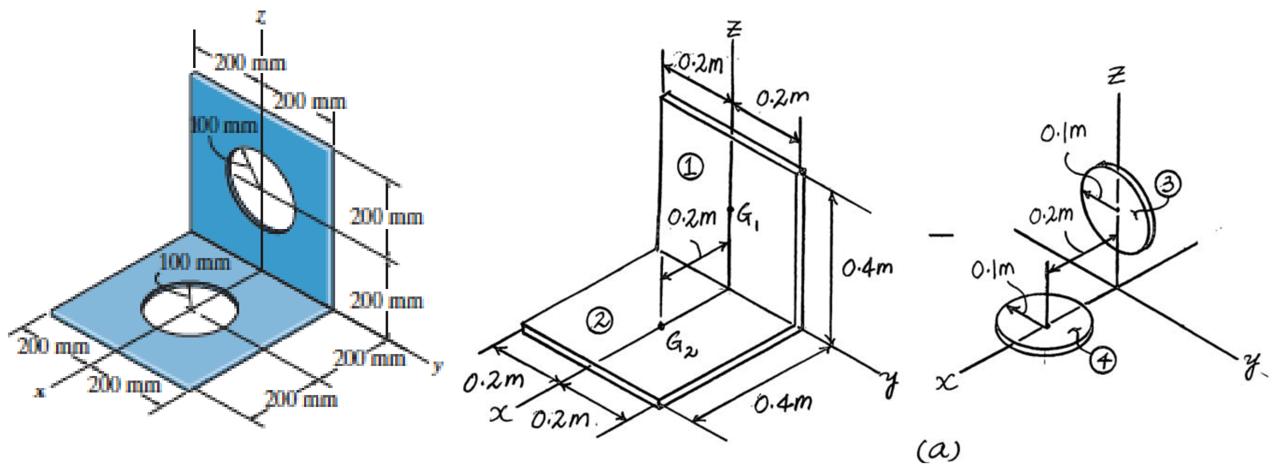

Figure P5.7: Problem 5.7

Solution

```
% composite parts: the thin plate can be subdivided into four segments
% segments 3 and 4 are holes and should be considered as negative parts
% mass for the segments
m1 = 0.4*0.4*10; % kg
m2 = m1;
% m1 = m2 = 1.6 kg
m3 = pi*0.1^2*10; % kg
m4 = m3;
% m3 = m4 = 0.1 pi kg
% mass moment of inertia of each segment about z-axis can be determined
% using parallel-axis theorem
% Izz = ICzz + m d^2

Izz = (1/12)*1.6*0.4^2 + ...
      (1/12)*1.6*(0.4^2+0.4^2)+1.6*0.2^2 - ...
      (1/4)*(0.1*pi)*0.1^2 - ...
      ((1/2)*(0.1*pi)*0.1^2+0.1*pi*0.2^2)
% Izz = 0.1131 (kg m^2)
```